Theoretical Physics

Electrostatic ion thrusters for space debris removal

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October 28, 2018

Abstract

The current levels of space debris are critical and actions are needed to prevent collisions. In this paper it is examined whether an electrostatic ion thruster can be powerful enough to slow down the debris in a sufficient manner. Furthermore, it is looked into whether the process can be repeated for a significant number of pieces by maneuvering between them. We conclude that the removal process seems possible although some improvements are needed. Maneuvering is costly but despite conservative assumptions, we estimate that about 800 pieces can be removed in one journey made by a satellite weighing ten tonnes of which nine are xenon.

Abstract

Den nuvarande nivån av rymdskrot är kritisk och åtgärder krävs för att förhindra kollisioner. I denna artikel undersöks det huruvida en elektrostatisk jonkanon är kraftfull nog för att bromsa skrot tillräckligt. Fortsättningsvis undersöks det om denna process är effectiv nog för att återupprepas för ett betydande antal bitar, inklusive manövrering bitarna emellan. Vi drar slutsatsen att processen verkar möjlig att genomföra även om vissa förbättringar behövs. Manövreringen är kostsam men trots konservativa antaganden uppskattar vi att ungefär 800 bitar kan tas ned under en resa av en satellit med vikt 10 ton varav nio ton är xenon.

Contents

1 Introduction

We are two students doing a Degree program in engineering physics at the Royal Institute of Technology, KTH. This paper on the removal of space debris is our bachelor's thesis and the main part of our bachelor's project. The idea is entirely our own but discussions have been held with our supervisor Prof. Christer Fuglesang.

We propose a system based on an electrostatic ion thruster positioned on a tracking spacecraft. The thruster will fire xenon ions at a piece of debris, causing it to lose speed and fall towards earth. We have chosen to focus on two key aspects:

- Is it possible to obtain enough thrust and firing-precision to bring down a single piece of debris using this method?

- Is the procedure, considering both maneuvering and firing, efficient enough to be extended to a large quantity of debris?

After a quick review of the background to the problem in section 2, the breaking requirements for a single piece will be examined in section 3.1. The following sections, 3.2 and 3.3, handles the precision and range of the thruster whereas the maneuvering of the tracking satellite, as well as the required fuel, will be dealt with in section 3.4 and 3.5. Calculated results will be presented in section 4 and discussed in section

5. At the very end an appendix containing calculations is provided.

2 Background

2.1 The space debris problem

The European Space Agency, ESA, defines space debris as

> "...all the inactive, man made objects, including fragments, that are orbiting Earth or reentering the atmosphere..."

and they warn how 10 km/s collisions with items larger than around 10 cm could lead to complete destruction of an active spacecraft [1]. It is shown in fig 1a) that impact velocities of around 10 km/s are to be expected if collision occurs. As of January 2017, US Space Surveillance System, the main source of large debris information, has published cataloguing of around 19 000 space objects larger than 5–10 cm, totalling close to 8000 metric tons in Earth orbit.[2] The majority of debris is found in low-earth-orbit and as seen in fig 1b), the highest density is found around 800 km. A collision will create new fragments that can cause new collisions in an avalanche effect known as *Kessler's syndrome* [3]. With every space mission potentially adding debris and risking collision it is increasingly important to deal with this issue.

2.2 The Dual-stage 4 grid ion thruster (DS4G)

The DS4G is a powerful concept engine, developed by the European space agency
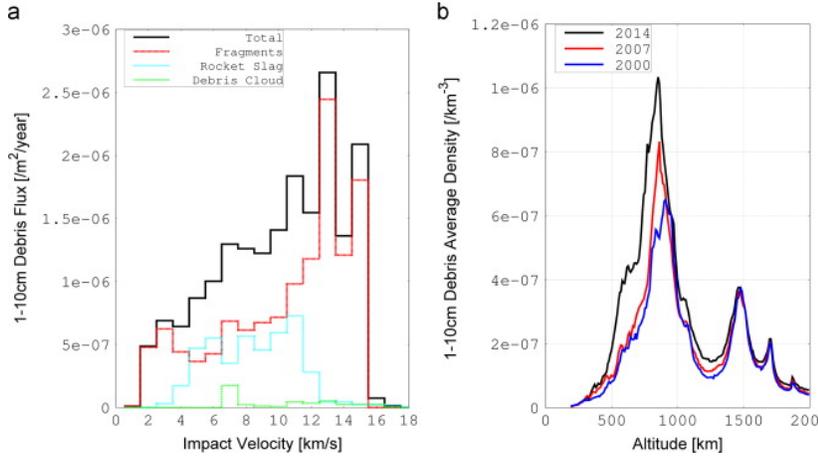
Figure 1: a) Results from simulations for debris 1-10cm performed with MASTER2009 code. The impact velocity and flux for 1–10 cm debris components calculated for the ISS orbit for 2008, here fragments (red-line) include debris from satellite explosions or collisions. (b) Recent evolution of 1–10 cm debris in low-earth-orbit altitudes.[4]

