Principles of Wireless Sensor Networks

https://www.kth.se/social/course/EL2745/

Lecture 9

Dynamic Distributed Estimation

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Course content

- Part 1
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  - Lec 2: Introduction to Programming WSNs
- Part 2
  - Lec 3: Wireless Channel
  - Lec 4: Physical Layer
  - Lec 5: Medium Access Control Layer
  - Lec 6: Routing
- Part 3
  - Lec 7: Distributed Detection
  - Lec 8: Static Distributed Estimation
  - Lec 9: Dynamic Distributed Estimation
  - Lec 10: Positioning and Localization
  - Lec 11: Time Synchronization
- Part 4
  - Lec 12: Wireless Sensor Network Control Systems 1
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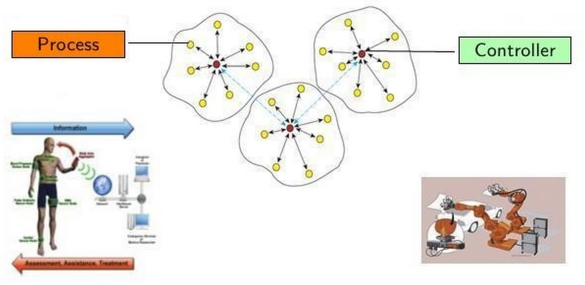
  - Lec 14: Summary and Project Presentations

Previous lecture

- Star and general topology
- Estimation from one sensor
- Distributed estimation in a start topology
- Distributed estimation in a general topology

Today's lecture



- Today we study how to perform dynamic estimation from erroneous or noisy measurements of the sensors
- "Dynamic" means that we take advantage of the time evolution of signals to build the estimators

Today's learning goals

- How to perform estimation of a dynamic signal from one sensor?
- How to perform estimation of a dynamic signal from many sensors?
- How to make a sensor fusion of a dynamic signal by the distributed Kalman filter?

Outline

- Dynamic estimation from one sensor
- Dynamic estimation from many sensors in a star topology
- Dynamic estimation from many sensors by the distributed Kalman filter

Outline

- Dynamic estimation from one sensor
- Dynamic estimation from many sensors in a star topology
  - Dynamic sensor fusion, centralized setup
  - Dynamic sensor fusion, centralized setup (drawbacks)
  - Dynamic sensor fusion, distributed Kalman filtering

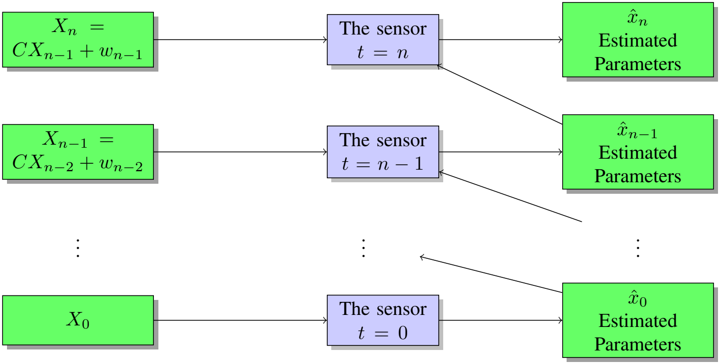


Figure: Illustration of how the fusion of sequential measurement works to combine measurements in one sensor.

- We want to combine many dynamic measurements in one sensor

Dynamic estimation from one sensor

Proposition 1

Consider a phenomenon $\mathbf{x}$ *evolving in time (indexed by* n*) according to*

$$\mathbf{x}_{n+1} = \mathbf{A}\mathbf{x}_n + \mathbf{w}_n$$

Dynamic estimation from one sensor

Proposition 1

Consider a phenomenon $\mathbf{x}$ *evolving in time (indexed by* n*) according to*

$$\mathbf{x}_{n+1} = \mathbf{A}\mathbf{x}_n + \mathbf{w}_n$$

Every time step sensor generates a measurement of the form

$$\mathbf{y}_n = \mathbf{C}\mathbf{x}_n + \mathbf{v}_n$$

- $\mathbf{w}_n$: *white zero mean Gaussian with covariance matrix* $E\{\mathbf{w}_n\mathbf{w}_n^T\} = \mathbf{Q}$
- $\mathbf{v}_n$: *white zero mean Gaussian with covariance matrix* $E\{\mathbf{v}_n\mathbf{v}_n^T\} = \mathbf{R}$
- $\mathbf{A}$: *a known nonsingular matrix*
- $\mathbf{C}$: *a known matrix*

