SolutionsExam SK2330 Optical Design 2011-05-27

Grading: 0–8p F, 9–11p Fx, 12–14p D, 15–18p C, 19–21p B, 22–24p A

1. a) On-axis and sagittal plots are the same, this excludes field curvature.
   On-axis and sagittal plots show a combination of spherical aberration and field curvature.
   All graphs are symmetrical (even) so there is no tilt, distortion, or coma.
   Tangential and sagittal plots differ, and the only remaining reason is astigmatism.
   $\Longrightarrow$ there will be spherical aberration, astigmatism, and defocus.

   b) On-axis, there is spherical aberration and defocus, $W = W_{040}y^4 + W_{020}y^2$. From the graph $W = 1$ for $y = 1$, so $W_{040} + W_{020} = \lambda$. For $y \approx 0.7$, $W = 0$ and thus $W_{020} = -(0.73)^2 W_{040}$. Solving the equation system yields $W_{040} = 1.96\lambda$ and $W_{020} = -0.96\lambda$. (Difficult to estimate well, answers may range e.g. from -1.2λ to -0.8λ for W_{020} and from 2.2λ to 1.8λ for W_{040}.) In the tangential plot there is also astigmatism, and $W = W_{040}y^4 + W_{020}y^2 + W_{222}y^2$. $W = -0.85$ for $y = 1$ gives $W_{040} + W_{020} + W_{222} = -0.85$, and solving for W_{222} yields $W_{222} = -1.85$. (Again, values may range between e.g. -1.8λ and -1.9λ.)

2. For the singlet lens, we have $W^I_{131} = \frac{1}{4}Lh^2K^2(EX+FY)$. We know that $K = 1/f$, $X = 1$, and $Y = -1$. Then $W^I_{131} = \frac{1}{4}Lh^2K^2(E - F)$. For the two lenses, $W^{II}_{131} = \frac{1}{4}Lh^2K_1^2(EX_1 + FY_1) + \frac{1}{4}Lh^2K_2^2(EX_2 + FY_2)$. We use $K_1 = K_2 = 1/2f = K/2$, $X_1 = X_2 = 1$, and $Y_1 = -1$. We calculate $Y_2 = (l'_2 + l_2)/(l'_2 - l_2) = (f + 2f)/(f - 2f) = -3$. This yields $W^{II}_{131} = \frac{1}{4}Lh^2K^2(E - 2F)/2$. As L is invariant and the same for the two cases, and h remains the same as the lenses are thin and close together, $W^{II}_{131}/W^I_{131} = \frac{1}{2}(E - 2F)/(E - F)$. Assuming a refractive index of e.g. 1.5 yields the quote as -1.5, so the absolute value of the coefficient will increase by around 50%.

3. Calculate the Abbe numbers $V_1 = 64.20$ for BK7 and $V_2 = 25.76$ for SF11. The total power is $K = 1/0.15\,\mathrm{m}^{-1} \approx 6.67\,\mathrm{m}^{-1}$, so we can calculate the powers of each lens as $K_1 = KV_1/(V_1 - V_2) \approx 11.14\,\mathrm{m}^{-1}$ and $K_2 = -KV_2/(V_1 - V_2) \approx -4.47\,\mathrm{m}^{-1}$. There are many curvatures c_1, c_2, c_3 that would fulfil these conditions. for example we can assume that the second lens is made from SF11, and that its last surface is flat ($c_3 = 0$). The $K_2 = (c_2 - c_3)(n_2 - 1) = c_2(n_2 - 1)$ and $c_2 = K_2/(n_2 - 1) \approx -5.69\,\mathrm{m}^-1$, so $r_2 = -176\,\mathrm{mm}$. The power of the first lens is $K_1 = (c_1 - c_2)(n_1 - 1)$ and solving for c_1 yields $c_1 = 15.85\,\mathrm{m}^-1$ so $r_1 = 63\,\mathrm{mm}$.

4. The F-number of the lens is $f/D = 200/16 = 12.5$, and the working F-number is even higher. For such large F-numbers the imaging is nearly diffraction limited, even for singlet lenses. The image distance l' is found from $1/f = 1/l' - 1/l$ for $f = 200\,\mathrm{mm}$ and $l = -300\,\mathrm{mm}$ to be $600\,\mathrm{mm}$. The highest image frequency will be $s'_lim = D/\lambda R = 16 \cdot 10^{-3}/632.8 \cdot 10^{-9}/600 \cdot 10^{-3}\,\mathrm{m}^{-1} \approx 42\,\mathrm{lines/mm}$. The given curve looks diffraction limited, but its highest image frequency is only just over $30\,\mathrm{lines/mm}$. This is not the MTF of the given lens.

5. a)

$$S_{IV} = -(H^2P_1 + H^2P_2) = H^2\left(c_1\Delta_1\left\{\frac{1}{n}\right\} + c_2\Delta_2\left\{\frac{1}{n}\right\}\right)$$
$$= H^2\left[c_1\left(\frac{1}{n} - 1\right) + c_1\left(1 - \frac{1}{n}\right)\right] = 0$$

The S_{IV} term is zero. We have used the fact the the Lagrange invariant is constant throughout the optical system, i.e., the same at the two surfaces. We have also used $c_1 = c_2$ as the two surfaces have equal curvatures.

b) $K = K_1 + K_2 - dK_1K_2/n = -d(n-1)(1-n)c_1^2/n = d(n-1)^2/nr^2 = 30 * 0.51679^2/1.51679/50^2\,\mathrm{mm}^-1$, so the focal length is $284\,\mathrm{mm}$.

c) The lens has positive focal power, and still zero S_{IV}. This is the part of the field curvature that is most difficult to get rid of (the remaining parts, and astigmatism, can be dealt with e.g. by a stop shift). So we get a positive lens without field curvature, which is valuable. Of course it still has spherical aberration, coma, distortion, and chromatic aberrations. It is also a very thick lens, which makes it expensive. (The lens is also a telephoto lens, effective focal power is longer than the lens system.)