A method for combining transaction- and valuation-based data in a property price index

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Abstract: This paper presents a method for combining transaction- and valuation-based data in a property price index. The methodology is devised for a world where observable transaction prices can be used to construct a price index that constitutes a noisy, unbiased signal of the "true" price index. It is furthermore assumed that valuations can be used to construct a market value index which does not contain noise but that suffers from so called appraisal "smoothing". The valuation-based index is thus assumed to lag the "true" value index and exhibit lower volatility. The model of the valuation-based index follows Geltner (1993). By regressing the transaction-based index on the valuation-based index (contemporaneous and lagged one period) it is possible to filter out the noise in the observable price index thus estimating the "true" price index. The method may be seen as a way of "de-smoothing" a valuation-based index without knowing the smoothing parameter beforehand. The methodology may also be used as a way of estimating the smoothing parameter.
**Introduction**

Price (or market value) indices for property markets are important for several reasons. Price indices are for example used as benchmarks by property owners and by investors as a means to compare average returns on property and alternative assets such as stocks and bonds. High quality price indices are also important in portfolio allocation decisions (indices can for example be used to calculate correlations between asset classes). Price indices are furthermore important in research on property markets. Research topics where price indices are used include property cycles and the relationship between property markets and other financial markets.

Unfortunately it is not a simple task to construct property price indices of high quality. Two important reasons for this are that properties are heterogeneous - different properties have different characteristics (size, age, technical amenities etc) - and that properties are transacted seldom. This means that there exists relatively few observable property prices during a given time period on a given market and that those prices are not directly comparable.

The difficulty of constructing price indices is less severe for certain types of property. Single-family homes is an example of a property type with relatively many sales where those properties that are sold also are relatively comparable. For this property type it is therefore comparatively easy to design a reliable index. For commercial properties, on the other hand, there may exist only a few transactions in a given year and market. In these conditions it may be impossible to construct a reliable index.

The difficulty of constructing an index is related to the level of aggregation. If the index is intended to capture the price level for properties in Europe we will most likely have enough transaction prices. This is likely also the case if we want to construct an index for Swedish offices. If we however want to construct an index for Stockholm CBD offices or single family homes in a particular parish of Stockholm there may not be enough data to construct a reliable index based on transactions.

One way of circumventing the problem of low liquidity is to make use of valuations instead of transaction prices. This approach depends heavily on the quality of valuations. If valuations are inaccurate this may not be a reliable way of obtaining a price index. As an index is an aggregate of many observations, inaccuracy of individual valuations is not necessarily problematic. Errors may cancel out. There is however research that suggests that valuations of properties lag behind and underestimate the volatility of actual value movements.
This valuation bias, popularly termed "appraisal smoothing", does not cancel out when valuations are aggregated (Geltner et al. 2003).

This paper presents a method for combining transaction- and valuation-based data in a price index. The point of the method is to at least partly provide a remedy for inherent problems in the two types of data: noise in transaction data and smoothing in valuation data. The methodology is devised for a world where there are at least some observable transaction prices that can be used to construct a price index that constitutes a noisy signal of the "true" price index (an index free of bias and noise). Furthermore, it is assumed that valuations from the population can be used to construct a noiseless but smoothed valuation index. This valuation index is a lagged, smoothed-out version of the "true" index. By regressing the price index on the valuation index (contemporaneous and lagged one period) it is possible to filter out the noise in the observable price index and hence estimate the "true" price index.

The nature of price indication data in property markets

An asset price index is an index that measures price movements in a population of assets. For some assets the construction of the index is fairly straightforward. For common stocks for example, we may simply collect price observations of every stock in the population for every time period, add them and divide by the price level in the chosen base period. Price data in property markets is generally not as easily transformed to a reliable index. For some markets there simply are too few transactions for this procedure to be feasible and when transaction data actually is available, heterogeneity of properties often makes it difficult to construct a reliable index.

Unless we control for differences in property characteristics, transaction prices are not comparable. Transaction price A may differ from transaction price B because the two transactions occur at different points in time and prices have changed or because property A and B are of different quality (property B may have a nice view for instance). Unless we can control for differing quality, heterogeneity will introduce noise in observed transaction prices. Hence, an index constructed by taking the average of transaction prices will be noisy. Noise will pose less of a problem the more transaction data that is available.

Heterogeneity may also introduce bias in an index. There are two reasons for this. First, the characteristics of properties may change systematically over time. If properties’ technical amenities are improved across a whole market for instance, we should observe price increases
due to quality improvement. For a given level of quality however, prices may have been constant.