Dynamic estimation from one sensor

Proposition 1

Then we have

$$\hat{\mathbf{x}}_{n|n-1} = \mathbf{A}\hat{\mathbf{x}}_{n-1|n-1}$$
$$\mathbf{P}_{n|n-1} = \mathbf{A}\mathbf{P}_{n-1|n-1}\mathbf{A}^T + \mathbf{Q}$$

- $\hat{\mathbf{x}}_{n-1|n-1}$: *estimate of* $\mathbf{x}_{n-1}$ *given* $\mathbf{z} = (\mathbf{y}_0, \ldots, \mathbf{y}_{n-1})$
- $\mathbf{P}_{n-1|n-1}$: *corresponding error covariance matrix*

Proof of proposition 1

Question: How to show that $\hat{\mathbf{x}}_{n|n-1} = \mathbf{A}\hat{\mathbf{x}}_{n-1|n-1}$?

Proof of proposition 1

Question: How to show that $\hat{\mathbf{x}}_{n|n-1} = \mathbf{A}\hat{\mathbf{x}}_{n-1|n-1}$?

Answer:
Use a well known result: MMSE estimate $\hat{\mathbf{x}}$ of a random variable $\mathbf{x}$ given a random variable $\mathbf{y}$ is $\mathsf{E}\{\mathbf{x}|\mathbf{y}\}$

$$\begin{aligned}
\hat{\mathbf{x}}_{n|n-1} &= \mathsf{E}\left\{\mathbf{x}_n|(\mathbf{y}_0,\ldots,\mathbf{y}_{n-1})\right\} \\
&= \mathsf{E}\left\{\mathbf{A}\mathbf{x}_{n-1} + \mathbf{w}_{n-1}|\mathbf{z})\right\} \\
&= \mathbf{A}\mathsf{E}\left\{\mathbf{x}_{n-1}|\mathbf{z}\right\} \\
&= \mathbf{A}\hat{\mathbf{x}}_{n-1|n-1}
\end{aligned}$$

Proof of proposition 1

Question: How to show that $\mathbf{P}_{n|n-1} = \mathbf{A}\mathbf{P}_{n-1|n-1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$?

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Answer:
By the definition of error covariance, we already have

$$\mathbf{P}_{n-1|n-1} = \mathsf{E}\{(\hat{\mathbf{x}}_{n-1|n-1} - \mathbf{x}_{n-1})(\hat{\mathbf{x}}_{n-1|n-1} - \mathbf{x}_{n-1})^{\mathsf{T}}\}$$

$$\begin{aligned}
\mathbf{P}_{n|n-1} &= \mathsf{E}\{(\hat{\mathbf{x}}_{n|n-1} - \mathbf{x}_{n})(\hat{\mathbf{x}}_{n|n-1} - \mathbf{x}_{n})^{\mathsf{T}}\} \\
&= \mathsf{E}\{(\mathbf{A}\hat{\mathbf{x}}_{n-1|n-1} - \mathbf{A}\mathbf{x}_{n-1} - \mathbf{w}_{n-1})(\mathbf{A}\hat{\mathbf{x}}_{n-1|n-1} - \mathbf{A}\mathbf{x}_{n-1} - \mathbf{w}_{n-1})^{\mathsf{T}}\} \\
&= \mathsf{E}\{\mathbf{A}(\hat{\mathbf{x}}_{n-1|n-1} - \mathbf{x}_{n-1})(\hat{\mathbf{x}}_{n-1|n-1} - \mathbf{x}_{n-1})^{\mathsf{T}}\mathbf{A}^{\mathsf{T}} + \mathbf{w}_{n-1}\mathbf{w}_{n-1}^{\mathsf{T}}\} \\
&= \mathbf{A}\mathbf{P}_{n-1|n-1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}
\end{aligned}$$

