Spectral Theory and Applications

Book of Abstracts

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conference in memory of Boris Pavlov (1936-2016)

List of talks

Abstracts

Singular selfadjoint perturbations of selfadjoint operators. Reverse approach

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Following [2] we call a perturbation of given unbounded selfadjoint operator A *singular* if A and corresponding perturbed selfadjoint operator A_1 have different domains. Starting from the M.G. Krein′s resolvent formula for the pair A, A_1 (or from the scattering matrix for A, A_1–pair whenever A has absolutely continuous spectrum) we discuss the problem of restoration of singular perturbations, which generate the given Q-function and matrix (operator) parameter in the A, A_1–Krein formula (or what is the same the given A, A_1–scattering matrix [1]).

References

[1] V. Adamyan and B. Pavlov, *Null–range Potentials and M.G. Krein's formula for generalized resolvents*, Zap. Nauchn. Sem. Leningrad. Otd. Matemat. Inst. im. V.F. Steklova **149** (1986), 7–23 (J. of Soviet Math. **42**(1988), 1537–1550).

[2] S. Albeverio and P. Kurasov, *Singular Perturbations of Differential Operators*, Cambridge University Press, 2000.

Scattering matrices and Dirichlet-to-Neumann maps

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In this talk we discuss a recent result on the representation of the scattering matrix in terms of an abstract Weyl function. The general result can be applied to scattering problems for Schrödinger operators with δ-type interactions on curves and hypersurfaces, and scattering problems involving Neumann and Robin realizations of Schrödinger operators on unbounded domains. In both applications we obtain formulas for the corresponding scattering matrices in terms of Dirichlet-to-Neumann maps. This talk is based on joint work with Mark Malamud and Hagen Neidhardt.

Boundary triplets approach to extensions of operator tensor products

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In many interesting problems of quantum physics (like the interaction of photons with electrons) the operators of the form of sum of tensor products occur. Up to this moment, the method of getting all self-adjoint extensions of such an operator has not been described. In particular, we consider a closed densely defined symmetric operator

$$S := A \otimes I_T + I_A \otimes T,$$

where A is a closed densely defined symmetric operator on the separable Hilbert space $\mathfrak{H}_A$ and T is a bounded self-adjoint operator acting on the separable infinite dimensional Hilbert space $\mathfrak{H}_T$

Our aim is to describe all self-adjoint extensions of S using the boundary triplet approach. More precisely, assuming that $\Pi_A = \{\mathcal{H}_A, \Gamma_0^A, \Gamma_1^A\}$ is a boundary triplet for A^* we construct a boundary triplet $\Pi_S = \{\mathcal{H}_S, \Gamma_0^S, \Gamma_1^S\}$ for S^*. In addition, using the γ-field $\gamma_(\cdot)$ and the Weyl function $M_A(\cdot)$ of the boundary triplet Π_A we express the γ-field $\gamma_S(\cdot)$ and Weyl function $M_S(\cdot)$ of Π_S.

As a demonstration of obtained theoretical results, we consider an example of Dirac and Schrödinger operator coupled to bosons.

Superexponential localisation near the defect in high-contrast 1-d homogenisation.

(Joint work with S. Cooper and K. Cherednichenko)
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We consider a high-contrast periodic homogenisation problem in 1-d with a finite size defect. In recent work by Cooper, Cherednichenko and Guenneau it was shown that in the homogenisation limit the spectrum of the corresponding purely periodic problem (i.e. without a defect) has a band-gap structure. In this work we study localised modes in the gaps of the spectrum of purely periodic problem arising due to the defect. We show that the limit problem for such modes is a Neumann boundary value problem on the defect with no contribution from the surrounding periodic media to the main order term. We also show that these modes decay superexponentially fast outside the defect. This give a drastic improvement for possible applications compared to the standard exponential decay known to exist in similar problems in higher dimensions.

