Modulation Transfer Function

The Modulation Transfer Function (MTF) is a useful tool in system evaluation. It describes if, and how well, different spatial frequencies are transferred from object to image. The MTF also relates the diffraction limit to the aberrations in a very clear way. So in order to understand MTF, a short introduciton to diffraction is needed.

Diffraction

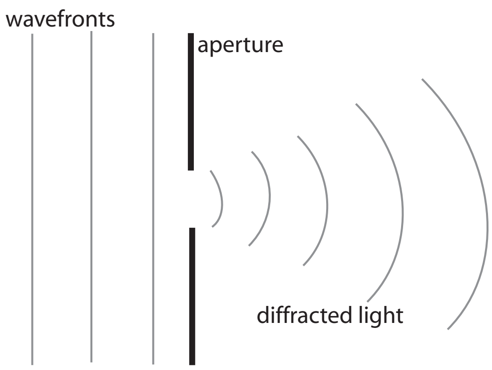

Figure 1: Illustration of diffraction of light passing through an aperture, seen as wavefronts.

Diffraction follows from the wave nature of light, and implies that any concentration of light, like a beam or light passing through an aperture, will spread. It places a fundamental limit on the spot size produced by a lens, and makes ideal imaging impossible. An illustration is shown in Fig. 1, where plane wavefronts incident on a hole are diffracted. For a closer study of diffraction, see e.g. the book by Goodman [1]. Diffraction will also affect the focal spot of a lens, considered in Fig. 2. The diffraction-limited intensity I at the focal spot will be given by

$$I \propto \left| \frac{2J_1(x)}{x} \right|^2 \quad (1)$$

where J_1 is the first-order Bessel function, $x = \pi Dr/\lambda R$, D the diameter of the lens aperture, λ the wavelength, and R the radius of convergence of the wavefront at the aperture. This is the Airy disc, a bright central area surrounded by dark and bright rings. The radius $r_A$of the Airy disc is given by the first zero of the Bessel function as $r_A = 1.22\lambda R/D$, and the diameter is

$$d_A = \frac{2.44\lambda R}{D}. \quad (2)$$

If the object is infinite, and visible light considered, the diameter in unit micrometers is roughly

$$d_A[\mu m] \approx 2.44 * 0.5 * F_w \approx F_w \quad (3)$$

where F_w is the working $F_\sharp$.

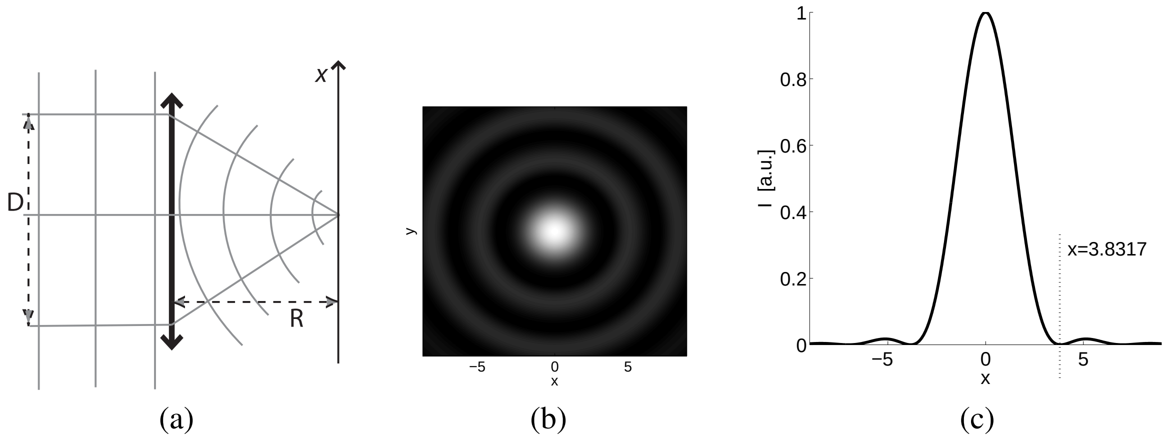

Figure 2: (a) A diffraction-limited lens system focuses light, (b) which forms an Airy disc (c) of Bessel intensity distribution.