An observation about $x_{n-1|n-1}$

$\hat{\mathbf{x}}_{n-1|n-1}$: estimate of $\mathbf{x}_{n-1}$ given $\mathbf{z} = (\mathbf{y}_0, \ldots, \mathbf{y}_{n-1})$?

$$
\begin{aligned}
\mathbf{y}_{n-1} &= \mathbf{C}\mathbf{x}_{n-1} + \mathbf{v}_{n-1} \\
\mathbf{y}_{n-2} &= \mathbf{C}\mathbf{x}_{n-2} + \mathbf{v}_{n-2} = \mathbf{C}(\mathbf{A}^{-1}(\mathbf{x}_{n-1} - \mathbf{w}_{n-2})) + \mathbf{v}_{n-2} \\
&= \mathbf{C}\mathbf{A}^{-1}\mathbf{x}_{n-1} + \left(\mathbf{v}_{n-2} - \mathbf{C}\mathbf{A}^{-1}\mathbf{w}_{n-2}\right) \\
&\vdots \\
\mathbf{y}_0 &= \mathbf{C}\mathbf{A}^{-(n-1)}\mathbf{x}_{n-1} + \left(\mathbf{v}_0 - \mathbf{C}\mathbf{A}^{-(n-1)}\mathbf{w}_{n-2} - \cdots - \mathbf{C}\mathbf{A}^{-1}\mathbf{w}_0\right)
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\end{aligned}$$

i.e., the overall linear system is given by

$$\underbrace{\begin{bmatrix} \mathbf{y}_{n-1} \\ \vdots \\ \mathbf{y}_0 \end{bmatrix}}_{\mathbf{z}} = \underbrace{\begin{bmatrix} \mathbf{C} \\ \vdots \\ \mathbf{C}\mathbf{A}^{-(n-1)} \end{bmatrix}}_{\mathbf{H}} \mathbf{x}_{n-1} + \underbrace{\begin{bmatrix} \mathbf{v}_{n-1} \\ \vdots \\ \mathbf{v}_0 - \cdots - \mathbf{C}\mathbf{A}^{-1}\mathbf{w}_0 \end{bmatrix}}_{\mathbf{u}}$$

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From Proposition 1 (Lecture 8)

$$\mathbf{P}^{-1}_{n-1|n-1}\hat{\mathbf{x}}_{n-1|n-1} = \mathbf{H}^{\mathsf{T}}\mathbf{R}_{\mathbf{u}}^{-1}\mathbf{z}\ \text{, where}$$
$$\mathbf{P}_{n-1|n-1} = \left(\mathbf{R}^{-1}_{\mathbf{x}_{n-1}} + \mathbf{H}^{\mathsf{T}}\mathbf{R}_{\mathbf{u}}^{-1}\mathbf{H}\right)^{-1}$$

Dynamic estimation from one sensor

Question: $\hat{\mathbf{x}}_{n|n}$, the MMSE estimate of $\mathbf{x}_n$ given $(\mathbf{y}_0, \dots, \mathbf{y}_{n-1}, \mathbf{y}_n) = (\mathbf{z}, \mathbf{y}_n)$

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Answer: Apply Proposition 2 (lec 8, Static Sensor Fusion) in a straightforward manner

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we **already know** $\hat{\mathbf{x}}_{n|n-1}$, i.e., the estimate of $\mathbf{x}_n$ given $\mathbf{z}$
we **already know** $\mathbf{P}_{n|n-1}$, the corresponding error covariance matrix

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we **need** the estimate of $\mathbf{x}_n$ given $\mathbf{y}_n$, denote it by $\hat{\mathbf{x}}$
we **need** the corresponding error covariance matrix, denote it by $\mathbf{M}$

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we **need** the corresponding error covariance matrix, denote it by $\mathbf{M}$
because $\mathbf{y}_n = \mathbf{C}\mathbf{x}_n + \mathbf{v}_n$, from Proposition 1 (Lecture 8), we have

$$\hat{\mathbf{x}} = \mathbf{M}\mathbf{C}^{\mathsf{T}}\mathbf{R}^{-1}\mathbf{y}_n \text{ , where } \mathbf{M} = \left(\mathbf{R}_{\mathbf{x}_n}^{-1} + \mathbf{C}^{\mathsf{T}}\mathbf{R}^{-1}\mathbf{C}\right)^{-1}$$