Lieb-Thirring inequalities for infinite gap Jacobi matrices

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In the talk, I'll explain how to establish Lieb-Thirring bounds on the discrete eigenvalues of Jacobi matrices for Schatten class perturbations under very general assumptions. The results apply, e.g., to perturbations of reflectionless Jacobi matrices with Cantor-type essential spectrum. As we'll see, the first step in the procedure is to obtain trace norm estimates by use of a Cauchy-type integral. The next and most technical issue is then how to estimate a variant of the m-function which has an absolute value under the integral sign. Once this is settled, the Lieb-Thirring bound for trace class perturbations follows from a version of the Birman-Schwinger principle due to Frank and Simon. If time permits, I'll discuss examples and non trace class perturbations as well. The talk is based on joint work with Maxim Zinchenko, UNM.

Resolvent estimates for high-contrast elliptic problems with periodic coefficients

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I will discuss the asymptotic behaviour of the resolvents $(A_\varepsilon + I)^{-1}$ of elliptic second-order differential operators A_ε in $\mathbb{R}^d$, $d \geq 2$, with periodic rapidly oscillating coefficients, as the period ε goes to zero. The class of operators covered in the discussion are the "double-porosity" case of coefficients that take high contrasting values of order one and of order ε^2 in different parts of the periodic reference cell. I shall describe a construction for the leading order term of the "operator asymptotics" of $(A_\varepsilon + I)^{-1}$ in the sense of operator-norm convergence and prove order $O(\varepsilon)$ remainder estimates. This is joint work with Kirill Cherednichenko.

Asymptotics of the Christoffel-Darboux kernel for generalized Jacobi measures

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Let μ be a finite Borel measure supported on the complex plane and let $p_n(\mu, z)$ be the n-th orthonormal polynomial with respect to μ. The goal of this talk is to study the asymptotic behavior of the Christoffel-Darboux kernel

$$K_n(\mu, z, w) = \sum_{k=0}^{n} \overline{p_k(\mu, z)} p_k(\mu, w)$$

around points where the measure exhibits a power type singularity, for example it is supported on the real line and for some x_0 in its support, μ is absolutely continuous in a neighbourhood U of x_0 with

$$d\mu(x) = w(x)|x - x_0|^\alpha dx, \quad x \in U$$

there for some $\alpha > -1$ and for some positive and continuous weight $w(x)$. Our results include
(i) universality limits

$$\lim_{n\to\infty} \frac{K_n(\mu, x_0 + a/n, x_0 + b/n)}{K_n(\mu, x_0, x_0)},$$

where μ is supported on a compact subset of the real line behaving like $d\mu(x) = |x - x_0|^\alpha dx$ in a neighbourhood of x_0,
(ii) limits of the type

$$\lim_{n\to\infty} \frac{1}{n^{\alpha+1}} K_n(\nu, z_0, z_0),$$

where ν is supported on a system of Jordan arcs and curves Γ behaving like $d\nu(z) = |z - z_0|^\alpha ds_\Gamma(z)$ and s_Γ denotes the arc length measure with respect to Γ. Part of the results was established in a joint paper with Vilmos Totik [1].

References

[1] T. Danka and V. Totik, *Christoffel functions for power type weights* (2015), to appear in Journal of the European Mathematical Society, arXiv identifier: *arXiv:1504.03968*

The Solvability of Differential Equations

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It was a great surprise Hans Lewy in 1957 showed that the tangential Cauchy-Riemann operator on the boundary of a strictly pseudoconvex domain is not locally solvable. Hörmander then proved in 1960 that almost all linear partial differential equations are not locally solvable. Nirenberg and Treves formulated their famous conjecture in 1970: that condition (PSI) is necessary and sufficient for the local solvability of differential equations of principal type. Principal type means simple characteristics, and condition (PSI) only involves the sign changes of the imaginary part of the principal symbol along the bicharacteristics of the real part. The Nirenberg-Treves conjecture was finally proved in 2006.

In this talk, we shall present some necessary condition for the solvability of differential operators that are not of principal type, instead the principal symbol vanishes of at least second order at the characteristics. Then the solvability may depend on the lower order terms, and one can define a condition corresponding to (PSI) on the subprincipal symbol. We show that this condition is necessary for solvability in some cases. The condition is not always necessary, for example effectively hyperbolic operators are always solvable with any lower order terms.