during the early 2000s, designed for long journeys with large spacecrafts. The idea was originally proposed by D.Fearn [5] and has thereafter been tested for ESA by C. Bramanti and colleagues [6]. The thruster was not designed for the purpose of removing space debris but possesses characteristics desired in order to do so.

An electrostatic thruster works by extracting ions from a plasma sheath and then accelerating them using an electrostatic potential. The DS4G thruster has, as its name suggests, two stages consisting of four grids instead of the conventional three gridded system. The extra grid allows the first stage to extract the ions from the plasma sheath before the second stage accelerates them. Stage 1, consisting of grid 1 and 2, has its potential limited to < 5 kV whilst stage 2, grid 3 and 4, can have potentials up to 80 kV. This allows for less beam curvature/divergence and higher velocities than conventional thrusters. Bramanti's team calculated that a single 20 cm diameter 4-gridded ion thruster could operate at 250 kW power to produce a 2.5 N thrust and a specific impulse of 19,300 s from Xenon propellant using 30 kV beam potential and 1 mm ion extraction grid separation [6].

3 Problem

The first question is whether ions exhausted from a Dual-stage 4 Grid engine will give enough thrust to slow down the debris to a lower orbit velocity where the atmosphere can finish the process. The magnitude of the thrust as well as the ability to focus it at the debris is of importance. Thereafter it will be examined whether the process is efficient enough so that a reasonable payload of xenon could bring down a significant number of debris. Both parts are heavily intertwined as a more concentrated beam would have a greater hit-

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percentage, giving better breaking abilities as well as minimizing waste. Better precision also allows for greater range, meaning less maneuvering and saving fuel.

Figure 2: Initial and resulting orbits after break impulse. (Not to scale)
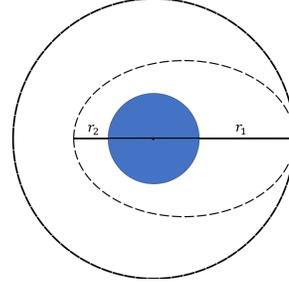
3.1 Required breaking, Δv

The targeted piece of debris is assumed to be in a circular orbit at a specific altitude and to satisfy the equation for specific orbital energy

$$\epsilon = \epsilon_k + \epsilon_p = v^2 - \frac{\mu}{r} = \frac{-\mu}{2a} \quad (1)$$

where v is the relative orbital speed, r is the distance from Earth's center, $\mu = G(m + M)$ is the sum of the gravitational bodies weighed with the gravitational constant, a is the semi-major axis and ϵ_k and ϵ_p are the kinetic and potential energies. Since μ and a are constants for two bodies in a specific orbit, the total energy ϵ is constant over time and for a non-circular orbit, speed varies with radius. Thus, if a circular orbit has its kinetic energy reduced at a certain point, the point will instead become a low speed outer point of a smaller elliptic orbit (see figure 2). The speed difference, Δv, necessary to move from a certain circular orbit to a desired elliptical orbit can be obtained by defining the original and desired orbits, solving for v_1 and v_2 and thereafter taking the difference (see Appendix 8.1 for details). The formula becomes

$$\Delta v = \sqrt{\mu} \left(\sqrt{\frac{r_2}{r_1(r_1 + r_2)}} - \sqrt{\frac{1}{2r_1}} \right) \quad (2)$$

where r_1 is the original orbital radius and r_2 is the smallest radius for the new elliptic orbit. Since Earth's mass, M, is many times larger than the mass of any piece of debris the approximation $\mu \simeq GM$ is made. Thus it is obtained that,

$$\Delta v = \sqrt{GM} \left(\sqrt{\frac{r_2}{r_1(r_1 + r_2)}} - \sqrt{\frac{1}{2r_1}} \right) \quad (3)$$

and Δv does not depend on the mass of the debris, m. A graph for Δv required to obtain a perigee altitude of 350 km as a function of starting altitude can be found as figure 5 in section 4.1