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from Proposition 2 (Leecture 8)

Result

$$\mathbf{P}_{n|n}^{-1}\hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \mathbf{M}^{-1}\hat{\mathbf{x}} \text{ , where}$$

$$\mathbf{P}_{n|n}^{-1} = -\mathbf{R}_{\mathbf{x}_n}^{-1} + \mathbf{P}_{n|n-1}^{-1} + \mathbf{M}^{-1}$$

Dynamic estimation from one sensor

- **Time** and **measurement** update steps of the **Kalman filter**
- **Kalman filter** can be seen to be a **combination of estimators**
- **Optimality** of the **Kalman filter** in the **minimum mean squared** sense

Outline

- Dynamic estimation from one sensor
- Dynamic estimation from many sensors in a star topology
  - Dynamic sensor fusion, centralized setup
  - Dynamic sensor fusion, centralized setup (drawbacks)
  - Dynamic sensor fusion, distributed Kalman filtering

Dynamic sensor fusion

Consider a phenomenon $\mathbf{x}$ evolving in time (indexed by n) according to the law

$$\mathbf{x}_{n+1} = \mathbf{A}\mathbf{x}_n + \mathbf{w}_n$$

Dynamic sensor fusion

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Every time step, sensor k generates a measurement of the form

$$\mathbf{y}_{n,k} = \mathbf{C}_k\mathbf{x}_n + \mathbf{v}_{n,k}$$

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- **Multiple sensors** that generate measurements about the random variable that is evolving in time

- **Question:** How to **fuse data** from all the sensors for an estimate of the state $\mathbf{x}_n$ at time step n?

Dynamic sensor fusion, centralized setup

- At every time step n, all the sensors **transmit** their measurements $\mathbf{y}_{n,k}$ to **a central node**
- The central node implements a fusion mechanism
- However, there are two reasons why this **may not be the preferred** implementation

(1) number of sensors increases $\Rightarrow$ computational effort required at the central node increases (bear some of the computational burden at sensors)

(2) the sensors may not be able to transmit at every time step (transmit local processed information rather that raw measurements)

Dynamic sensor fusion, centralized setup (transmitting local estimates)

- Assume that the sensors can transmit at every time step

- Reducing the computational burden at the central node?

Dynamic sensor fusion, centralized setup (transmitting local estimates)

Let $\mathbf{y}_k = (\mathbf{y}_{0,k}, \mathbf{y}_{1,k}, \ldots, \mathbf{y}_{n,k})$ denote the measurements from sensor k that is used to estimate $\mathbf{x}_n$

Dynamic sensor fusion, centralized setup (transmitting local estimates)

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Potential method 1

The overall linear system is given by

$$\underbrace{\begin{bmatrix} \mathbf{y}_{n,k} \\ \mathbf{y}_{n-1,k} \\ \vdots \\ \mathbf{y}_{0,k} \end{bmatrix}}_{\mathbf{y}_k} = \underbrace{\begin{bmatrix} \mathbf{C}_k \\ \mathbf{C}_k \mathbf{A}^{-1} \\ \vdots \\ \mathbf{C}_k \mathbf{A}^{-n} \end{bmatrix}}_{\mathbf{H}_k} \mathbf{x}_n + \underbrace{\begin{bmatrix} \mathbf{v}_{n,k} \\ \mathbf{v}_{n-1,k} - \mathbf{C}_k \mathbf{A}^{-1} \mathbf{w}_{n-1} \\ \vdots \\ \mathbf{v}_{0,k} - \cdots - \mathbf{C}_k \mathbf{A}^{-1} \mathbf{w}_0 \end{bmatrix}}_{\mathbf{v}_k}$$

Dynamic sensor fusion, centralized setup (transmitting local estimates)