Rational operator functions and applications to electromagnetics

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Operator functions with a rational dependence on a spectral parameter describe a large number of processes in science. Here we study functions that in particular have applications in electromagnetic field theory. The applications include dielectric resonators and photonic crystals where the ω-dependent material properties are of Drude-Lorentz type. Let M_ℓ, $\ell = 1, 2, \ldots, L$ denote bounded linear operators in a Hilbert space $\mathcal{H}$ and denote by A a self-adjoint operator with compact resolvent that is bounded from below. In this talk we consider rational operator functions of the form

$$\mathcal{S}(\omega) = A - \omega^2 - \sum_{\ell=1}^{L} \frac{M_\ell}{c_\ell - i d_\ell \omega - \omega^2}, \quad \operatorname{dom} \mathcal{S}(\omega) = \operatorname{dom} A, \quad \omega \in \mathbb{C} \setminus P,$$

where P is the set of poles of the rational function and c_ℓ, d_ℓ are non-negative constants. The operator function is self-adjoint in ω^2 if the the damping d_ℓ is set to zero and we establish then variational principles and provide optimal two-sided estimates for all the eigenvalues of $\mathcal{S}$. A new enclosure of the numerical range is studied for $L = 1$ and $d_1 > 0$. The derived enclosure is optimal given the numerical ranges of A, B. We apply the new theory to wave propagation in photonic crystals and discretize the operator function with a high order finite element method. Several examples illustrate the two-sided estimates of the eigenvalues in the self-adjoint case and the enclosure of the numerical range for the case $L = 1$, $d_1 > 0$.

The first part of the talk is based on a joint work with Heinz Langer and Christiane Tretter and the second part of the talk is based on a joint work with Axel Torshage.

Spectral problem with applications in quantum complex analysis

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Let U, V be a Weyl pair $UV = q^2VU$ of unbounded self-adjoint operators, realized in $L_2(\mathbb{R})$ as $U = e^{\alpha}Q$ and $V = e^{\beta}P$, where P , Q are canonical operators and α, β are real positive. Operator $L = U + U^{-1} + V$ plays important role in the theory of quantum group $SL_q(2, \mathbb{R})$, cluster algebras, quantum Liouville model, quantum Teichmuller theory. We show that L has simple continuous spectrum in the interval $[2, \infty]$ and construct its resolvent and complete family of eigenfunctions. The talk is based on a joint article with L. Takhtajan.

Spectral analysis of lattice-of-rod photonic crystals

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We consider photonic crystals consisting of periodically ordered and infinitely long rods. The goal is to design a crystal in such a way that (specially polarized) light with given frequencies can not propagate in the medium. In order to study whether the motion of electromagnetic waves with a given frequency in the crystal is possible or not, the spectra of two differential operators with periodic coefficients have to be analyzed.

We are going to show the following main result: given a finite number of frequencies it is possible to design a photonic crystal of the above form with high contrast dielectric permittivity ε such that waves with a polarization of transverse magnetic type and such frequencies can not propagate in the crystal. This result is connected with the spectrum of the operator $-\varepsilon^{-1}\Delta$ in $L^2(\mathbb{R}^2)$. The situation for modes with transverse electric polarization is different. We show that for any frequency, which is not too large, there exists an electromagnetic wave with this frequency that can propagate in the crystal. This behavior is connected with the spectral properties of $-\mathrm{div}(\varepsilon^{-1}\mathrm{grad})$ in $L^2(\mathbb{R}^2)$. These results fit to experimental observations obtained for such crystals with simple geometries [1].

The talk is based on a joint work with V. Lotoreichik.

References:

[1] J. D. Joannopoulos, S. G. Johnson, J. N. Winn, and R. D. Meade. *Photonic Crystals: Molding the Flow of Light (Second Edition).* Princeton University Press, 2 edition, 2008.

