MATEMATISKA INSTITUTIONEN
KTH
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HT2014

First set of hand-in-problems for SF 2741, Enumerative Combinatorics.

The problems are due **October 14** at 10.15. You may discuss the problems with other students in the class, but with no one else. You may not copy solutions you have found elsewhere. If you discuss with other student(s) in the class you should mention the name(s) for each problem. Each and every one should write down your own solution in your own words.

Write your name in the upper right corner of each paper. Staple in the upper left corner. **Maximal credit will be given only to complete and clear solutions.**

1. Problem 1.12 in the book. (Hint: One way is to use Euler's formula for planar graphs.)
2. Give an explicit formula for the Stirling numbers of the second kind $S(n, n-3)$. (Hint: See the book for $S(n, n-2)$.)
3. Problem 1.26 in the book. (Hint: Find a recursion.)
4. Problem 1.114ab in the book.
5. Problem 1.33a in the book.
6. Problem 1.112 in the book.
7. Call an integer composition of n **symmetric** if it is the same when read bakwards as when read forwards. E.g. $7 = 2+1+1+1+2$. Let c_n = total number of ones in all symmetric compositions of n. E.g. for $n = 4$, $1+1+1+1$ and $1+2+1$ gives $c_4 = 6$. Find and prove a formula for c_n. (Hint: Start with considering even n.)
8. Let $p(n|X)$ denote the number of integer partitions of n satisfying condition X. Prove the following identity for integer partitions:

$$p(n|\text{distinct parts, largest part odd}) = p(n|\text{distinct parts, largest part even}) + e_n,$$

where $e_n = \begin{cases} -1, & \text{if } n = j(3j+1)/2, \text{ for some postitive integer } j \\ 1, & \text{if } n = j(3j-1)/2, \text{ for some postitive integer } j \\ 0, & \text{otherwise.} \end{cases}$

(Hint: Use the involution φ from the proof of Euler's pentagonal theorem (1.8.7).)

Lycka till!

Petter and Svante