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- Process noise $\mathbf{w}_n$ appears in the noise $\Rightarrow$ the **measurement noises** $\mathbf{v}_k$ are **not independent** as desired
- $\mathbf{v}_k$ are not independent $\Rightarrow$ the **noise is correlated**
- $\Rightarrow$ **Proposition 2 (Lecture 8) does not apply** for combining local estimates

Dynamic sensor fusion, centralized setup (transmitting local estimates)

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Let $\mathbf{y}_k = (\mathbf{y}_{0,k}, \mathbf{y}_{1,k}, \ldots, \mathbf{y}_{n,k})$ denote the measurements from sensor k that is used to estimate $\mathbf{x}_n$

Potential method 2: Estimate of $\mathbf{x}_0$ known $\Rightarrow$ estimate of $\mathbf{x}_n$ known

The overall linear system is given by

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Dynamic sensor fusion, centralized setup (transmitting local estimates)

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- The **measurement noises** $\mathbf{v}_k$ are **independent** as desired
- $\Rightarrow$ **Proposition 2 (lec 8) does apply** for combining local estimates
- **Vectors** transmitted from sensors are **increasing in dimension** as the time step n increases

Dynamic sensor fusion, centralized setup (drawbacks)

- Practically, it is not feasible to combine local estimates from method 2 to obtain the global estimate
- i.e., lots of communication overhead
- If there is no process noise, then the method 1 will work
- However, in general it is not possible

Dynamic sensor fusion: Distributed Kalman filtering

- **Recall:** Sequential Measurements from One Sensor

Dynamic sensor fusion: Distributed Kalman filtering

- **Recall:** Sequential Measurements from One Sensor
- Random variable evolution: $\mathbf{x}_{n+1} = \mathbf{A}\mathbf{x}_n + \mathbf{w}_n$
- Measurements: $\mathbf{y}_n = \mathbf{C}\mathbf{x}_n + \mathbf{v}_n$, where $\mathsf{E}\{\mathbf{v}_n\mathbf{v}_n^\mathsf{T}\} = \mathbf{R}$

Dynamic sensor fusion: Distributed Kalman filtering

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- We have

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$$\mathbf{P}_{n|n}^{-1}\hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \mathbf{C}^\mathsf{T}\mathbf{R}^{-1}\mathbf{y}_n$$

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$$\hat{\mathbf{x}} = \mathbf{M}\mathbf{C}^\mathsf{T}\mathbf{R}^{-1}\mathbf{y}_n \text{ , where } \mathbf{M} = \left(\mathbf{R}_{\mathbf{x}_n}^{-1} + \mathbf{C}^\mathsf{T}\mathbf{R}^{-1}\mathbf{C}\right)^{-1}$$

$$\mathbf{P}_{n|n}^{-1}\hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \mathbf{C}^\mathsf{T}\mathbf{R}^{-1}\mathbf{y}_n$$

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \mathbf{C}^\mathsf{T}\mathbf{R}^{-1}\mathbf{C}$$

- The requirements from individual sensors are derived by the equations above

Dynamic sensor fusion: Distributed Kalman filtering

Proposition 2

Consider a random variable $\mathbf{x}_n$ *evolving in time as* $\mathbf{x}_n = \mathbf{A}\mathbf{x}_{n-1} + \mathbf{w}_{n-1}$ *being observed by* K *sensors in every time step* n*. Suppose they generate measurements of the form* $\mathbf{y}_{n,k} = \mathbf{C}_k \mathbf{x}_n + \mathbf{v}_{n,k}$*. Then the global error covariance matrix and the estimate are given in terms of the local covariances and estimates by*

$$\mathbf{P}^{-1}_{n|n} = \mathbf{P}^{-1}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1} \right)$$

$$\mathbf{P}^{-1}_{n|n}\hat{\mathbf{x}}_{n|n} = \mathbf{P}^{-1}_{n|n-1}\hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1}\hat{\mathbf{x}}_{n,k|n-1} \right)$$