Essential spectrum of singular matrix differential operators

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An analysis of the essential spectrum of singular matrix differential operators of mixed order is presented. The essential spectrum is described explicitly and consists of possibly two branches one of which appears due to cancellations in the formal determinant of the operator matrix while the other may appear due to the singularity of the matrix coefficients near the boundary. We discuss criteria for the absence and presence of the latter. The results were obtained in collaboration with P. Siegl and C. Tretter.

The string density problem and nonlinear wave equations

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Classical objects in spectral theory are the differential equation

$$-y'' = z\,\omega\,y, \quad x \in [0, L), \tag{1}$$

(here $L \in (0, \infty]$, ω is a positive Borel measure on $[0, L)$ and z is a spectral parameter) and the Weyl–Titchmarsh m-function, which encodes all the spectral information about (1). In a series of papers in the 1950s, M. G. Krein investigated the direct and inverse spectral problems for this equation. Viewing these problems as a natural generalization of investigations of T. Stieltjes on continued fractions to the moment problem, M. G. Krein identified the totality of all possible m-functions with the class of the so-called Stieltjes functions in a bijective way. Recently, the string density problem has come up in connection with some completely integrable nonlinear wave equations (e.g., the Camassa–Holm equation) for which the string spectral problem serves as an underlying isospectral problem. In contrast to the KdV equation, the Camassa–Holm equation possesses peaked solitons, called peakons, and models breaking waves. The latter happens when ω is a signed measure, i.e., the string is *indefinite*.

In this talk, we review the direct and inverse spectral theory for indefinite strings and relate it to the conservative Camassa–Holm flow. As one of our main results we are going to present the indefinite analog of M. G. Krein's celebrated solution to the string density problem. A special attention will be given to multi-peakon solutions.

The talk is based on joint work with Jonathan Eckhardt.

Three one-dimensional quantum particles scattering problem with short-range repulsive pair potentials. To the question of absolutely continuous spectrum eigenfunctions asymptotics justification

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We consider the quantum scattering problem of three one-dimensional particles with repulsive short-range pair potentials. For clarity we restrict ourself by the case of finite pair potentials. The absence of singular continuous spectrum of the corresponding Schroedinger operator for the broad class of pair potentials was proved earlier in known works [1], [2]. Nevertheless the Mourre techniques do not allow to describe the asymptotics of absolutely continuous spectrum eigenfunctions. In our work we prove the existence of the resolvent limit values on absolutely continuous spectrum regardless of Mourre results and construct them explicitly.

It allows us following the known procedure to derive the absolutely continuous spectrum eigenfunctions asymptotics, offered earlier in [3]. Our approach, close to the foundational work of L.D.Faddeev devoted to three-dimensional particles [4], specifically uses the ideas of Schwarz alternating method [5],

[6].

References

[1] E.Mourre, Commun.Math.Phys., 78, 391-408, (1981)

[2] P.Perry, I.M.Sigal, B.Simon, Annals of Mathematics, 114, 519-567, (1981)

[3] Buslaev, V.S. and Levin, S.B.,

Amer.Math.Soc.Transl. (2)v.225, pp.55-71, (2008)

[4] L.D.Faddeev, Mathematical aspects of the three-body problem of the quantum scattering

theory. Daniel Davey and Co., Inc., (1965).

[5] K.Moren, Methods of Hilbert spaces. PWN, Warszawa (1967, translated from Polish).

[6] A.M.Budylin, V.S.Buslaev, St.Petersburg Math.J., vol.7, n.6, 925-942 (1996).

Bounds for eigenfunctions of the Laplacean on noncompact Riemannian manifolds

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We deal with eigenvalue problems for the Laplacian on noncompact Riemannian manifolds M of finite volume. Sharp conditions ensuring $L_q(M)$ and $L_\infty(M)$ bounds for eigenfunctions are exhibited in terms of either the isoperimetric function or the isocapacitary function of M. This is a joint work with Andrea Cianchi (Florence)

The H-class for the transfer matrix sequence for block-Jacobi operators. — How to generalize the scalar-Jacobi results?