3.2 Beam divergence

One of the most important factors, both concerning range and efficiency, is the divergence of the beam. Coletti, Gessini and Gabriel [7] has derived that the diffraction angle of the beam can be expressed as

$$\alpha = 0.62\ S\left[\frac{P}{P_0} - 0.4\frac{r_2}{r_1}\frac{\Gamma^2}{\lambda(1+\Gamma)} + 0.53\frac{r_2}{r_1} - 1\right] +$$

$$+\ 0.31\ S\left(\frac{P}{P_0}\right)\left[1 + \frac{t_2}{t_1} + 0.35\frac{r_2}{r_1}\left(\lambda + \frac{d_3 + t_3 + t_2}{d_1}\right)(1 + 0.5\Gamma)^{-1.5}\right] \quad (4)$$

where $\Gamma = (V_2 - V_3)/(V_1 - V_2)$ is the ratio between the acceleration and extraction voltage, $\lambda = d_2/d_1$ is the ratio between the first and second stage grid gaps, t_i, $i = 1, 2, 3$, r_i, $i = 1, 2$, are geometrical properties of the tube and $S = r_1/d_1$. Given equation 4 they could then calculate the optimal value of the perveance ratio P/P_0 to allow for zero beam divergence. The perveance, P, is a measurement of how the much the space charge affects the beam's motion. It is a constant dependent on the geometrical properties of the thruster. P_0 is a constant with value $P_0 = \frac{4}{9}\epsilon_0\left(\frac{2e}{m}\right)^{1/2}d_1^2$ where m is the mass of the ions in the beam and e the electronic charge.

$$P = \frac{2.335 \cdot 10^{-6}A}{d_{cg}^2[1 + \frac{1}{\mu}(\frac{d_{cp}}{d_{cg}})^{3/2}]^{4/3}}\ [\text{A/V}^{3/2}] \quad (5)$$

where A is the cross-sectional area, d_{cg} is the distance between cathode and grid and d_{cp} is the distance cathode to plates.

Perveance increases with increasing area of the tube and decreasing distance between the plates and the grid. Coletti, Gessini and Gabriel argue that there are two limitations on the minimal spacing between the plates. The first is the maximum electric field that can be applied without causing arcing of the beam and the second is the engineering limit. They further argue that a reasonable minimum value on the grid spacing is 0.5mm as opposed to the 1mm used by Bramanti [6]. Important to note is that the diffraction angle of the beam can be reduced to zero and it is from now on assumed that the beam is parallel when fired.

Sedlaček [8] calculated that given a perveance P the beam diameter of an electron beam at a distance z can be calculated using

$$2.09\sqrt{\frac{r_b}{b} - 1} = \frac{z}{b}\sqrt{\kappa P} \quad (6)$$

where b is the initial radius of the beam, $\kappa = \frac{1}{2\pi\epsilon_0\eta^{1/2}} = 3.034 \cdot 10^4\ [\text{V}^{3/2}/\text{A}]$ and r_b is the beam radius which is defined as the position of the outermost ion. The beam diameter was calculated for an idealized beam only affected by the repulsive forces of the electrons. For an optimal beam of electrons with the data measured by [6] the beam radius is about 20 cm at a distance of $z = 10$ m and about 7 m at a distance of $z = 100$ m. The principle is the same for a xenon beam. Despite the mass of the xenon ions being greater by a factor 10^5 compared to the electrons the spread of the beam will increase compared to the electron beam. This is due to the $+8$ charge of the ions causing the space charge to be

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many times greater. However, when firing ions it is also necessary to fire equal charge of the opposite sign to neutralize the satellite. Kaganovich et.al [9] suggest that it is possible to neutralize a beam with the help of electrons by 50-90% depending on how the electrons are fired in comparison to the beam.