Dynamic sensor fusion
Distributed Kalman filtering

Proof: Note that overall linear system is given by

$$\begin{bmatrix} \mathbf{y}_{n,1} \\ \vdots \\ \mathbf{y}_{n,K} \\ \mathbf{z}_1 \\ \vdots \\ \mathbf{z}_K \end{bmatrix} = \begin{bmatrix} \mathbf{C}_1 \\ \vdots \\ \mathbf{C}_K \\ \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_K \end{bmatrix} \mathbf{x}_n + \begin{bmatrix} \mathbf{v}_{n,1} \\ \vdots \\ \mathbf{v}_{n,K} \\ \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_K \end{bmatrix} \triangleq \begin{bmatrix} \mathbf{y}_n \\ \mathbf{z} \end{bmatrix} = \begin{bmatrix} \mathbf{C} \\ \mathbf{H} \end{bmatrix} \mathbf{x}_n + \begin{bmatrix} \mathbf{v}_n \\ \mathbf{u} \end{bmatrix}$$

Lets now simplify $\mathbf{C}^{\mathsf{T}}\mathbf{R}^{-1}\mathbf{y}_n$

$$\mathbf{C}^{\mathsf{T}}\mathbf{R}^{-1}\mathbf{y}_n = \begin{bmatrix} \mathbf{C}_1^{\mathsf{T}} \cdots \mathbf{C}_K^{\mathsf{T}} \end{bmatrix} \begin{bmatrix} \mathbf{R}_1^{-1} & 0 & \cdots & 0 \\ 0 & \mathbf{R}_2^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{R}_K^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{y}_{n,1} \\ \vdots \\ \mathbf{y}_{n,K} \end{bmatrix}$$

$$= \sum_{k=1}^{K} \mathbf{C}_k^{\mathsf{T}}\mathbf{R}_k^{-1}\mathbf{y}_{n,k}$$

$$= \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1}\hat{\mathbf{x}}_{n,k|n-1} \right)$$

$$\mathbf{C}^{\mathsf{T}}\mathbf{R}^{-1}\mathbf{C} = \sum_{k=1}^{K} \mathbf{C}_k^{\mathsf{T}}\mathbf{R}_k^{-1}\mathbf{C}_k$$

$$= \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$

Dynamic sensor fusion
Distributed Kalman filtering

Recap:

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$

$$\mathbf{P}_{n|n}^{-1}\hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1}\hat{\mathbf{x}}_{n,k|n-1} \right)$$

Based on the result above → **two architectures** for dynamic sensor fusion

- **Method 1: more** computation at the fusion center, **less** communication overhead
- **Method 2: less** computation at the fusion center, **more** communication overhead

Dynamic sensor fusion
Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$
$$\mathbf{P}_{n|n}^{-1} \hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

Dynamic sensor fusion
Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$
$$\mathbf{P}_{n|n}^{-1} \hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

Dynamic sensor fusion
Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$

$$\mathbf{P}_{n|n}^{-1} \hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

fusion center

Dynamic sensor fusion
Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$

$$\mathbf{P}_{n|n}^{-1} \hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

fusion center

Dynamic sensor fusion
Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1}\right)$$
$$\mathbf{P}_{n|n}^{-1}\hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1}\hat{\mathbf{x}}_{n,k|n-1}\right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{0,1|0}^{-1}, \hat{\mathbf{x}}_{0,1|0}$

$\mathbf{P}_{0,2|0}^{-1}, \hat{\mathbf{x}}_{0,2|0}$

fusion center

Dynamic sensor fusion
Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}^{-1}_{n|n} = \mathbf{P}^{-1}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1} \right)$$
$$\mathbf{P}^{-1}_{n|n} \hat{\mathbf{x}}_{n|n} = \mathbf{P}^{-1}_{n|n-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}^{-1}_{0,1|0}, \hat{\mathbf{x}}_{0,1|0}$
$\mathbf{P}^{-1}_{0,2|0}, \hat{\mathbf{x}}_{0,2|0}$ $\longmapsto$

fusion center

Dynamic sensor fusion
Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$
$$\mathbf{P}_{n|n}^{-1} \hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{0,1|0}^{-1}, \hat{\mathbf{x}}_{0,1|0}$
$\mathbf{P}_{0,2|0}^{-1}, \hat{\mathbf{x}}_{0,2|0}$ $\longmapsto$

fusion center

$\mathbf{P}_{1,1|0} = \mathbf{A}\mathbf{P}_{0,1|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$