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Consider classical "scalar" self-adjoint Jacobi operator J acting in the Hilbert space $l^2(N, C)$. The n-th transfer matrix $T_n(\lambda)$ for J is the 2 by 2 matrix which shifts the "conditions at the point n" of any solution of the generalized eigenequation for J and λ onto its "conditions at the point $n+1$". The notion of the H-class (H for "Homogeneous") of sequences of 2 by 2 matrices was introduced in [MMdouble] to describe the case, when all non-zero solutions behave asymptotically in the same way.

As showed in [MMWeyl], under some regularity assumptions on the weight sequence for J

$$H(J) := \{\lambda \in R : \{T_n(\lambda)\} \in H\} \subset \sigma_{ess}(J). \tag{2}$$

The following result based on Gilbert-Pearson-Khan subordination theory is proved in [MMdouble]

Theorem
If $\{T_n(\lambda)\} \in H$ for any $\lambda \in G$ — an open subset of the real line, then J is absolutely continuous in G, and the closure of G is contained in $\sigma_{ac}(J)$.

Moreover, for many classes of Jacobi operators with "sufficiently regular coefficients"

$$\sigma_{ess}(J) = \sigma_{ac}(J) = \overline{H(J)} = \sigma_{ac}(J). \tag{3}$$

My talk is devoted to the preliminary trials to generalize this kind of results onto block-Jacobi operators. So, the Jacobi matrix terms are now "blocks"— d by d scalar matrices, transfer matrices $T_n(\lambda)$ are $2d$ by $2d$ scalar matrices and the operator J acts in the space $l^2(N, C^d)$. First I propose a certain natural way of defining the H class for sequences of $2d$ by $2d$ matrices. With this definition, my unique generalization proved so far is the result of (2) type, i.e. $H(J) \subset \sigma_{ess}(J)$ in the block case with some extra assumptions. Then I present some simple examples showing the difference between the equality (3) for the case $d = 1$ and the situation for cases $d > 1$. I formulate some open questions and conjectures concerning the proper generalizations of the above theorem and the equalities (3) for the general d-block case.

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References:

[**MMdouble**] M. Moszyński, *Spectral properties of some Jacobi matrices with double weights*, J. Math. Anal. Appl. **280** (2003), 400–412.

[**MMWeyl**] M. Moszyński, *Weyl sequences and the essential spectrum of some Jacobi operators*, Journal of Operator Theory **67** no. 1 (2012), 237/226256.

On the detectable subspace structure and applications

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The structure of the so-called detectable subspace of the non-Hermitean operators subspace structure to be discussed.Applications to the Hain-Lutz operators are planned to be considered.

New formulae for the first-order traces for ODO

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Consider an operator $\mathbb{L}$ generated by differential expression

$$lu \equiv u^{(\ell)} + \sum_{k=0}^{\ell-2} p_k(x)u^{(k)}$$

(here $p_k \in L_1(0,1)$) and by ℓ boundary conditions

$$U_{\nu 0}(u) + U_{\nu 1}(u) = 0, \qquad \nu = 1, \dots, \ell. \tag{$*$}$$

Here (for any ν at least one of the coefficients α_ν, γ_ν is not zero)

$$U_{\nu 0}(u) = \alpha_\nu u^{(k_\nu)}(0) + \sum_{j=0}^{k_\nu-1} \alpha_{\nu j} u^{(j)}(0), \quad U_{\nu 1}(u) = \gamma_\nu u^{(k_\nu)}(1) + \sum_{j=0}^{k_\nu-1} \gamma_{\nu j} u^{(j)}(1),$$

The system $(*)$ is assumed Birkhoff regular and normalized.