3.3 Curvature due to Earth's magnetic field

When firing charged particles in space, the curvature due to Earth's magnetic field has to be accounted for. The curving force is determined by Lorentz's formula,

$$\vec{F} = q\vec{v} \times \vec{B}. \tag{7}$$

Considering an arbitrary xenon ion, its velocity can be split into two components, one parallel to, and one perpendicular to the magnetic field lines, $v_\parallel$ and $v_\perp$ respectively. The perpendicular component will be affected by equation 7 and form a circular trajectory of radius r_c according to the relation with the centrifugal force $F = \frac{mv^2}{r}$,

$$r_c = \frac{mv_\perp}{qB}. \tag{8}$$

On the contrary, the parallel component will not be affected by the magnetic field and thus remain in its original direction. The total velocity will therefore result in a trajectory the shape of a helix,

$$\vec{r}(t) = (r_c \cos(\omega t), -r_c \sin(\omega t), v_\parallel t) \tag{9}$$

where $\omega = \frac{qB}{m}$ and the z-axis is placed along the magnetic field lines which are approximated as straight and infinite. The range is limited in the radial direction by the diameter of the circle defined by equation

8, $2r_c$. This value can be maximized by letting $v_\perp = v$ but will remain finite. Along the field lines however the range is infinite, although utilizing this range is not necessarly desirable. As shown above, despite zero beam divergence, spreading will still occur and limit us to a certain firing range, here denoted L. The helix shape gives

$$L^2 = r^2\phi^2 + \frac{v_\parallel^2 r^2 \phi^2}{v_\perp} \tag{10}$$

where ϕ is the curved angle around the z-axis ($\phi = 2\pi$ would mean orbiting the z-axis once). Rearranging and using equation 8 yields

$$\phi = \frac{LqB}{mv} \tag{11}$$

meaning that for a fixed L, the curve of the trajectory is independent of the ration between $v_\perp$ and $v_\parallel$ as long as the beam is not parallel to the magnetic field (see appendix 8.2 for details).

Continuing, the vector from thruster to target is denoted as $\vec{d}$. It is now clear that the projection of $\vec{d}$ onto the xy-plane is $d_{xy} = 2r_c|\sin\left(\frac{\omega L}{2v}\right)|$ and that $\vec{d}_z = \frac{v_\parallel L}{v}$. This yields

$$d^2 = 4r_c^2 \sin^2\left(\frac{\omega L}{2v}\right) + \left(\frac{v_\parallel L}{v}\right)^2 =$$
$$= \frac{4m^2v^2 \sin^2(\beta)}{q^2 B^2} \sin^2\left(\frac{qBL}{2mv}\right) + L^2 \cos^2(\beta) \tag{12}$$

where $\beta \in [0, \pi]$ is the firing angle of the beam measured between $\vec{v}$ and $\hat{z}$.

Considering instead the angle between $\vec{d}$ and $\hat{z}$, $\theta \in [0, \pi]$, the distance is calculated to be

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$$d^2 = \frac{L^2}{\cos^2(\theta)}\left(1 - \frac{\omega^2 r_c^2}{v^2}\right) = \frac{L^2}{\cos^2(\theta)}\left(1 - \frac{v_\perp^2}{v^2}\right) \tag{13}$$

Equating (12) and (13) allows for an expression of the firing angle as a function of the position of the debris to be derived.

$$\beta = \arcsin\left(\sqrt{\frac{L^2 q^2 B^2 (1 - \cos^2(\theta))}{4m^2 v^2 \cos^2(\theta)\sin^2(\frac{qBL}{mv}) - L^2 q^2 B^2 \cos^2(\theta) + L^2 q^2 B^2}}\right) \tag{14}$$

The angle θ will be a function of time since there is a limit on the mass output per second in the DS4G-thruster. To calculate the exact direction, dependent on time, in which to fire will require a careful analysis of the trajectory of both debris and satellite and precise knowledge of the engineering limits of the engine. This is left out in this report.