Dynamic sensor fusion
Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$
$$\mathbf{P}_{n|n}^{-1}\hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1}\hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{0,1|0}^{-1}, \hat{\mathbf{x}}_{0,1|0}$
$\mathbf{P}_{0,2|0}^{-1}, \hat{\mathbf{x}}_{0,2|0}$

$\longmapsto$

fusion center

$$\mathbf{P}_{1,1|0} = \mathbf{A}\mathbf{P}_{0,1|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{1,1|0} = \mathbf{A}\hat{\mathbf{x}}_{0,1|0}$$

Dynamic sensor fusion
Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}^{-1}_{n|n} = \mathbf{P}^{-1}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1} \right)$$
$$\mathbf{P}^{-1}_{n|n}\hat{\mathbf{x}}_{n|n} = \mathbf{P}^{-1}_{n|n-1}\hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1}\hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}^{-1}_{0,1|0}, \hat{\mathbf{x}}_{0,1|0}$
$\mathbf{P}^{-1}_{0,2|0}, \hat{\mathbf{x}}_{0,2|0}$ $\longmapsto$

fusion center

$$\mathbf{P}_{1,1|0} = \mathbf{A}\mathbf{P}_{0,1|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{1,1|0} = \mathbf{A}\hat{\mathbf{x}}_{0,1|0}$$
$$\mathbf{P}_{1,2|0} = \mathbf{A}\mathbf{P}_{0,2|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$

Dynamic sensor fusion
Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$
$$\mathbf{P}_{n|n}^{-1} \hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{0,1|0}^{-1}, \hat{\mathbf{x}}_{0,1|0}$

$\mathbf{P}_{0,2|0}^{-1}, \hat{\mathbf{x}}_{0,2|0}$

$\longmapsto$

fusion center

$$\mathbf{P}_{1,1|0} = \mathbf{A}\mathbf{P}_{0,1|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{1,1|0} = \mathbf{A}\hat{\mathbf{x}}_{0,1|0}$$
$$\mathbf{P}_{1,2|0} = \mathbf{A}\mathbf{P}_{0,2|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{1,2|0} = \mathbf{A}\hat{\mathbf{x}}_{0,2|0}$$

Dynamic sensor fusion
Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$
$$\mathbf{P}_{n|n}^{-1}\hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1}\hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{0,1|0}^{-1}, \hat{\mathbf{x}}_{0,1|0}$

$\mathbf{P}_{0,2|0}^{-1}, \hat{\mathbf{x}}_{0,2|0}$

$\longmapsto$

fusion center

$$\mathbf{P}_{1,1|0} = \mathbf{A}\mathbf{P}_{0,1|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{1,1|0} = \mathbf{A}\hat{\mathbf{x}}_{0,1|0}$$
$$\mathbf{P}_{1,2|0} = \mathbf{A}\mathbf{P}_{0,2|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{1,2|0} = \mathbf{A}\hat{\mathbf{x}}_{0,2|0}$$
$$\mathbf{P}_{1|0} = \mathbf{A}\mathbf{P}_{0|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$

Dynamic sensor fusion
Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$
$$\mathbf{P}_{n|n}^{-1}\hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1}\hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{0,1|0}^{-1}, \hat{\mathbf{x}}_{0,1|0}$

$\mathbf{P}_{0,2|0}^{-1}, \hat{\mathbf{x}}_{0,2|0}$

$\longmapsto$

fusion center

$$\mathbf{P}_{1,1|0} = \mathbf{A}\mathbf{P}_{0,1|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{1,1|0} = \mathbf{A}\hat{\mathbf{x}}_{0,1|0}$$
$$\mathbf{P}_{1,2|0} = \mathbf{A}\mathbf{P}_{0,2|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{1,2|0} = \mathbf{A}\hat{\mathbf{x}}_{0,2|0}$$
$$\mathbf{P}_{1|0} = \mathbf{A}\mathbf{P}_{0|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{1|0} = \mathbf{A}\hat{\mathbf{x}}_{0|0}$$

Dynamic sensor fusion
Distributed Kalman filtering (method 1)