Second, properties of different characteristics may transact at different points in time. If high quality properties typically transact in certain time periods, failing to control for this may lead us to believe that prices have increased more than they actually have during these periods. Note that if we had continuous price data for every property, this would not be a problem. Heterogeneity and low liquidity thus together make it difficult to create indices.

It should be noted that what we mean by bias may depend on what use the index is intended for. For some applications it may not be necessary or even desirable to control for all types of differences in characteristics. One may for instance want to construct an index for which depreciation and improvements are not controlled for. This, along with other basic issues regarding index construction, is discussed in more detail by Wang and Zorn (1997).

The fact that property markets are search markets is another source of noise in property transaction data (Fisher et al. 2007). Transaction prices are the outcomes of negotiations between buyers and sellers. For any transaction the outcome of the transaction process is just one realization of many possible outcomes. The actual selling price can be viewed as a random variable distributed around the market value (where I think of the market value as the expectation of the selling price in a normal transaction, i.e. no forced sales for example). To exemplify, the price may end up below market value if the buyer has an exceptionally skilled negotiator at the negotiation.

A substantial literature has addressed index construction methodology and has suggested solutions to the inherent problems. The repeat sales regression (first developed by Bailey et al., 1963) is a method for producing an index that compares prices of houses that have transacted at least twice during the period for which the index is constructed. The regression model is constructed so as to compare the transaction price for the same property at two (or more) transactions. The methodology thus at least in part avoids the problem of heterogeneity.

There are three main problems with this type of index. First, the method requires plenty of transaction data and is therefore not feasible for many property markets. Only properties that have transacted at least twice during the index period can be used. Second, in its simplest form, the method does not adjust for the fact that the properties in the index may change over time (depreciation, renovations etc). Later literature has suggested ways of dealing with this problem (Case and Quigley, 1991, is one example). Third, the method necessarily means that we build the index on properties that transact often (properties that have transacted only once
during the index period will not enter the regression). These properties may not be representative of the population. One study that investigates this problem is Englund et al. (1999). Their study shows that Swedish single family homes that are transacted often typically are of lower quality (small lots etc).

Another way to design transaction-based indices that controls for differences in characteristics is to use a hedonic regression model. In the hedonic approach, a property is viewed as a composite good: When buying a property one is really buying a set of goods. The hedonic approach aims to find the marginal contribution of each of these goods or characteristics on the value of the composite good (in our case a property). This is achieved by regressing the transaction price of a property on a number of its characteristics (location, area, age etc). By introducing time dummies in the regression, it is possible to capture the price level in different time periods while the included property characteristics control for heterogeneity. An alternative approach is to estimate a hedonic regression for each time period and revalue a representative property each time period using each respective period’s characteristics prices. Miles et al. (1990) and Webb et al. (1992) are examples of studies where a hedonic methodology is used.

Clapp and Giacotto (1992) suggested an efficient way of controlling for heterogeneity among properties. They argue that valuations of each respective property provide an excellent heterogeneity control. Using valuations as a control for differing characteristics is an attractive idea for two reasons: They are likely to capture very much of the heterogeneity and they are fairly easy to obtain unlike other controls that may require collection of an extensive array of property attributes. Fisher et al. (2007) present a new quarterly index for commercial property that uses this technique. As with repeat-sales methods the hedonic method is only feasible when there is plenty of data. For the hedonic approach not only transaction data is needed but also data on the characteristics of the properties in the index.

A completely different approach to constructing price (or value) indices for property is to use valuations instead of observed transaction prices. A valuation-based index is constructed by revaluing the same sample of properties each time period. Valuation-based indices thus in part avoid the problem of heterogeneity. However, assuming that the properties in the sample change in quality over time, this should be taken into account.

Using valuations as a means of tracking price (or value) movements hinges critically on the nature and quality of valuations. There is a fairly substantial literature that shows that valuations are prone to a certain type of bias. More specifically, a number of articles suggest that valuations tend to lag actual prices and also tend to smooth out actual price movements,
so called "appraisal smoothing" (Geltner et al. 2003, Diaz and Wolverton 1998, Fisher et al. 1999 and Fisher and Geltner 2000). This phenomenon can be shown to be the result of optimal valuer behaviour (Quan and Quigley, 1989 and 1991, Childs et al. 2002) but is not optimal from an index-construction point of view as smoothing in individual valuations is likely to spread to an aggregate index. If smoothing is present in valuations, an index based on valuations will simply not show actual price movements but movements in valuations. The phenomenon will be dealt with in some detail in the following.

