Solutions SK2330 Optical Design 2013-08-26 8-13 FB52

Grading limits: 0-10p F, 10p or over Fx, 12p or over D, 15p or over C, 19p or over B, 22–24p A

1. a) All graphs are equal, so the only possible aberrations are spherical aberration and defocus. The shape indicates both are present.
   b) The wavefront aberration is $W = W_{040}(x^2 + y^2)^2 + W_{020}(x^2 + y^2)$, so the transverse aberration in the y direction is $TA_y = -2l/h_p(2W_{040}y^3 + W_{020}y)$ for $x = 0$. Reading values from the graph, for example $TA_y \approx 75\lambda$ for $y = -1$ and $TA_y \approx -20\lambda$ for $y = -0.5$, and solving the equation system yields $W_{040} \approx 3.07\lambda$ and $W_{020} \approx -3.13\lambda$. Results may vary depending on values read from the graphs. Graphs were drawn using $W_{040} = 3\lambda$ and $W_{020} = -3\lambda$.

2. a) The Seidel sum is $S_{IV} = -H^2 (K_1/n_1 + K_2/n_2 + K_3/n_3)$ (using the fact that H is constant throughout the system, and that the S_{IV} term of a thin lens is not affected by the stop shift). Here 1 refers to the first part of the doublet, 2 to the second part of the doublet, and 3 to the field lens. K_1 and K_2 can be found from the conditions for an achromatic doubled as $K_1 = V_1/(V_1 - V_2)K_D = 10.58\,\mathrm{m}^{-1}$ and $K_2 = V_2/(V_2 - V_1)K_D = -5.5821\,\mathrm{m}^{-1}$, where $K_D = 5\,\mathrm{m}^{-1}$ id the power of the doublet. Hence S_{IV} will be zero if $K_3/n_3 = -3.59\,\mathrm{m}^{-1}$. I choose BK7, as this is a standard glass and the low refractive index will reduce the power required (other choices with other motivations are readily accepted). Then the radius of curvature is -95 mm.
   b) The size of the field lens will limit the image size, and hence the field of view.

3. a) Using the paraxial law of refraction, $n_2u_2 = n_1u_1 - hc(n_2 - n_1)$, we find that $u_2 = -hc(n_2 - 1)/n_2$. The ray height will be zero when $0 = h + u_2d$, i.e., when $d = n_2r/(n - 1) = 159\,\mathrm{mm}$. b) The angle of incidence is given by $\sin i = h/r$, the refraction angle by Snell's law as $\sin i' = \sin(i)/n_2$, and from geometry we see that $|u_2| = i - i'$. The sought distance is $d = r + x$, where $x = \sin i'/\sin|u_2| \cdot r$ which gives $d = 159\,\mathrm{mm}$. Even if this surface should have a lot of spherical aberration, this particular ray is close to the optical axis and the difference between paraxial and exact ray-tracing is small.

4. With this object distance the magnification is -0.5, so the spatial frequencies in the image will be $50\,\mathrm{mm}^{-1}$, $100\,\mathrm{mm}^{-1}$, and $150\,\mathrm{mm}^{-1}$. Searching the graphs for the highest value of the MTF at those frequencies, we find that system (a) is better in the first case, system (c) in the second case, and system (d) in the third case. (Note: the weird MTF shape in (d) can be obtained e.g. using an annular aperture.) (Note: The diffraction limit is actually of no interest to this task, the only thing that matters is the actual MTF.)

5. First of all, the question was wrongly phrased, the light should have been monochromatic. Anybody mentioning chromatic aberrations can pride themselves with be-

ing attentive! And anybody getting stuck because of this gets compensated with extra points... The smallest spot would occur when aberrations and diffraction are balanced so that they are approximately equal. Disregarding chromatic aberrations, the only aberration of interest is spherical aberration. As an estimation is asked for, we're free to use thin lens theory. The transverse aberration for a ray at the edge of the pupil will be $TA = -4W_{040}f/h = -4W_{040}/hK$, where $W_{040} = 1/32 \cdot h^4K^3(AX^2 + BXY + CY^2 + D)$. Using $X = 1$ if the lens is turned the right way, and $Y = -1$ for object at infinity, this becomes $TA = -1/8 \cdot h^3K^2(A - B + C + D)$. Inserting $A - B + C + D = 28/3$ and $K = 5\,\mathrm{m}^{-1}$ yields $|TA| = 25 * 28/(8 * 3) \cdot h^3$. Finally, defocusing to the best focus reduces the spot size to a fourth of this, $|TA| \approx 7.3 \cdot h^3$. The spot size radius due to diffraction is $r_a = 1.22\lambda/(2hK) \approx 6.1 \cdot 10^-8 \cdot 1/h$ assuming a wavelength of $500\,\mathrm{nm}$. Setting the diffraction and aberration spot size radii equal yields $h \approx 10\,\mathrm{mm}$, with a spot size radius of around $7\,\mu\mathrm{m}$. (Sanity check: singlet lenses become diffraction limited around an F-number or 10, and the spot diameter should be roughly equal to the F-number, i.e., $10\,\mu\mathrm{m}$. The answer is reasonable.)