Say $n = 1$....what will happen?

$$\mathbf{P}^{-1}_{n|n} = \mathbf{P}^{-1}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1} \right)$$
$$\mathbf{P}^{-1}_{n|n}\hat{\mathbf{x}}_{n|n} = \mathbf{P}^{-1}_{n|n-1}\hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1}\hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

- The fusion center now have knowledge about the terms in red for the previous step

Dynamic sensor fusion
Distributed Kalman filtering (method 1)

Say $n = 1$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$
$$\mathbf{P}_{n|n}^{-1} \hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

fusion center

$\mathbf{P}_{1,1|1}^{-1}, \hat{\mathbf{x}}_{1,1|1}$
$\mathbf{P}_{1,2|1}^{-1}, \hat{\mathbf{x}}_{1,2|1}$ $\longmapsto$

Dynamic sensor fusion
Distributed Kalman filtering (method 1)

Say $n = 1$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$

$$\mathbf{P}_{n|n}^{-1}\hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1}\hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{1,1|1}^{-1}, \hat{\mathbf{x}}_{1,1|1}$

$\mathbf{P}_{1,2|1}^{-1}, \hat{\mathbf{x}}_{1,2|1}$

$\longmapsto$

fusion center

$\mathbf{P}_{2,1|1} = \mathbf{A}\mathbf{P}_{1,1|1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$

Dynamic sensor fusion
Distributed Kalman filtering (method 1)

Say $n = 1$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$

$$\mathbf{P}_{n|n}^{-1} \hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{1,1|1}^{-1}, \hat{\mathbf{x}}_{1,1|1}$

$\mathbf{P}_{1,2|1}^{-1}, \hat{\mathbf{x}}_{1,2|1}$

$\longmapsto$

fusion center

$$\mathbf{P}_{2,1|1} = \mathbf{A}\mathbf{P}_{1,1|1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$

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$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

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sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{1,1|1}^{-1}, \hat{\mathbf{x}}_{1,1|1}$

$\mathbf{P}_{1,2|1}^{-1}, \hat{\mathbf{x}}_{1,2|1}$

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fusion center

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$$\hat{\mathbf{x}}_{2,1|1} = \mathbf{A}\hat{\mathbf{x}}_{1,1|1}$$

$$\mathbf{P}_{2,2|1} = \mathbf{A}\mathbf{P}_{1,2|1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$

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$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

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sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{1,1|1}^{-1}, \hat{\mathbf{x}}_{1,1|1}$

$\mathbf{P}_{1,2|1}^{-1}, \hat{\mathbf{x}}_{1,2|1}$

$\longmapsto$

fusion center

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sensor 1 measurements

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what we want to estimate

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sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{1,1|1}^{-1}, \hat{\mathbf{x}}_{1,1|1}$
$\mathbf{P}_{1,2|1}^{-1}, \hat{\mathbf{x}}_{1,2|1}$

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fusion center

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Dynamic sensor fusion
Distributed Kalman filtering (method 1)

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sensor 1 measurements

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what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{1,1|1}^{-1}, \hat{\mathbf{x}}_{1,1|1}$
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fusion center

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Dynamic sensor fusion
Distributed Kalman filtering (method 1)

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$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

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$\mathbf{P}_{1,1|1}^{-1}, \hat{\mathbf{x}}_{1,1|1}$

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Dynamic sensor fusion
Distributed Kalman filtering (method 2)

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Dynamic sensor fusion
Distributed Kalman filtering (method 2)

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$
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key idea:

- The term $\mathbf{P}_{n|n-1}^{-1} \hat{\mathbf{x}}_{n|n-1}$ can be written in terms of contributions from individual sensors
- The term $\mathbf{P}_{n|n-1}^{-1}$ can be written in terms of contributions from individual sensors
- Allows the fusion center to form the estimate by summing the results sent from the sensors
- Try it or look it up

Summary

Today we have studied:

- Dynamic estimation from one sensor
- Dynamic estimation from many sensors
- Dynamic sensor fusion, distributed Kalman filtering

Next Lecture

- Application of Lecture 8 and 9 to Positioning and Localization in WSNs