A valuer that tries to estimate the market value of a property uses transaction prices from properties that are as comparable as possible to the property that is being valued. Ideally these comparable sales (comps) should (1), come from properties that are identical to the property being valued, (2), the transactions should have occurred very recently (ideally at the same moment that we are making the valuation) and (3), we should have access to plenty of them. This will typically not be the case as properties are heterogeneous and transact seldom. The valuer will have to make do with less perfect data. The data that is available to the valuer will contain noise (due to heterogeneity) and it will not be completely up to date (old transactions). We can think of the value estimate as a simple average of the transaction prices from comparable sales. If we use only very recent comps the value estimate will be up to date but noisy due to the fact that we have very few comps in the average, perhaps only one comparable sale. As we include older and older comps the number of comps in the average will be larger reducing the effect of noise. The value estimate will however be less up to date the farther back in time we go. Thus there will be a trade-off between noise and bias depending on how far back the valuer decides to go. Using only recent comparable sales will give an estimate that contains a lot of noise but very little bias. Using comparable sales from a longer time period will result in less noise but more bias.

How far back it is optimal to go depends on what use the valuation is intended for. If we aim for as small error as possible in the individual value estimate it may be optimal to go quite far back as this will reduce the noise. If we want to have an estimate with as little bias as possible it may be optimal to use only very recent comps. If we want to value an entire portfolio of properties for example it is arguably better to have unbiased but noisy estimates of the individual properties as noise will filter out in the aggregate.

This description of the valuers problem is simplified (valuations are usually not simple averages of comps) and is meant to give an intuitive explanation for appraisal smoothing. The general idea is that valuers use old information and that this behaviour is justified. Quan and Quigley (1991) have studied the valuers problem more formally. They find that, given a
number of assumptions, it is optimal\(^1\) for the valuer to behave according to the following model:

\[ v_{it} = \alpha_{it} p_{it} + \left(1 - \alpha_{it}\right) v_{it-1} \]  
(1)

Where \( v_{it} \) is the valuation of property \( i \) in time \( t \), \( p_{it} \) are (noisy) contemporaneous comparable sales and \( v_{it-1} \) is the valuation in the previous period. \( \alpha_{it} \) is a parameter that tells us how much weight is given to current information relative to how much weight is given to old information. A large \( \alpha_{it} \) corresponds to much weight being given to contemporaneous information and vice versa. Note that previous valuations (\( v_{it-1}, v_{it-2} \) etc) will follow the same model. It can easily be shown that formula (1) is simply a weighted average of the current and all previous comparable sales (i.e. \( p_{is} \) where \( s = t, t-1, t-2, \ldots \)) with lower and lower weights the farther back we go (see formulas (16) and (17) below). \( \alpha_{it} \) is usually written without time or individual subscripts but they are included here in order to emphasize that alpha may differ over time as well as for different properties.

An index based on valuations that follow the pattern in formula (1) will be smoothed. For many applications this is problematic and research has therefore been devoted to the question of how to derive unbiased price indices from valuation-based indices. Two groups of solutions are the “zero-autocorrelation” method and the “reverse engineering” method. The zero-autocorrelation method builds on the idea that returns in property markets should be unpredictable. Using this assumption it is possible to back out ”true” (non-autocorrelated) returns through a regression where autocorrelated return is filtered out. Once ”true” returns have been calculated these can be used to calculate ”true” price levels. Blundell and Ward (1987) proposed this technique and a number of articles have used and/or developed the method (Fisher et al. 1994, Cho et al. 2003 and Brown and Matysiak, 1998). The main caveat of the method is the problematic assumption of zero-autocorrelation in returns, which may not hold.

The ”reverse engineering” method is related to model (1) and was proposed by Geltner (1993). Geltner (1993) argues that if individual valuations follow the pattern in formula (1), a valuation-based index will be well described by the following model:

\[ \text{footnote}(1) \text{ It is optimal in the sense that if } v_{it} \text{ is chosen in accordance with this formula, it will converge to the true market value of the property faster than any other simple linear valuation rule.} \]
\[ V_t = \alpha \tilde{P}_t + (1-\alpha) V_{t-1} \]

where \( V_t \) and \( V_{t-1} \) are the valuation-based index levels at time \( t \) and \( t-1 \) respectively and \( \tilde{P}_t \) is a price (or market value) index level at time \( t \) (the tilde is merely there to distinguish \( \tilde{P}_t \) from a different price index \( P_t \) below). Note that \( V_t \) and \( V_{t-1} \) are observable. \( \tilde{P}_t \) on the other hand is here regarded as a non-observable component of \( V_t \). If (2) holds and if we know \( \alpha \) it is possible to construct a price index by backing out ("reverse engineering") \( \tilde{P}_t \) from formula (2):

\[ \tilde{P}_t = \frac{V_t}{\alpha} \frac{(1-\alpha)V_{t-1}}{\alpha}. \]

(3) is just a simple manipulation of formula (2). Geltner furthermore argues that the noise in \( p_{it} \) will largely diversify away in their aggregate counterpart \( \tilde{P}_t \) so that \( \tilde{P}_t \) may be viewed as a "true" (unbiased, noiseless) price index. We may also assume that \( \tilde{P}_t \) contains noise and employ some noise-reduction technique.