Diffraction limit

Definition. If the spot size caused by aberrations is smaller than or approximately equal to the diffraction spot, the system is diffraction limited.

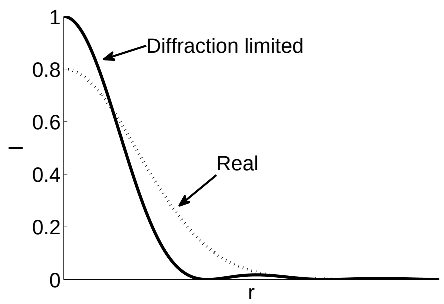

Figure 3: A diffraction-limited and an aberrated focal spot intensity distribution. The aberrated spot is broadened, and consequently its peak intensity is lower than the diffraction-limited peak intensity.

A measure of image quality is the Strehl ratio. Two example intensity curves are shown in Fig. 3. One shows a diffraction-limited intensity distribution, the other the corresponding real distribution where both diffraction and aberrations are included. When the intensity distribution is broadened due to aberrations, the peak intensity I_{ideal} at $r = 0$ is lowered to I_{real}. The definition of the Strehl ratio is their ratio,

$$\text{Strehl ratio} = \frac{I_{\text{real}}}{I_{\text{ideal}}}\,. \tag{4}$$

By experience, it is known that for a Strehl ratio of 0.8 or above, an observer is unable to separate the real image from the ideal one. Thus a system is considered diffraction limited if it has a Strehl ratio above 0.8. It can be shown (using defocus as the aberration) that this is identical to the Rayleigh criterion demanding $W \leq \frac{\lambda}{4}$ for a diffraction-limited system.

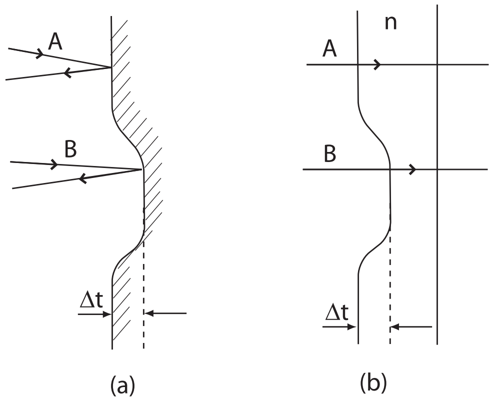

Figure 4: Surface quality requirements: OPD due to a dent of thickness Δt in an optical surface for (a) a mirror and (b) a lens. The lens surfaces may be curved, but locally they will seem flat.

This criterion can be used to find the demanded surface quality of different optical components. In Fig. 4, the effect of a non-smooth surface, i.e., a dent in an otherwise smooth surface, is shown for a mirror and for a lens. For the mirror, the optical path difference (OPD) between rays A and B is $OPD = 2\Delta t \leq \frac{\lambda}{4}$, so the surface quality demand for a mirror is $\Delta t \leq \frac{\lambda}{8}$. For a lens, on the other hand, the optical path difference is $OPD = \Delta t(n-1) \leq \frac{\lambda}{4}$, and assuming a typical refractive index of 1.5 we have a surface quality demand of $\Delta t \leq \frac{\lambda}{2}$. Note that the demand is different for mirrors and lenses!

Resolution

According to the Rayleigh criterion, two points are resolved if the central maximum of one point is over the first zero of the second. Using the known size of the Airy disc, we find that two points are resolved if they are a distance d apart, where

$$d = \frac{1.22\lambda R}{D}\,. \tag{5}$$

This holds for diffraction-limited systems. The resolution angle for a diffraction-limited system is

$$\theta_r = \frac{d}{R} = \frac{1.22\lambda}{D}\,. \tag{6}$$

Modulation transfer function (MTF)

First, we define the contrast C at spatial frequency s as

$$C(s) = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \tag{7}$$

where $I_{\max}$ and $I_{\min}$ are illustrated in Fig. 5. When an object is imaged, it is likely that the contrast will go down or even go to zero, depending on how well this particular frequency