At impact, only the force-component perpendicular to the trajectory of the debris will contribute to retardation. Thus, a maximum range with the angle of incidence considered is of interest but this is highly dependent of the trajectory of the targeted debris. The criterion reads

$$\frac{\vec{v}_{Xe} \cdot \vec{v}_{debris}}{v_{Xe} v_{debris}} \geq \cos\alpha \tag{15}$$

for allowed angle of attack α.

3.4 Maneuvering

Since the allowed firing distance is limited, the satellite must come close to the debris. When maneuvering satellites, two changes are considered; the change of altitude and the change of angle. A change of altitude is relatively cheap in fuel and is performed by the Hohmann maneuver. The maneuver utilizes two velocity changes in order to change altitude, the first to leave the current orbit and a second to exit the elliptical transfer orbit and enter a new circular orbit.

$$\Delta v_1 = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{2r_2}{r_1 + r_2}} - 1\right) \tag{16}$$

$$\Delta v_2 = \sqrt{\frac{\mu}{r_2}}\left(1 - \sqrt{\frac{2r_1}{r_1 + r_2}}\right) \tag{17}$$

where r_1 is the initial orbital altitude and r_2 is the new orbital altitude.

Changing the angle, i, which is either the angle between the orbit and the equatorial latitude, inclination, or the longitude angle, right ascension of the ascending node (RAAN), is significantly more costly. The velocity change is

$$\Delta v_i = 2v_{initial} \sin\left(\frac{\theta}{2}\right) \tag{18}$$

where θ is the absolute value of the difference in angle, $\theta = |i_{new} - i_{initial}|$. See
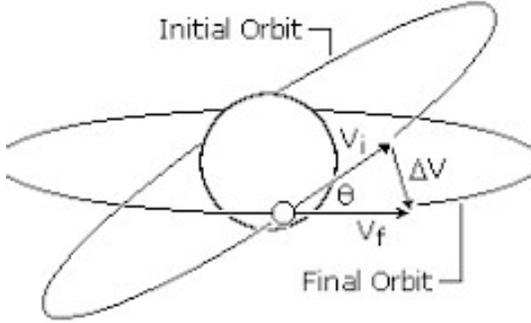
figure 3 for illustration.

Figure 3: The inclination angle θ is marked in the image. RAAN is the corresponding longitude.

Furthermore, due to the conservation of momentum, the satellite will accelerate itself when firing ions at the debris. Considering the firing process as a constant thrust, F_{thrust}, over time, t. The change in velocity becomes

$$\Delta v = \frac{F_{thrust} \cdot t}{m} \qquad (19)$$

where m is the mass of the spacecraft and considered constant during the relatively small impulse. This single change in velocity will have similar effect as on the debris; an elliptical orbit will be created, however this one being larger than the previous circular one. Thus, equation 3 can be used and solved for r_2. The change in altitude as a function of Δv can be found in figure 4. As equation 3 only depends on the mass of Earth, this plot looks the same for both satellite and debris. The difference is found in equation 19 where $m_{satellite}$ could be 10,000 times larger than m_{debris}.
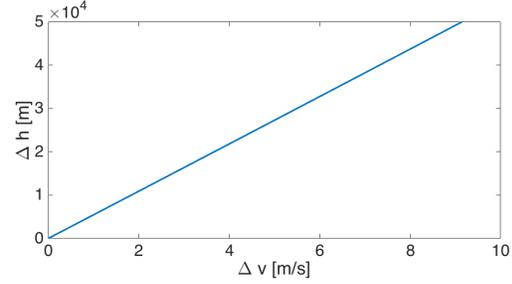

Figure 4: Change in orbital altitude, Δh as a function of changed velocity. Initial altitude $h_1 = 800\,000$ [m].

3.5 Xenon usage

To estimate the consumption of xenon, both the xenon fired at debris as well as the xenon used for maneuvering have to be considered. Using the specific impulse, I_{sp}, as well as the standard gravity, $g_0 = 9.80665$ m/s^2, the thrusters mass-flow can be expressed as

$$\dot{m} = \frac{F_{thrust}}{g_0\ I_{sp}}. \qquad (20)$$

Furthermore, the rocket equation uses the same g_0 and I_{sp} when relating the velocity difference to the fuel required to perform a maneuver

$$\Delta v = v_e \ln\left(\frac{m_0}{m_f}\right) = g_0 I_{sp} \ln\left(\frac{m_0}{m_f}\right). \qquad (21)$$

Here, v_e is the exhaust velocity of the propellant, m_0 is the initial mass including propellant, and m_f is the final mass of the satellite.