The i subscripts have been dropped in formula (2) and (3) in order to emphasize that \( \tilde{P}_t \), \( V_t \) and \( V_{t-1} \) are measured at the index level in these formulas and that \( \alpha \) when used in this way usually is assumed to be constant (an assumption that may not hold).

One of the main problems with reverse engineering is that we must estimate \( \alpha \), which is inherently difficult as we do not have access to the "true" price index and probably not the valuers’ comps (\( p_{it} \) in formula (1)) either. One of the few studies on the subject is Clayton et al. (2001). The difficulty of obtaining \( \alpha \) is aggravated by the fact that \( \alpha \) may vary over time and over different properties (empirical support for this can be found in Brown and Matysiak (1998)) and that \( \alpha \) on the individual property level not is necessarily immediately transferable to the aggregate (index) level (Bond and Hwang, 2007).

So far we have discussed how both valuations and transaction prices are imperfect measures of price movements in property markets. There are, however, other more indirect indicators of property prices. One prominent example is prices on stocks of listed property companies (or REITS). These prices refer to indirectly owned property which means that they cannot be used as price indicators for the direct property market without adjustment (or at
least not without caution). Property stocks are for example usually leveraged assets. This has to be taken into account as we usually create property indices for properties as such, not leveraged property holdings (which does not preclude that the properties in indices are owned by leveraged owners). Empirical research has also found that property stock prices move partly independently from the directly owned property market (Chau et al. 2001).

Ling et al. (2000) and Fu (2003) are two examples of articles that present methods of using indirect indicators for computing price indices. Both articles make use of latent variable models. With this type of model it is possible to calculate an unobservable "latent" variable with the help of a number of observable "indicator" variables. Applied to property price indices, the latent variable is the "true" value index while valuations and property stock prices may be used as indicator variables.

**Proposed index construction method**

In short, the setting is as follows. It is assumed that indications of current market value can be obtained from two sources; transaction prices and valuation data. The transaction prices are assumed to be unbiased estimates of market value but contain a lot of noise. The valuation data on the other hand is assumed to suffer from the effects of appraisal smoothing (lag, lower volatility).

Assume that there are three indices in the market, two of them observable and one unobservable. First we have the unobservable "true" price index, $I_t$, that we want to estimate. There is also a transaction-based index, $P_t$, which is built on noisy transaction price data. It is assumed that the price index $P_t$ is dispersed around the "true" market value index:

$$P_t = I_t + u_t$$

where $u_t$ is a random error distributed around the market value index and $E(u_t) = 0$. $u_t$ is assumed to be uncorrelated with $I_s$ where $s = \ldots, t+2, t+1, t, t-1, t-2 \ldots$. The variance of $u_t$ may differ in different time periods. In words, $P_t$ is a noisy measure of $I_t$.

Assume furthermore that we have a valuation-based index, $V_t$. This index is built on individual appraisals. The individual appraisals are assumed to follow the pattern discussed above (formula (1)). It is furthermore assumed that this pattern carries through to the index so that we have
\[ V_t = \alpha I_t + (1 - \alpha)V_{t-1}, \quad (5) \]

where \( \alpha \) is the smoothing parameter. In words, the valuation series \( V_t \) provides a "smoothed" but noiseless signal of \( I_t \). Regarding the behaviour of \( I_t \) and \( V_t \), the presented setting is the same as Geltner (1993). One could use "reverse engineering" on the valuation-based index \( V_t \) presuming that we have an idea of the value of \( \alpha \).

After considering the set-up, the following question may arise: Why does the price index \( P_t \) contain noise while the signal of "true" value in the valuation-based index does not? In the presented set-up, the individual valuation is built on noisy price information and the previous valuation, but when we combine valuations in an index, the noise in the price information filters out. Why can we not simply collect the price information that valuers use and create a transaction-based index free of noise? The noise filters out in the valuation-based index – why not in the transaction-based index?

