

Adam Zdunek “A computational form of Spencer’s theory for anisotropic finite hyperelasticity”

R.S. Rivlin is one of the main contributors to nonlinear continuum mechanics: His work on the mechanics of rubber (in the 1940s and 50s) established the basis of isotropic finite hyperelasticity [1]. A.J.M. Spencer extended that work to anisotropic finite hyperelasticity [2]. He introduced extended invariant integrity bases describing fibre reinforced materials (in the 1970s and 80s). It took almost 40-years before the computational form of Rivlin’s work was formulated by Simo Taylor and Pister [3] and others. Using separate volumetric– isochoric (unimodular) deformation measures and induced conjugate spherical and deviatoric stress-like variables they introduced the frequently used three-field Hu-Washizu principle for isotropic nearly-incompressible finite hyperelasticity (rubber elasticity, soft tissue ground substance, plasticity etc). The extension and a computational setting of Spencer’s formulation of combined almost inextensible transversely isotropic and nearly incompressible finite hyperelasticity by Zdunek, Rachowicz and Eriksson [4], is presented. It relies on the construction of a unimodular and simply stretch-free deformation measure. Separate measures for volume change and volume-preserving stretch in the preferred direction are added. The kinematic set induces the classical orthogonal decomposition into spherical and deviatoric stresses. It further induces a new decomposition of the deviatoric stresses into fibre tension and stresses orthogonal to the fibre tension. The fully constrained case dictates decoupled reactive and work-performing stress responses. The presented formulation properly predicts a trivial ground substance stress response in the inextensible direction, contrary to the simple volumetric–isochoric theory. The formulation [4] is coherent with the algebraic–geometric constraint manifold based theory by Podio-Guidugli, [5], Carlson et al [6]. Notably, the limiting reactive stresses are orthogonal to the work performing stresses, as opposed to the classic Lagrange multiplier formulation, see eg. [2]. Finally, the known inability of the standard volumetric-isochoric theory to predict proper compressible anisotropic deformations is addressed and resolved, see Figure 1, and Vergori et al [7].



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Zdunek obtained a M.S. in chemical engineering in 1975 at Lund University of Technology (LTH), Sweden. In 1995 he obtained his Ph.D. and in 2004 his docent degree, both at the Royal Institute of Technology (KTH), Sweden. He worked as a research engineer in rubber engineering at Trelleborg AB from 1975-1979, a consultant in structural dynamics and acoustics at IFM Akustikbyrån AB from 1979-1983, and since 1983 he has been with the Swedish Defence Research Agency, FOI in Stockholm. He is currently holding a position as Dept. Research Director at the Information & Aeronautical Systems department. His research area up to the millennium was computational solid mechanics (CSM) and since then it is enlarged to include computational electromagnetics (CEM) using adaptive hp-FE methods.

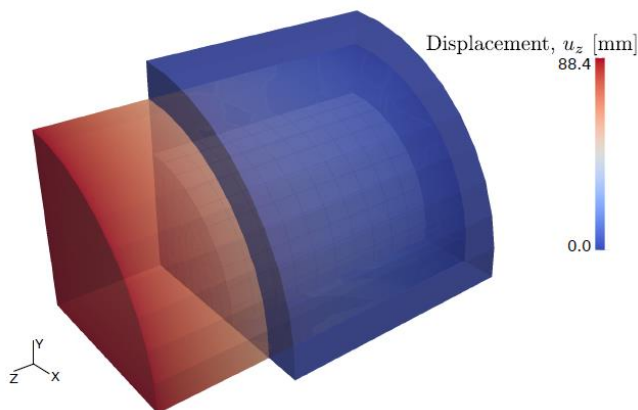


Figure 1: Isostatic loading of a compressible axially inextensible solid circular cylinder. Brick element mesh and deformed configurations. White cylinder – undeformed configuration. Red cylinder – deformed configuration volumetric–isochoric Standard Reinforcing Material (SRM) model. Blue cylinder – deformed configuration Generalised SRM model (GSRM). The SRM model violates the axial inextensibility, while the GSRM model [4], obeys it.

References

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