An arbitrary maneuver when targeting a piece of debris includes both a change of orbital altitude, using the Hohmann maneuver, equations 16 and 17, and also an

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adjustment of inclination angle, equation 18. Denoting initial and final orbital radii as r_1 and r_2, the angle change θ and the satellite mass m_0, the xenon consumption, m_{Xe} is

$$m_{Xe} = m_0 \left(1 - e^{-(\Delta v_1 + \Delta v_2 + \Delta v_i)/g_0 I_{sp}}\right) \tag{22}$$

using the rocket equation, (21), for each of the velocity changes.

Once in position, the cannon will fire until sufficient momentum, Δp, has been transfered. Assuming the thruster force, F, is constant, using $\dot{p} = F$, and introducing a hit-factor, f_h (this represents the percentage of the original thrust that makes it to the target), the amount of xenon used is,

$$m_{Xe-firing} = \frac{Ft}{g_0 I spf_h} = \frac{\Delta p}{g_0 I_{sp} f_h}. \tag{23}$$

Adding equations yields the total fuel used.

4 Results

4.1 Required Δv

In figure 5 the required velocity losses are plotted for different starting altitudes, equation 3, within low earth orbit. In figure 1 it is clear that the highest density of debris is found, with a sharp peak, at altitudes around 800 km corresponding to $\Delta v = 86$ m/s. The inner altitude $h_2 = 350$ km is chosen as an altitude where the atmosphere is dense enough to finish the process within a few months [10].
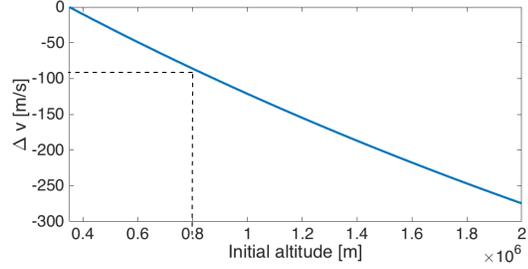

Figure 5: Required Δv to achieve a lower orbital radius $r_2 = R + 3.5 \cdot 10^5$m, as a function of initial altitude, $h_1 = r_1 - R$, where R is Earth's radius (see figure 2). The x-axis covers all LEO altitudes above 350 km and the dotted line follows the altitude with greatest debris density, 800 km.

4.2 Aiming

Using the fact that the magnetic field strength at 800 km altitude has roughly 40 nT [11] as its maximum near the poles and the exhaust velocity is 200.000 m/s, a burst perpendicular to the field will, according to equation 8, allow for a curvature radius of $\sim$ 850.000 m. Thus, the double radii maximum range is irrelevant compared to the range due to beam divergence. Furthermore, using equation 11 with $L = 100$ meters, the curve will be about 0.007° regardless of firing angle. Thus the magnetic field can be neglected until longer range is achieved.

Assuming a 1% hit factor the allowed firing distance for an electron beam, also assuming uniformly distributed space charge, is about 40 m. Based on the data provided in 3.2 it is assumed that the spreading of the xenon beam can be made lower than that of an electron beam, since the mass is many magnitudes greater and the space-charge
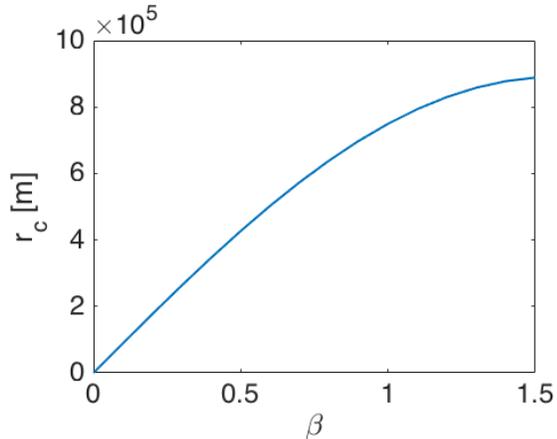
can be significantly neutralized.