The set-up implicitly assumes that the price information that valuers have access to is richer than the price information available to the person constructing the index. This requires some motivation. First of all, the information available to valuers may be costly or impractical for the index-constructor to acquire. It may for example be the case that the data are not collected in one place or that the raw data needs extensive processing before use. Secondly, valuers may have access to information that simply is not available to the index-constructor. Some transaction prices may not be disclosed publicly but leak to valuers. Some transactions are part of a larger deal that includes other assets as well. In this type of deal the implicit transaction price of the property may not be known to the public but to valuers. Furthermore, the noisy price information that valuers use may not be actual transaction prices. Knowledge of deals that did not happen, rumours etc may be seen as part of the noisy price information used by valuers. Despite this argument one may argue that the "true" price index component in (5) should include an error term. The effects of allowing for this are discussed in a subsequent section (equations (25) and (26)).

Simulation (A) in Figure 1 shows visually how \( I_t, P_t \) and \( V_t \) relate to each other. In this simulation \( I_t \) is assumed to follow a random walk:

\[ I_t = I_{t-1} + v_t, \quad (6) \]

\[ v_t \sim N(0,1) \quad (7) \]
It was constructed by generating 25 random numbers \( v_i \) and then using formula (6). \( P_t \) was generated using formula (4) where \( u_i \sim N(0,4) \). \( V_t \) was constructed using formula (5). \( \alpha \) was set equal to 0.4. The figure illustrates that \( P_t \) is a noisy (more volatile) version of \( I_t \) and that \( V_t \) is a smoothed (less volatile, lagging) version of \( I_t \).

\[
I_t = \frac{V_t}{\alpha} - \frac{(1-\alpha)V_{t-1}}{\alpha}
\]  

(8)

Equation (8) is a description of how \( I_t \) is related to \( V_t \) and \( V_{t-1} \) where \( I_t \) is expressed as a linear function of \( V_t \) and \( V_{t-1} \). Of course, \( I_t \) is not literally \textit{driven} by \( V_t \) and \( V_{t-1} \). (8) merely shows how variation in \( I_t \) can be captured with \( V_t \) and \( V_{t-1} \) if we assume that equation (5) holds. Assuming that we can observe the three variables we could estimate (8) by OLS. If we were to regress \( I_t \) on \( V_t \) and \( V_{t-1} \) we would be able to capture all variation in \( I_t \) since \( I_t \) only
"depends" on $V_t$ and $V_{t-1}$. The coefficient for $V_t$ would equal $1/ \alpha$ and the coefficient for $V_{t-1}$ would equal $-(1-\alpha)/\alpha$. If we included an intercept in the regression it would equal zero. I use the word “depend” here in the sense that the variation in $I_t$ can be captured by $V_t$ and $V_{t-1}$.

Now, we can observe $V_t$ and $V_{t-1}$ but not $I_t$. We can however observe $P_t$ which is just a noisy measure of $I_t$:

$$P_t = \frac{V_t}{\alpha} - \frac{(1-\alpha)V_{t-1}}{\alpha} + u_t \tag{9}$$

I have simply inserted the right-hand side of equation (8) instead of $I_t$ in equation (4) in order to arrive at (9). Model (9) is possible to estimate since we have assumed that $P_t$ and $V_t$ are observable. We would then run the following regression model:

$$P_t = \beta_0 + \beta_1 V_t + \beta_2 V_{t-1} + e_t \tag{10}$$

where we know from (9) that the true parameters are $\beta_0 = 0, \beta_1 = 1/ \alpha, \beta_2 = -(1-\alpha)/\alpha$ and that $e_t = u_t$. Assuming that $u_t$ is uncorrelated with $V_t$ and $V_{t-1}$ the coefficients for the explanatory variables will be unbiased. In other words, their expected values are their respective true population counterparts:

$$E(\hat{\beta}_0) = 0, \tag{11}$$
$$E(\hat{\beta}_1) = 1/ \alpha, \tag{12}$$
$$E(\hat{\beta}_2) = -(1-\alpha)/\alpha \tag{13}$$

We can obtain predicted $P_t$:

$$\hat{P}_t = \hat{\beta}_0 + \hat{\beta}_1 V_t + \hat{\beta}_2 V_{t-1} \tag{14}$$

The expected value of $\hat{P}_t$ given $V_t$ and $V_{t-1}$ is:
\[
E(\hat{P}_t | V_t, V_{t-1}) = E\left(\hat{\beta}_0 + \hat{\beta}_1 V_t + \hat{\beta}_2 V_{t-1} | V_t, V_{t-1}\right) \\
= \beta_0 + \beta_1 V_t + \beta_2 V_{t-1} \\
= \frac{V_t}{\alpha} - \frac{(1 - \alpha) V_{t-1}}{\alpha} \\
= I_t
\]  

In words, predicted \( P_t \) is an unbiased estimate of \( I_t \). As the number of observations increases, the coefficients are better and better estimated and the predicted \( P_t \) will come closer and closer to \( I_t \).

