Solutions Exam SK2330 Optical Design 2010-05-25

No devices that allow for communication with the outside world, or that would permit installation of commercial optical design software, are permitted (i.e., no computers). Any other material such as books, notes, and calculators may be used.

Grading: 0–8p F, 9–11p Fx, 12–14p D, 15–18p C, 19–21p B, 22–24p A

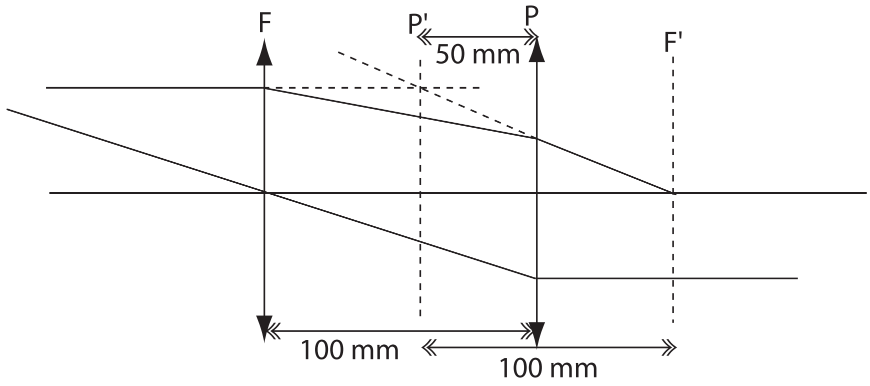
1. An achromatic doublet should fulfil $K_1 = V_1/(V_1 - V_2)K = 21.96\,m^{-1}$ and $K_2 = V_2/(V_2 - V_1)K = -11.96\,m^{-1}$. Lens A has $K_1 = (n_1 - 1)(c_1 - c_2) = 25.6\,m^{-1}$ and $K_2 = (n_2 - 1)(c_2 - c_3) = -15.6\,m^{-1}$. Lens A is not an achromat, don't pick this lens! Lens B has $K_1 = 21.97\,m^{-1}$ and $K_2 = -11.96\,m^{-1}$. Lens B is an achromat, pick this one! (Lens B also has very low spherical aberration, if anybody bothered checking.) Your friend must have made a mistake on lens A.

2. a) All four graphs have the same third-order shape, so there is spherical aberration (the only on-axis aberration). The off-axis tangential ray fan has been shifted upwards, which means there is also distortion.
   b) Spherical aberration: $\epsilon_y = -4W_{040}l'/h(x^2 + y^2)y$, pick e.g. $\epsilon_y = -20\,\mu m$ for $x = 0$, $y = 1$. That gives $W_{040} = 0.25\,\mu m$. Distortion: The curve is shifted $5\,\mu m$ upwards, so $\epsilon_y = -l'/hW_{311}\eta^3 = 5\,\mu m$ for $\eta = 1$ gives $W_311 = -0.25\,\mu m$.

3. a) The object distance is calculated from image distance and focal length to be -107 mm. The magnification is $m = l'/l = 14$ times, so the spatial frequencies in the image plane are 3.6 lines/mm and 1.1 lines/mm. The limiting frequency of the linear relation is $s_{lim} = (D/2)/(0.61\lambda R) \approx 22$ lines/mm. The contrast will be $C = 1 - 3.6/22 \approx 0.83$ for the first grating and $C = 1 - 1.1/22 \approx 0.95$ for the second.
   b) The first has visibility 0, lens not suitable to image this grating. Second grating has visibility around 0.6, this one is fine.

4. a) 1:1 imaging, so Y=0. That means coma is zero if $EX + FY = EX = 0$, since $E \neq 0$ we must have $X = 0$. The lens is equiconvex, from $K = (n - 1)(c_1 - c_2)$ we find the radii of curvature $r_1 = 60$ mm and $r_2 = -60$ mm.
   b) On-axis, we need to consider spherical aberration. $\epsilon_y = -4W_{040}l'/h(x^2 + y^2)y$, maximum value for $x = 0$, $y = 1$. $\epsilon_y = -4W_{040}2f/h = -1/(Kh)S_1 = -1/4h^3K^2D \approx 89\,\mu m$. The spot radius is $89\mu m$.

5. a) The effective power is $K = K_1 + K_2 - dK_1K_2 = 10\,m^{-1}$, so the effective focal length is 100 mm. The distances to the principal planes are $\delta = K_2d/K = d = 100$ mm and $\delta' = -K_1d/K = -50$ mm. The planes are shown in figure. Note that the first principal plane is at the second lens, and the first focal plane at the first lens.
   b) Thin lens theory can be used, stop shift on second lens has no effect since spherical aberration and coma are zero. We have $W_{222} = \frac{1}{2}S_{III}$ (astigmatism) and $W_{220} = \frac{1}{4}(S_{III} + S_{IV})$ (field curvature). We also have $S_{III} = L^2(K_1 + K_2)$ and $S_{IV} = \frac{2}{3}L^2(K_1 - K_2)$. The curvature of the sagittal image plane is (Axner, 11.67) $c_s = -4W_{220}/H^2$, so insertion and $L = -H$ yields $c_s = -\frac{5}{3}(K_1 + K_2) = -25\,m^{-1}$ which means $r_s = -40$ mm. Similarly for the tangential plane, $c_t = -4(W_{220} +$



$W_{222})/H^2 = -\frac{11}{3}(K_1 + K_2) = -55\,m^{-1}$ and $r_t = -18\,mm$. These are steep curvatures! (The expressions for the curvatures could also be derived from the longitudinal aberration and the sag formula.)

c) Sharp curvatures of image surfaces, astigmatism and field curvature dominate and should be removed. There are many options, but some examples are either a curved image surface or a field lens close to the image plane (if we curve the sagittal image plane backwards, we could get either medial or tangential image plane flat). Another option is to voluntarily introduce equal and opposite amounts of coma in the two doublets. That way the total coma will still be zero, but via the stop shift there will be astigmatism introduced. This can be used to reduce astigmatism and flatten one of the image surfaces.