Figure 6: The curvature radius due to the magnetic field, r_c, as a function of different firing angles, β measured in radians.

4.3 Xenon usage

Figure 1 shows the peak density of debris as approximately 10^{-6}pieces/km^3. Based on this, an assumption of the distance needed to travel from piece to piece can be made. Based on the peak debris density it can be calculated that at 800 km altitude there are approximately 650 pieces of debris per km altitude of the size 1-10 cm in diameter. Most of the debris is concentrated around certain orbits and it is assumed, based on figure 7 that there are around 400 pieces of debris in the interval $i = [90°, 100°]$. Also, according to figure 7 the debris is uniformly spread across the other angle, RAAN. Based on this, it is assumed that the average change in inclination angle required to reach a piece is $\approx 3°$. This answers to an average mass output, using equation 19, of the n:th maneuver being $m_n \approx 0.0013m_0$, where m_0 is the current mass of the spacecraft, needed to adjust

the orbit from one piece to another. In comparison this is approximately the same mass needed to change altitude from 800 to 850 km. It is noted that moving in the radial direction instead of the azimuthal is more efficient. Therefore, the overall xenon usage might be reduced by first adjusting the altitude to minimize θ. It is furthermore not sufficient to be in the same orbit as the piece, the thruster also has to be within firing range. Precision maneuvering and a safety margin is added as a factor 2, bringing mass output to $m = 0.0025m_0$.
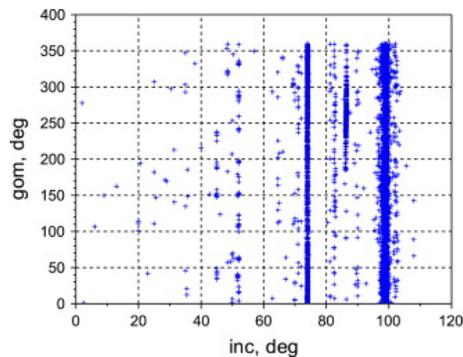

Figure 7: Space debris dispersion of RAAN for different inclinations. Circa 900 pieces covered. [12]

The Xenon needed to bring down the debris is calculated according to 23 with the hit-factor, $f_h = 0.01$, as argued in 3.2. The velocity of the debris at the altitude 800 km is roughly 5 km/s and it is assumed that the angle of attack presented in equation 15 is never worse than 60° so that mass needed will increase by a factor 2 at most. Using these assumptions the mass output per kilogram debris will be 1.07. A piece is assumed to weigh on average 1 kg and the mass output for breaking will therefore be 1.07 kg per piece.

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5 Discussion

Removing space debris with a focused xenon-beam is beneficial in many ways as suggested by previous calculations. First of all, it does not require physical contact with the debris (although decreasing firing distance would certainly improve performance). Furthermore, there is already existing research and knowledge regarding xenon-fueled thrusters which is of essence as modifications to existing equipment is needed. The method is applicable on a variety of debris as it relies on the basic principle momentum conservation (although breaking larger debris would require a greater mass flow). Considering that the debris is distributed mainly at certain orbits, smart maneuvering might allow a reduction in fuel consumption and so more debris can be brought down.

5.1 Limitations

Tracking debris from Earth is today possible using radar systems but maneuvering is nevertheless the main contributer to mass output. Positioning will be the most demanding part of the process.

Since the hit factor is low, even for short firing distances, the firing process takes about 28 h (based on the current hit factor and distance) and so the satellite would be required to follow the debris closely over a long time. The model used to calculate Δv was based on the assumption that the momentum was transferred instantaneously. For the current time required this is clearly wrong and would result in extensive setbacks such as higher required Δv.

This will also require additional maneuvering which has not been taken into account when calculating xenon usage. In order to shorten the time needed the hit-factor and the mass flow would have to be increased. A more focused burst would not only lead to less maneuvering but less fired xenon as well. If possible, it might be beneficial to position the satellite closer than 40 m initially, in order to increase the hit factor and therefore reduce both firing and maneuvering.