We obtain a new formula of regularized traces which generalizes a well known formula of Gelfand and Levitan. In the simplest case of **separated** boundary conditions $\ell = 2m$,

$$U_{\nu 1}(u) \equiv 0, \quad \nu = 1, \dots, m; \qquad U_{\nu 0}(u) \equiv 0, \quad \nu = m+1, \dots, 2m,$$

this formula reads as follows:

Let $q \in L_1(0,1)$ be such that $\int_0^1 q = 0$ and Let λ_n and $\widetilde{\lambda}_n$ be eigenvalues, respectively, of the operators $(-1)^m\mathbb{L}$ and $(-1)^m(\mathbb{L}+q)$. Then, under some additional regularity assumptions for q at a neighborhood of the endpoints, the following formula holds:

$$\sum_{n=1}^{\infty} (\widetilde{\lambda}_n - \lambda_n) = -q(0) \cdot \Big(\frac{m}{2} - \frac{1}{4} - \frac{\varkappa_0}{2m}\Big) - q(1) \cdot \Big(\frac{m}{2} - \frac{1}{4} - \frac{\varkappa_1}{2m}\Big), \tag{4}$$

where $\varkappa_0 = \sum_{\nu=1}^{m} k_\nu$, $\varkappa_1 = \sum_{\nu=m+1}^{2m} k_\nu$.

Now let q be a finite measure (also regular at a neighborhood of the endpoints). In the case $\ell \geq 3$ formula (4) holds if we use Cesaro summation. However, for $\ell = 2$ it changes drastically, and terms *quadratic* with respect to q arise in the right-hand side. In a particular case, the last phenomenon was discovered by Savchuk and Shkalikov in 2001.

BIBLIOGRAPHY

1. A.I. Nazarov, D.M. Stolyarov, P.B. Zatitskiy, The Tamarkin equiconvergence theorem and a first-order trace formula for regular differential operators revisited: Journ. of Spectral Theory, **4** (2014), 365-389.

2. E.D. Gal'kovskii, A.I. Nazarov, in preparation.

Super singular perturbations

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Quantum graph model of Helmholtz resonator

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The problem of resonances and resonant states attracted great attention starting from famous Lord Rayleigh work. But rigorous mathematical description of the problem was given at the end of 20-th century. Particularly, it became clear that resonances are eigenvalues of some dissipative operator. A few models and asymptotic approaches to the problem were developed on the background of this operator treatment. One of the intriguing question in this problem is: What is a domain Ω which gives one the completeness of the resonant states in $L_2(\Omega)$? It is proved only for some examples of particular problems. There is an interesting relation between the scattering problem and Sz-Nagy functional model. More precisely, the completeness is related with the factorization of the scattering matrix (characteristic function for the functional model). We use this relation in the present paper. Namely, we consider the simplest, one-dimensional, model of the Helmholtz resonator and investigate the scattering matrix for this quantum graph. We consider the Schrödinger operator on the graph Γ consisting from three edges $\Omega_1 \cup \Omega_2 \cup \Omega_3$ (see Fig.) coupled at vertex V. Here Ω_1 and Ω_2 are semi-axis, Ω_3 is a segment, $\partial\Gamma = V_0$.

Definition. *The Schrödinger operator H on Γ acts as $-\frac{d^2}{dx^2}$ at each edge Ω_i. The operator has the following domain:*

$$\begin{aligned} &dom\ H = \{\psi \in C(\Gamma) \cap W_2^2(\Gamma \setminus V); \psi_1(-0) = \psi_2(+0) = \psi_3(L-0), \\ &-\psi_1'(-0) + \psi_2'(+0) - \psi_3'(L-0) = \alpha\psi_2(0), \psi_3(0) = 0.\} \end{aligned} \tag{5}$$

Here W_2^2 is the Sobolev space, $\frac{d\psi_i}{dx_i}(V)$ is the derivative of the solution at the vertex V of edge Ω_i in the outgoing direction from the vertex.

Theorem. *The system of resonant states of the Schrödinger operator H on the graph Γ is complete in $L_2(\Omega_3)$.*

Resonant states completeness for zero-width slit model of the Helmholtz resonator

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We deal with a model of the Helmholtz resonator, resonator with point-like boundary window. Lax-Phillips approach is used. Resonances (quasi-eigenvalues) are eigenvalues of some dissipated operator.. The corresponding resonant states belong to $L_2(\Omega)$ for any bounded Ω. An interesting quaestion appeared: What is the maximal domain Ω such that the resonant states are complete in $L_2(\Omega)$? The result is given by the following theorem.

