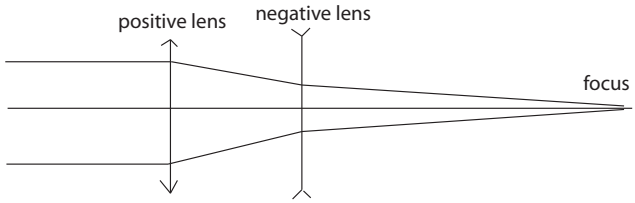
Solutions Exam SK2330 Optical Design 2010-08-26

No devices that allow for communication with the outside world, or that would permit installation of commercial optical design software, are permitted (i.e., no computers). Any other material such as books, notes, and calculators may be used.

Grading: 0–8p F, 9–11p Fx, 12–14p D, 15–18p C, 19–21p B, 22–24p A

1. The spatial frequency in image space is $1/0.05\,m^{-1} = 20\,m^{-1}$ or $1/0.10\,\mathrm{m}^{-1}$ depending on how you interpret the text. This solution will be for the first. The fence is approximated by a sinusoidal grating. The object distance $l = -500$ m is large, so $l' = f = 35$ mm. The magnification is $M = 35 \times 10^{-3}/500 = 7 \times 10^{-5}$, so the image-space frequency is $s_f = 20/M = 2.86 \times 10^5\,m^{-1}$. The MTF at this frequency should be $C'/C = 0.4/0.8 = 0.5$. The limiting frequency is $s_{lim} = D/2/(0.61\lambda R)$, and since $MTF = 0.5$ we know that $s_{lim} = 2s_f$ by the approximate linear relationship. This yields $D = 4 \times 0.61 s_f \lambda R$, and for $R = 35 \times 10^{-3}$ m and the wavelength approximated by $\lambda = 500 \times 10^{-9}$, we get $D \approx 1$ cm.

2. a) Use system below. Distance between lenses can be adjusted so that longitudinal chromatic aberration is removed.



b) The longitudinal chromatic aberration of a thin lens is $C_L = h^2K/V$, and is not affected by stop shifts (Axner, p. 582-583). So the longitudinal chromatic aberration of this system is $C_L = h_1^2K_1/V_1 + h_2^2K_2/V_2 = K_1(h_1^2/V_1 - h_2^2/V_2)$, where 1 refers to the positive lens and 2 to the negative. Longitudinal chromatic aberration is canceled if $h_1^2/V_1 = h_2^2/V_2$, which yields $h_1 = 1.588h_2$. If the distance between lenses is d, equiangular triangles yield $h_1 = f_1/(f_1 - d)h_2$, so we know that $f_1/(f_1 - d) = 1.588$ and so $d = 0.37f_1 = 37$ mm.

c) Effective focal power of the system is $K = K_1 + K_2 - K_1K_2d/n$, for $n = 1$ and $K_2 = -K_1$ we have $K = dK_1^2 = 0.37/f_1$, so $f = 1/K = f_1/0.37 = 270$ mm.

3. a) No aberrations on optical axis, so no spherical aberration or defocus. Only linear graphs, so no coma. Field curvature present since sagittal line is inclined, astigmatism present since tangential line is more inclined than sagittal. Distortion present since line is moved vertically in tangential graph compared to sagittal.

b) Field curvature: $\epsilon_x = -2l'/hn \times W_{220}\eta^2 x$, and for $\eta = 1$ and $x = 1$ we have from graph $\epsilon_x = -10\,\mu m$. This gives $W_{220} = -\epsilon_x h/2l' \approx 0.33\mu$ m. Distortion: at $y = 0$ in tangential graph we see line is moved $10\mu m$ upwards, so $\epsilon_y = -l'W_{311}/h =$

$10\mu m$ gives $R_{311} = -0.67\mu m$. Astigmatism: If contributions form field curvature and distortion are removed, we have $\epsilon_y = -2l'/h * W_{222} = -10\mu m$, so $W_{222} = 0.33\mu m$.

4. a) Spherical aberration the only relevant, since the imaging is on-axis in monochromatic light.

   b) The spherical aberration coefficient for a thin lens is $S_I = h^4K^3/4 \times (AX^2 + BXY + CY^2 + D)$. In 1:1 imaging, Y=0. For the equi-convex lens, $X = 0$ and $S_I = h^4K^3/4 \times D$. For the plano-convex lens, $X = \pm 1$ depending on which way you turn it, so $S_I = h^4K^3/4 \times (A + D)$. Since A and D are positive, the equi-convex lens will be better, and its aberrations $D/(A + D)$ times smaller. Using $A = (n + 2)/n(n - 1)^2 \approx 9.333$ and $D = n^2/(n - 1)^2 \approx 9.000$, we find the aberration for the equiconvex lens is only 51% of the planoconvex.

5. The marginal ray angle is $u = 0.26$ from the definition of NA, and the principal ray angle is $\bar{u} = 0.76$ (45 degrees expressed in radians). the thickness is d=0.01 m. (3p for getting this far!) Using the relation between Seidel and wavefront aberration coefficients (Axner 12.27) we get the aberration polynomial

$$W = -\frac{1}{2}ud\frac{n^1 - 1}{n^3}\left(\frac{1}{4}u^3(x^2 + y^2)^2 + u^2\bar{u}y(x^2 + y^2) + u\bar{u}^2y^2 + \frac{1}{2}u\bar{u}^2(x^2 + y^2) + \bar{u}^3y\right) \quad (1)$$

   which for sagittal plane is

$$W_x = -2.11 \times 10^{-6}x^4 - 3.90 \times 10^{-5}x^2\,[m]. \quad (2)$$

   When sketching the graph, neglect the fourth-order term which is tiny in comparison. For the tangential plane,

$$W_y = -2.1 \times 10^{-6}y^4 - 2.5 \times 10^{-5}y^3 - 1.2 \times 10^{-4}y^2 - 2.4 \times 10^{-4}y\,[m]. \quad (3)$$

   When sketching the graph, neglect the third-and fouth-order terms which are small in comparison.