Figure 2 shows simulation (B) which is similar to simulation (A) in Figure 1 but in which I have also included \( \hat{P}_t \) which is predicted \( P_t \) from a regression where \( P_t \) is regressed on \( V_t \) and \( V_{t-1} \) (regression model (10)). As is evident from the figure, the predicted \( P_t \) comes close to \( I_t \).

![Figure 2](image-url)  

Figure 2. Simulation (B) of a "true" value index (\( I_t \)), and an estimation of \( I_t \) (predicted \( P_t \)) using transactions-based and valuation-based indices.
We do not actually have to assume that \( u_t \) is uncorrelated with \( V_t \) and \( V_{t-1} \). It follows from previously made assumptions: (i) the assumption that \( u_t \) is uncorrelated with \( I_t \) in all time periods and (ii) the assumed model of the appraisal-based index, equation (5). To see this, note that equation (5) implies that \( V_t \) can be expressed as a function of the current and lagged values of \( I_t \). We have (equation (5) restated):

\[
V_t = \alpha I_t + (1 - \alpha)V_{t-1}
\]  

(16)

Insertion of \( \alpha I_{t-1} + (1 - \alpha)V_{t-2} \) instead of \( V_{t-1} \), \( \alpha I_{t-2} + (1 - \alpha)V_{t-3} \) instead of \( V_{t-2} \) and so on yields:

\[
V_t = \alpha I_t + (1 - \alpha)\alpha I_{t-1} + (1 - \alpha)^2 \alpha I_{t-2} + (1 - \alpha)^3 \alpha I_{t-3} + \ldots
\]

(17)

Equation (17) shows that \( V_t \) is a function of \( I_s \) where \( s = t, t-1, t-2, \ldots \) which are all uncorrelated with \( u_t \) by assumption. Hence, \( u_t \) is uncorrelated with \( V_t \). The same argument holds for \( V_{t-1} \).

The reader may object that estimating \( P_t \) on \( V_t \) and \( V_{t-1} \) results in biased coefficient estimates due to simultaneity (the argument might be that prices drive valuations, not the other way round). Then we have to remember what we are trying to achieve with regression equation (10). The point of the regression is not to test a causal relationship. The point is instead to reduce the noise in the \( P_t \) observations (or to get rid of the lagging/smoothing behaviour in \( V_t \) if you will). \( \beta_1 \) and \( \beta_2 \) should not be thought of as measuring causal effects but rather the linear relationship between \( P_t, V_t \) and \( V_{t-1} \). We know from the assumptions that we have made that this relationship follows formula (9).

How can valuations completely capture "true" price movements in this setting? In order to give an intuitive explanation why this may be the case let us start with the basic model of how the valuation-based index relates to the "true" price index:

\[
V_t = \alpha I_t + (1 - \alpha)V_{t-1}
\]

(18)

The formula shows that \( V_t \) contains both the "true" price \( I_t \) scaled down by a factor \( \alpha \) and the previous valuation \( V_{t-1} \). Thus, by scaling up the "true" price component and getting rid
of the $V_{t,1}$ component we have the "true" price. This is exactly what happens when we regress $P_t$ on $V_t$ and $V_{t-1}$. From (15) we have that:

$$E\left(\hat{P}_t | V_t, V_{t-1}\right) = \frac{V_t}{\alpha} - \frac{(1-\alpha)V_{t-1}}{\alpha}$$  \hspace{1cm} (19)

The first term in this expression may be thought of as the term that scales up the $I_t$ component of $V_t$. To see this note the following:

$$\frac{V_t}{\alpha} = \frac{1}{\alpha} \left(\alpha I_t + (1-\alpha)V_{t-1}\right) = I_t + \frac{(1-\alpha)V_{t-1}}{\alpha}$$  \hspace{1cm} (20)

Subtracting the "previous-valuation-component", $\frac{(1-\alpha)V_{t-1}}{\alpha}$, from $\frac{V_t}{\alpha}$ we get:

$$\frac{V_t}{\alpha} - \frac{(1-\alpha)V_{t-1}}{\alpha} = I_t + \frac{(1-\alpha)V_{t-1}}{\alpha} - \frac{(1-\alpha)V_{t-1}}{\alpha} = I_t$$  \hspace{1cm} (21)

Relaxing assumptions

The proposed method relies on a number of assumptions. If these assumptions are fulfilled, the index construction method works well in the sense that it produces an unbiased estimate that converges to the "true" index series. Of course, the assumptions may not be fulfilled or at least may not be completely fulfilled. The rest of the paper discusses how the results are affected if the assumptions are not fulfilled.

