Solutions SK2330 Optical Design 2013-05-21 14-19 FD41

Grading limits: 0-10p F, 10p or over Fx, 12p or over D, 15p or over C, 19p or over B, 22–24p A

1. a) No on-axis aberrations means there is no spherical aberration. The second-order curve for $\eta = 1$, $y = 0$ indicates there is field curvature, with $W_{220} = 2\lambda$ found, e.g., at $x = 1$. The fourth graph indicates there is coma and possibly astigmatism. Using $x = -1$ gives $W_{220} + W_{222} - W_{131} = 6\lambda$, while $x = 1$ gives $W_{220} + W_{222} + W_{131} = -2\lambda$, and solving the equation system gives $W_{222} = 0$ and $W_{131} = -4\lambda$.
   b) Draw graphs. On-axis graphs are zero (no on-axis aberrations). The graph for $\eta = 1$, $y = 0$ is given by $\epsilon_x = -2W_{220}xR/h = -50\lambda x$ for R=150 mm and d=12 mm. The graph for $\eta = 1$, $x = 0$ is given by $\epsilon_y = -2W_{220}yR/h - 3W_{131}y^2R/h = -50\lambda y + 150\lambda y^2$.

2. a) The Seidel sum is $S_{IV} = -H^2 \left[c_1(1/n - 1) + c_2(1 - 1/n)\right] = -1.72H^2$ (using the fact that H is constant throughout the system). This is negative.
   b) The power of a thick lens is $K = K_1 + K_2 - dK_1K_2/n$, where $K_1 = c_1(n - 1)$ and $K_2 = c_2(1 - n)$. Insertion yields $K = 5 \cdot 10^{-3}\,\mathrm{m}^{-1} \approx 0$, so the lens har no power.
   c) As positive lenses have positive field curvature, this is an excellent compensation element: no power, but negative field curvature. Disadvantages are e.g. that it's thick and thus both bulky and expensive, plus that it might add other aberrations.

3. As the F-number is over 10, we assume the system is diffraction limited (at least on-axis). White light gives a central wavelength around $\lambda \approx 550\,\mathrm{nm}$. Both the letters and the building are approximated as sinusoidal gratings. (a) The resolution limit in object space is given by $s = D/\lambda R \approx 1.1/(550 \cdot 10^{-9} \cdot 681 \cdot 10^3)\,\mathrm{m}^{-1} \approx 2.9\,\mathrm{m}^{-1}$. No matter how you estimate them, the spatial frequencies of the letters on a license plate are higher than this, and thus the letters are not resolved and their contrast is 0. (Not the result you would expect from some TV crime series, but maybe good for your integrity?) (b) The building has spatial frequency $s \approx 3/100\,\mathrm{m}^{-1} \approx 0.03\,\mathrm{m}^{-1}$. As this is around a tenth of the resolution limit, it's clearly in the linear part of the MTF. The linear limiting frequency is $2.4\,\mathrm{m}^{-1}$, so the image contrast will be $C' = 1 - 0.03/2.4 \approx 1$.

4. The spherical aberration coefficient of a singlet lens of power K is $S_I = \frac{1}{4}h^4K^3(AX^2 + BXY + CY^2 + D)$. If a plano-convex lens is turned the right way and object is at infinity, $X = 1$ and $Y = -1$. This yields $S_I = \frac{1}{4}h^4K^3(A - B + C + D)$.
   Splitting the power on two lenses, each of power $K_1 = K_1 = K/2$, instead gives the spherical aberration coefficient $S_I = \frac{1}{4}h^4(K/2)^3(AX_1^2 + BX_1Y_1 + CY_1^2 + D + AX_2^2 + BX_2Y_2 + CY_2^2 + D)$. Using $X_1 = 1$, $Y_1 = -1$, and finding $Y_2 = (l_2' + l_2)/(l_2' - l_2) = (f + 2f)/(f - 2f) = -3$ yields $S_I = \frac{1}{32}h^4K^3(A - B + C + D + AX_2^2 - 3BX_2 + 9C + D)$. Taking the derivative and setting it to zero yields the minimum at $2AX_2 - 3B = 0$ or $X_2 = 2A/3B$. Assuming the refractive index is 3/2, this yields $X_2 \approx 2.14$. Insertion yields $S_I \approx \frac{1}{4}h^4K^3 \cdot 1.8$ for the split system,

compared to $S_I \approx \frac{1}{4}h^4K^3 \cdot 9.3$ for the singlet system, so the spherical aberration has been reduced by a factor of 5.

5. System A is a doublet with a large F-number but a larger field of view. As can also be seen from the drawing, the system should be dominated by field curvature. So it has wavefront aberrations II.
   System B is a complicated system with a medium F-number and the same field of view as A. As also indicated by the drawing, we expect this system to perform best. So it has wavefront aberrations IV, which at least on-axis indicate diffraction-limited performance.
   System D is a singlet lens turned the wrong way around, with a low F-number. There should be huge amounts of spherical aberration, so it has wavefront aberrations I (check the scale, 200 wavelengths!).
   System C is a doublet used at a lower F-number and smaller field of view than A. As the only remaining wavefront diagram is III, it must be this one. We see the achromat is not completely compensated for spherical aberration (as can be expected from a cemented doublet, unless the materials of the lens are perfectly matched).