Theorem. Let Ω^{in} be convex bounded domain in $\mathbb{R}^3$, $-\Delta$ be the model operator corresponding to point-like window in the boundary. Then, the set of the resonant states of the operator $-\Delta$ forms a basis in $L_2(\Omega^{in})$.

A relation with Sz.-Nagy model is analyzed.

Corollary. Let a characteristic function S be the S-matrix for scattering by a convex obstacle with point-like window. Then S is the Blaschke (Blaschke-Potapov) product.

Decorated quantum graph (hybrid manyfold) with two infinite is considered. Resonant states completeness is discussed.

Recovering quantum graph spectrum from vertex data

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We discuss the question to what extent spectral information of a Schrödinger operator on a finite, compact metric graph subject to standard (Kirchhoff) matching conditions can be recovered from a corresponding Titchmarsh-Weyl function on the boundary of the graph or another set of selected vertices. In contrast to the case of ordinary or partial differential operators, the knowledge of the Titchmarsh-Weyl function is in general not sufficient for recovering the complete spectrum of the operator (or the potentials on the edges). Positive results depending on geometric properties of the graph are presented.

Locality of the heat kernel

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We will discuss what it means for the heat kernel to be local. We will define this in a broad metric space setting and look for general conditions that guarantee locality. This will include cases like Riemannian manifolds and quantum graphs, where locality is known to hold but also various other spaces. We will also discuss locality of the Wiener measure of the associated Brownian motion. This is still work in progress.

The zero-two law for cosine families

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For strongly continuous semigroups the following dichotomy result is well-known. If $\limsup_{t\to 0^+} \|T(t) - I\| < 1$, then the semigroup T is uniformly continuous. We present recent developments concerning the analogous question for *operator cosine families*. In particular, we show that

$$\limsup_{t\to 0^+} \|C(t) - I\| < 2 \qquad \Longrightarrow \qquad \lim_{t\to 0^+} \|C(t) - I\| = 0,$$

for a strongly continuous cosine family C. Furthermore, we will discuss related laws and generalizations.
The work is based on joint work with Hans Zwart.

[1] J. Esterle. *A short proof of the zero-two law for cosine families*, Arch. Math. 105(4): 381–387, 2015.

[2] W. Chojnacki. *On cosine families close to scalar cosine families*, J. Aust. Math. Soc. 99(2): 166–174, 2015.

[3] F.L. Schwenninger, H. Zwart. *Zero-two law for cosine families*, J. Evol. Eq. 15(3): 559-569, 2015.

[4] F.L. Schwenninger, H. Zwart. *Less than one implies zero*, Studia Math. 229(2): 181-188, 2015

PT-Symmetric Operators with Parameter. The Problem on Similartity to Self-adjoint Ones.

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We consider PT-symmetric Sturm-Liouville operators

$$T(\varepsilon) = -\frac{d^2}{dx^2} + \varepsilon P(x), \quad \varepsilon > 0,$$

in the space $L_2(-a, a)$, $0 < a \leqslant \infty$, where P is subject to the condition $P(x) = -\overline{P(-x)}$. The spectra of these operators are symmetric with respect to the real axis and discrete, provided that the interval $(-a, a)$ is finite and P is not a singular potential. We will show that the spectrum of the operator $T(\varepsilon)$ is real for sufficiently small values of the parameter ε and in this case $T(\varepsilon)$ is similar to a self-adjoint operator. For large values of ε the complex eigenvalues do appear and the number of non-real eigenvalues increases as $\varepsilon \to \infty$.

The aim of the talk is to cast some light to the following problems: How the eigenvalues of $T(\varepsilon)$ do behave when the parameter changes? Is it possible to evaluate or to calculate the critical value ε_0 of the parameter, such that $T(\varepsilon)$ are similar to self-adjoint operators for all $\varepsilon < \varepsilon_0$? We will find an explicit answer for some particular potentials.