Price process

In the presentation of the methodology, the process of the "true" price index was not discussed and no assumptions were made about what it looks like. In other words, the index construction method is not dependent on a particular process of the "true" price index. Simulation (C) was made to illustrate this. $I_t$ is assumed to follow an ARMA(1,1) process:
\[
I_t = 0.5I_{t-1} + v_t + v_{t-1} \tag{22}
\]

\[
v_t \sim N(0,1) \tag{23}
\]

\(V_t\) and \(P_t\) are constructed in the same way as in simulation (A) and (B) but \(\alpha\) is assumed to be 0.3 in this simulation.

![Graph](image)

Figure 3. Simulation (C) of a "true" value index \((I_t)\), and an estimation of \(I_t\) (predicted \(P_t\)) using transactions-based and valuation-based indices. \(I_t\) is assumed to follow an ARMA(1,1) process.

As in simulation (B), predicted \(P_t\) follows \(I_t\) closely: the methodology is not sensitive to the process of the "true" price index. The simulation serves a second purpose. In this simulation, 200 observations were generated instead of 25 observations as in simulation (B). This means that when regressing \(P_t\) on \(V_t\) and \(V_{t-1}\) in this simulation, coefficients are estimated with more accuracy. Consequently, predicted \(P_t\) follows \(I_t\) more closely than in simulation (B) illustrating the fact that the more observations, the better the proposed methodology works.

Valuer model

The assumption of how the valuation index behaves, equation (5), is explicitly used in the derivation of the index construction method. In general, therefore, the method does not
work unless this assumption holds. The method may however still work as an approximation even if equation (5) does not hold in a strict sense. Whether the approximation is reasonable or not depends on exactly how reality deviates from equation (5). As the true behaviour of \( V_t \) may deviate from equation (5) in countless ways it is impossible to give an exhaustive discussion of what happens when model (5) is invalid. This section will discuss some possible deviations.

First, one may think of several models that share important traits with model (5) but deviate in some sense. Model (24) is one such example:

\[
V_t = \alpha_1 I_t + \alpha_2 I_{t-1} + (1 - \alpha_1 - \alpha_2) I_{t-2} \tag{24}
\]

This model will lag the "true" index and will smooth out its movements just like model (5). The difference between the models is the weights and the fact that model (5) goes further back in time. Model (24) is motivated for example if we think that valuers do not go as far back in time as suggested by model (5).

A simulation was run where the "true" price index is assumed to follow a random walk as in simulation (B), \( P_t \) is generated as in simulation (B) and \( V_t \) is now assumed to follow model (24) with weights chosen to be \( \alpha_1 = \alpha_2 = 1 - \alpha_1 - \alpha_2 = 1/3 \). The results of simulation (D) are shown in figure 4. As expected, the results are not as good as in the previous simulations. The methodology does however not collapse completely. There is little lagging and much of the noise is eliminated. If we have more observations the results are even better. Simulation (D) was made with 25 observations. Appendix A shows the results when the simulation is made with 1000 observations. While the results for model (24) are encouraging, they cannot be generalized. Simulation (D) does however show that the methodology does not necessarily collapse if model (5) is not true.
An alteration to model (5) that makes sense intuitively is to assume that instead of $I_t$ in model (5) we have $I^*_t$, which is $I_t$ plus random noise $n_t$:

$$V_t = \alpha I^*_t + (1-\alpha)V_{t-1}$$  \hfill (25)$$

$$I^*_t = I_t + n_t$$  \hfill (26)$$

The rationale for this model is that maybe not all of the noise from the individual valuations is filtered out when valuations are aggregated into an index. If (25) holds the true population model of $P_t$ is:

$$P_t = \frac{V_t}{\alpha} - \frac{(1-\alpha)V_{t-1}}{\alpha} - n_t + u_t$$  \hfill (27)$$
If we regress $P_t$ on $V_t$ and $V_{t-1}$ when the true population model is equation (27) the coefficient estimates will be biased as $V_t$ is correlated with the error term in equation (27). This can be seen from equation (25) and (26): $V_t$ is a function of $n_t$. In general therefore, this type of deviation from the assumptions is problematic. Three simulations were made in order to see how problematic. The simulations are all similar to simulation (B) except that $V_t$ is constructed using formula (25) and (26). They differ between each other in how large the variance of $n_t$ is. Simulation (E) has the lowest variance of $n_t$, 0.0625, which can be compared with each time periods innovation in $I_t$ which has a variance of 1. When the variance of $n_t$ is this low the problem associated with this type of deviation is relatively small (see figure 5).