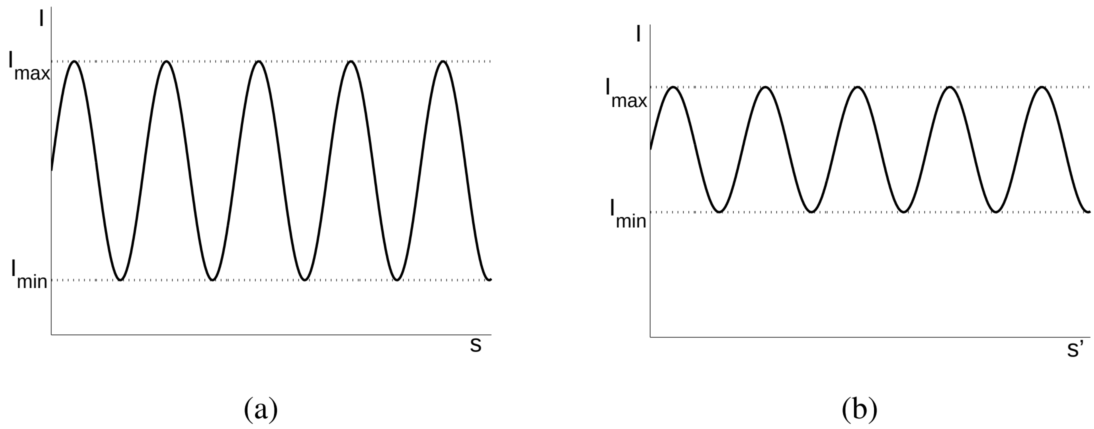

Figure 5: Illustration of modulation, or contrast, in (a) the object and (b) the image.

is transferred to the image plane. An important characteristic of an optical system is this capability of transferring object information to the image, while loosing as little as possible. Thus we define the modulation transfer function (MTF) as

$$MTF(s) = \frac{C'(s')}{C(s)}, \tag{8}$$

where $C(s)$ is the object contrast and $C'(s')$ the image contrast. A value of 1 means that frequency is perfectly transferred to the image plane, which is the best possible outcome in a passive system. A value of 0 means that frequency will not appear at all in the image. The object- and image-space spatial frequencies are related as $s' = s/|M|$ where M is the magnification. This definition of MTF, based on image quality, is identical to that obtained from diffraction theory (see, e.g., Goodman [1]). Diffraction theory can be used to determine the MTF for a diffraction-limited system, and the result is shown in Fig. 6. A diffraction-limited MTF is linear for most of its extent, going from an MTF of 1 for $s' = 0$ towards an MTF of 0 for

$$s'_{\text{lin}} = \frac{NA}{0.61\lambda} = \frac{D}{1.22\lambda R}. \tag{9}$$

For high frequencies, the MTF deviates from its linear behaviour and instead ends up at a limiting frequency of

$$s'_{\text{lim}} = \frac{NA}{0.5\lambda} = \frac{D}{\lambda R}. \tag{10}$$

The shape of the curve comes from a convolution of two circles. Note that you can calculate limiting frequencies in the object plane or in the image plane, as it suits you best. If the distance between object and lens is l (we assume a real object, so l is negative) and the distance between lens and image is l', the magnification will be $M = l'/l$. Then in the object plane, the limiting frequency would be $s_{\text{lim}} = D/\lambda|l|$, and in the image plane it would be $s'_{\text{lim}} = D/\lambda l' = D/\lambda|l||M| = s_{\text{lim}}/|M|$. Just keep in mind which space you're in, so you don't mix them!

An aberrated system will have a lower MTF than a diffraction-limited system, as illustrated in Fig. 5.

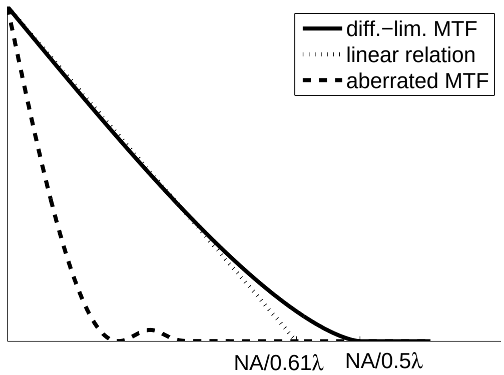

Figure 6: MTF curves for a diffraction-limited system, its linear approximation, and an aberrated system.

References

[1] J.W. Goodman, *Introduciton to Fourier Optics* (McGraw-Hill, Singapore, 1996).