The talk is based on the joint papers with S.Tumanov.

Large Deviations and a Conjecture of Lukic

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I will begin by discussing the role of sum rules in the spectral theory of orthogonal polynomials and the conjectures of Simon and Lukic for higher order sum rules in OPUC. Then I will discuss the new approach of Gamboa, Nagel and Roualt obtaining sum rules form the theory of large deviations. Finally, I will describe some recent joint work with Breuer and Zeitouni on using sum rules to partially prove a new case of Lukic's conjecture.

The Interior Exterior Duality - in Memory of Boris Pavlov

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Consider a finite and simply connected domain in the plain, bounded by a sufficiently well behaved boundary. Requiring any standard boundary conditions on the boundary, the spectrum of the Laplacian in the interior is discrete. The Laplacian in the exterior (subject to the same boundary conditions) has a continuous spectrum and a corresponding scattering operator. It describes scattering from the obstacle defined by the boundary. There is an intimate connection between the spectrum in the interior, and the properties of the scattering operator in the exterior. This is just one of the examples of the general phenomenon of the interior – exterior duality which interested the late Pavlov, and which I shall review and discuss from different points of view and settings.

Spectral analysis of two doubly infinite Jacobi operators

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Two simple second order difference equations of the form

$$q^{n-1}u_{n-1} - zu_n + q^n u_{n+1} = 0$$

and

$$u_{n-1} + (\alpha q^n - z)u_n + u_{n+1} = 0$$

where $q \in (0,1)$ and $\alpha \in \mathbb{R}$, have been studied several times in past mainly from the point of view of the theory of orthogonal polynomials, continued fractions, indeterminate moment problem and special functions.

In this talk, we consider the corresponding two doubly-infinite Jacobi matrices and provide a detailed analysis of their spectral properties. Unlike the respective semi-infinite matrices, the spectrum of both doubly infinite Jacobi matrices is obtained fully explicitly. We also emphasize the close connection between spectral properties of these operators and special functions: Jacobi theta functions, certain q-exponential function and the Hahn-Exton q-Bessel function. If time remains, we will discuss also some properties of these operator in the non-selfadjoint setting with complex parameters q and α.

On the Ambartsumian theorem for quantum graphs

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Enclosure of the numerical range of a class of nonselfadjoint rational operator functions

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Let $A, B : \mathcal{H} \to \mathcal{H}$ be selfadjoint operators in a Hilbert space $\mathcal{H}$ and assume that B is bounded. Define the rational operator function

$$\mathcal{S}(\omega) = A - \omega^2 - \frac{\omega^2}{c - id\omega - \omega^2}B, \quad \text{dom } \mathcal{S}(\omega) = \text{dom A}, \quad \omega \in \mathbb{C} \setminus \text{P},$$

where P is the set of poles of the rational function. Rational operator functions have been studied extensively in the case $d = 0$ and $\mathcal{S}$ is then selfadjoint in ω^2. In this talk we present results for the nonselfadjoint case $d > 0$ and derive a new enclosure of the numerical range of $\mathcal{S}$. Contrary to the numerical range of $\mathcal{S}$, the presented enclosure can be computed exactly given only the numerical ranges of A and B. The enclosure is optimal and many characteristics of the numerical range of $\mathcal{S}$ can be obtained by just investigating the enclosure. Inspired by the definition of pseudospectra, we introduce a pseudonumerical range and study an enclosure of this set. The enclosure of the pseudonumerical range provides an upper bound on the norm of the resolvent of $\mathcal{S}$.

This talk is based on a joint work with Christian Engström.

Variational principles for eigenvalues in spectral gaps and applications

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In this talk variational principles for eigenvalues in gaps of the essential spectrum are presented which are used to derive two-sided eigenvalue bounds. Applications of the results include the Klein-Gordon equation, even when complex eigenvalues occur, and a spectral problem related to 2D photonic crystals.

(joint work with M. Langer/H. Langer and with C. Engstroem/H. Langer)