![Figure 5](image_url)

Figure 5. Simulation (E) of a "true" value index ($I_t$), and an estimation of $I_t$ (predicted $P_t$) using transactions-based and valuation-based indices. $V_t$ is assumed to follow model (25) and the variance of $n_t$ is 0.0625.

If the variance of $n_t$ is 0.5625 as in simulation (F) there are bigger problems as can be seen from figure 6. Appendix B shows the results when the variance of $n_t$ is 6.25. When the variance is this high, the predicted $P_t$ follows $V_t$ rather than $I_t$. This simulation is however not included as a practical example but rather to show that the estimate of $P_t$ is biased towards $V_t$. 
The results show that the effect of this type of noise depends critically on the variance of the noise.

Figure 6. Simulation (F) of a "true" value index ($I_t$), and an estimation of $I_t$ (predicted $P_t$) using transactions-based and valuation-based indices. $V_t$ is assumed to follow model (25) and the variance of $n_t$ is 0.5625.

Constant alpha

The proposed model implicitly assumes that the smoothing parameter $\alpha$ does not change over time. Quan and Quigley (1991) showed in a theoretical model that $\alpha$ can be expected to be different in different market conditions. This is intuitively appealing since different periods exhibit differences in transaction volume and hence the number of comps that valuers can use. Brown and Matysiak (1998) show empirical evidence that $\alpha$ differs over time and circumstances. A simulation (G) was made in order to see what happens when $\alpha$ changes over time. In the simulation, $\alpha$ follows a simple process: for the first 13 time periods,
\( \alpha \) is 0.4, for the latter 12 time periods \( \alpha \) is 0.2. Except for the changing \( \alpha \) the simulation is similar to simulation (B).

![Graph showing simulation (G) of a "true" value index (\( I_t \)), and an estimation of \( I_t \) (predicted \( P_t \)) using transactions-based and valuation-based indices. The smoothing parameter \( \alpha \) shifts over time in this simulation.](image)

The simulation, shown in figure 7, shows that the method is sensitive to changing \( \alpha \). For the first part of the index, true market movements are exaggerated while the opposite is true for the latter part. This stems from the fact that \( \alpha \) is estimated at 0.3 or the average \( \alpha \) over the time period. Consequently \( \alpha \) is underestimated for the first half of the period and overestimated for the second half. This in turn has the effect that movements in \( I_t \) is exaggerated in the first half and the other way round in the second half. Simulation (G) has shown but one way in which \( \alpha \) may change but has demonstrated that the method is sensitive to this assumption. A feasible remedy to this problem is to use a rolling regression technique.
Conclusion

This paper presents a method for combining transaction- and valuation-based data in a price index. The point of the method is to at least partly provide a remedy for inherent problems in the two types of data: noise in transaction data and smoothing in valuation data. The methodology is devised for a world where the observable transaction prices can be used to construct a price index that constitutes a noisy signal of the “true” price index. Furthermore, it is assumed that valuations can be used to construct a market value index which is a noiseless but smoothed version of the “true” index.

By regressing the observable price index on the valuation index (contemporaneous and lagged one period) it is possible to filter out the noise in the observable price index. If there are many observations, the predicted observable price index comes very close to the “true” price index. The method may be seen as a way of “de-smoothing” a valuation-based index. The advantage that this method gives compared to earlier de-smoothing techniques is that it does not require us to know the smoothing parameter beforehand. On the contrary, the methodology may be seen as a way of estimating the smoothing parameter.

The paper discusses some of the assumptions made. It is shown that the method is insensitive to the “true” price process. The model of the valuation index is a more crucial assumption but it is demonstrated that deviation from the model assumed is not necessarily critical. It is furthermore pointed out that over time varying smoothing of the valuation index is problematic. This may however be remedied by a rolling regression technique.
References


Simulation of "true" value index \(I_t\), and estimation of \(I_t\) (predicted \(P_t\)) using transactions-based and valuation-based indices. \(V_t\) is assumed to follow model (24). The simulation is based on 1000 observations.
Simulation of a "true" value index ($I_t$), and an estimation of $I_t$ (predicted $P_t$) using transactions-based and valuation-based indices. $V_t$ is assumed to follow model (25) and the variance in $n_t$ is 6.25.