The assumptions made when deciding the hit factor should be considered to be conservative. It was assumed that the space-charge was uniformly distributed over the cross sectional area of the beam. This is definitely a poor estimate considering how the radius was defined. The beam will be denser towards the center, but the exact distribution remains unknown. Notwithstanding, beam spreading was assumed only due to Coulomb forces and factors like Maxwellian velocity distribution are neglected. This suggests that the spreading might not be too underestimated. Still, we believe losses to be smaller in real application.

5.2 Modifications

The DS4G has shown performance desirable for the purpose of this report, mainly in terms of high exhaust velocity and mass output per second. It is important to keep in mind that the DS4G engine is not yet in use. It has however, in several experiments, proven to be of interest for future space missions. The modifications needed for our use of the engine has not yet been discussed. Throughout this report it has

been assumed that it is possible to make the beam completely parallel in accordance with equation 4. However changing the perveance means changing the geometrical properties of the thruster. If and how this affects performance has been left out and calculations have been performed under the assumption that the effect on performance is, or can be made, negligible.

An engineering improvement which would rapidly increase the hit factor is the neutralization of the beam. In theory (once again assuming only Coulomb forces affect the spread) a neutralized beam would not spread, clearly increasing the allowed firing distance. Neutralizing more than the 90% suggested by [9] could greatly improve performance, especially considering that spreading depends on the distance fired squared.

6   Summary and Conclusions

The results of this paper shows that it seems possible to bring down one piece of debris using an electrostatic ion thruster provided that certain improvements are made. The calculations made in this report has not been optimistic and most calculations has been a worst-case scenario. Despite this, if at a distance of 40 m from the debris the mass output is approximately 1.07 kg per kg debris. Even if the distance needs to be increased it would still be possible to bring down a single piece of debris due to the high amount of xenon able to bring to space. It is further concluded that this method can be repeated in order to bring down multiple pieces of debris. In this worst case scenario it was estimated that, with 9 tonnes of xenon and a total satellite weight of 10 tonnes, about 800 pieces of debris with the average size of 1 kg can be brought down using this method. A report by NASA [13] suggests that only 5 pieces a year need to be removed in order to keep the debris at a stable level which has been shown possible.

7   Acknowledgements

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8 Appendix

8.1 Δv

The specific energy is

$$\epsilon = v^2 - \frac{\mu}{r} = \frac{-\mu}{2a} \tag{24}$$

where ϵ is the total energy, a is the semi-major axis, r the radius, v the velocity and $\mu = G(M + m)$ is the sum of the masses weighed by gravitational constant. Defining outer circular orbit gives,

$$\epsilon_1 = v_1^2 - \frac{\mu}{r_1} = \frac{-\mu}{2a_1} \implies v_1 = \sqrt{\frac{-\mu}{2r_1} + \frac{\mu}{r_1}} = \sqrt{\frac{\mu}{2r_1}} \tag{25}$$

since $a = r_1$. For the inner elliptical orbit $2a = r_1 + r_2$ which yields,

$$\epsilon_2 = v_2^2 - \frac{\mu}{r_1} = \frac{-\mu}{2a_2} \implies v_2 = \sqrt{\frac{-\mu}{r_1 + r_2} + \frac{\mu}{r_1}} = \sqrt{\frac{r_2\mu}{r_1(r_1 + r_2)}}. \tag{26}$$

The difference is,

$$\Delta v = v_1 - v_2 = \sqrt{\frac{\mu}{2r_1}} - \sqrt{\frac{r_2\mu}{r_1(r_1 + r_2)}}. \tag{27}$$

8.2 The curvatures independence of firing angle

A helix has the curve length

$$L^2 = (r\phi)^2 + \left(\frac{dz}{d\phi}\phi\right)^2 \tag{28}$$

where ϕ is the rotation angle. Since $\frac{dz}{d\phi} = \frac{v_\parallel r}{v_\perp}$, rearranging gives

$$\phi^2 = \frac{L^2}{r^2 + \frac{v_\parallel^2 r^2}{v_\perp^2}} = \frac{L^2 v_\perp^2}{r^2(v_\parallel^2 + v_\perp^2)} = \frac{L^2 v_\perp^2}{r^2 v^2}. \tag{29}$$

Using equation 8, $r = \frac{mv_\perp}{qB}$ yields

$$\phi = \frac{LqB}{mv}. \tag{30}$$

which does not depend on aiming.