AUTOMATIC CONTROL

KTH

EL2745 Principles of Wireless Sensor Networks

Exam 14:00–19:00, October 24, 2014

Aid:

Printed slides from the course, reading material such as ‘An Introduction to Wireless Sensor Networks” or similar text approved by course responsible are approved; Mathematical handbook (e.g., “Beta Mathematics Handbook” by Råde & Westergren) and pocket calculators are approved. The course compendium, your notes to the exercise lectures, other textbooks, handbooks, exercises, solutions, smartphones, tablets, etc. may **not** be used.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page and write on one side per sheet.
- Each answer must be motivated.
- Specify the total number of handed in pages on the cover.
- Each subproblem is marked with its maximum credit.

Grading:

Grade A: ≥ 43, Grade B: ≥ 38

Grade C: ≥ 33, Grade D: ≥ 28

Grade E: ≥ 23, Grade Fx: ≥ 21

Results:

The results will be available on your “my pages” between one and two weeks from the exam.

Responsible: Carlo Fischione, `carlofi@kth.se`

Good Luck!

1. **Probability of error at the message level**

   In a Wireless Sensor Network (WSN), messages are transmitted over a Rayleigh channel and received with an Additive White Gaussian Noise (AWGN) receiver noise. The message is a frame of size f bits.

(a) [1p] Describe the physical propagation reason leading to the the Rayleigh fading.

(b) [3p] Compute the average probability of error for a Rayleigh fading channel given the error probability of AWGN channel model. [hint: At high SNR regimes one can use the approximation $(1+x)^{1/2} \sim 1 + x/2$.]

(c) [3p] Compute the probability p that the message is correctly received.

(d) [3p] Assume that the received signal level at the receiver decays inversely with the squared of the distance, i.e.,

$$\text{SNR} \approx \frac{\alpha E_b}{N_0 d^2},$$

where suppose that $E_b/N_0 = 100$ and $\alpha = 0.1$. For messages of size 10 bits, characterize the farthest distance to place a receiver such that the probability of successfully message reception is at least $p = 0.9^{10} \approx 0.35$.

2. **Analysis of CSMA based MAC in WSNs**

   In this exercise you will have to evaluate the performance of slotted carrier sense multiple access (CSMA) protocol with fixed contention window size of IEEE 802.15.4. Assume a network of N sensor nodes with a single channel and all the nodes are in the communication range of each other. Let M be the fixed contention size, t_{slot} be the time slot duration, t_d the required time to transmit the data packet with $t_d < t_{\text{slot}}$.

(a) [1p] In which modality of IEEE 802.15.4 is such a slotted CSMA used?

(a) [3p] Describe the slotted CSMA mechanism in less than one page.

(a) [3p] Define P_s as the probability of having a successful transmission after a contention round with M maximum window size and N contenders. Let $p_s(m)$ be the probability of success at slot m. Find $p_s(m)$ and P_s.

(a) [3p] Let P_c be the probability of collision after contention round and $p_c(m)$ be the probability of collision at slot m. Propose an analytical model to calculate P_c and $p_c(m)$. Recall that a collision happens at slot m, if at least two sensor nodes pick the same slot m to transmit given that nobody has selected a previous slot.

3. **Shortest path routing in WSNs**

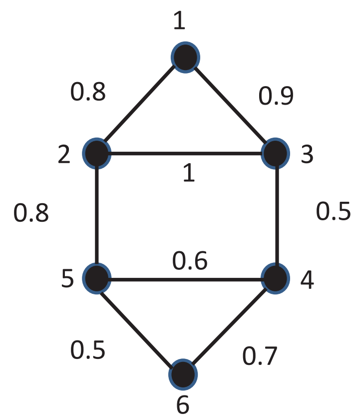

Figure 1: A sample WSN topology. Node 1 is the sink and link qualities (PRR) are depicted on each arc

In Wireless Sensor Networks (WSNs), a routing tree can be built based on the expected number of transmissions (ETX). Here, a minimum spanning tree (MST) minimizes the ETX for each node. The ETX over a link is a function of the packet reception rate (PRR) for the link between the pair of nodes (i, j). Note that PRR is directional and the rate of packet reception for links (i, j) and (j, i) can be different. Having the values of PRR of direct neighbors available at each node, in a recursive fashion nodes can build a routing tree that minimizes the ETX toward the sink.

(a) [2p] Given the PRR over the link (i, j), calculate the average number of required transmissions to send a packet successfully over the link.

(b) [3p] Develop a sketch of the algorithm and the required equations to build the routing tree based on ETX metric.

(c) [3p] Write down the Dijkstra algorithm to find the minimum shortest path routing.

(d) [2p] Consider Figure 1 and assume the PRR is bidirectional (links are undirected) where the values of the PRR are given on the arcs. Find the MST based on ETX metric and Dijkstra algorithm.

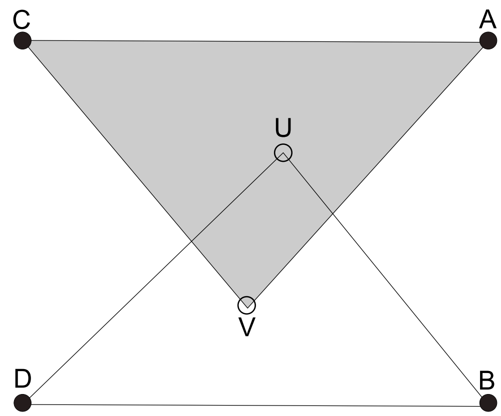

Figure 2: Four node multilateration.

4. **Collaborative multilateration**

Consider a network with a number of nodes. Suppose that node U can estimate ranges only for nodes A, C, and V, and node V can estimate ranges only for nodes B, D, and U.

(a) [3p] Assume that measurements from A, C and V include a timing offset that corresponds to an error of Δm, which is common for every measurement. Suppose the positions of A, C and V are $(0, 3.0)$, $(4.0, 0)$ and $(4.0, 3.0)$, respectively, and the measurements are 3.5, 4.5 and 5.5 respectively. Find the position of U and offset Δ.

Now consider Figure 2, where the unknown locations are U and V. To estimate the positions of U and V, begin with an initial guess at the position of U from either the centroids of the known positions in immediate range, or via the topology. Then multilateration is performed using the locations of all neighbors (estimated or known) to refine the positions, in a sequence that proceeds until locations stabilize. This time, suppose positions of A, B, C, and D are $(1, 1)$, $(1, -1)$, $(-1, 1)$ and $(-1, -1)$ respectively. The squared distances from U to A, C and V are 1.25, 1.25 and 1 respectively, while the squared distances from V to B, D and U are 1.25, 1.25 and 1 respectively.

(b) [1p] Using simple geometry, find the position of node U and V.

(c) [2p] Now, assume that the position is found by an iterative algorithm. Compute the first estimate $\hat{U}_0$ and $\hat{V}_0$ of the positions of U and V as the centroids of the nodes they can hear that have known position.

(d) [4p] Consider again the previous item. Assume perfect range measurements. Iteratively calculate by multilateration the positions of U and V, in the order U and V, after one iteration, which we denote by $\hat{U}_1$, $\hat{V}_1$. Discuss what can happen by continuing the iterations.

5. **Networked Control System**

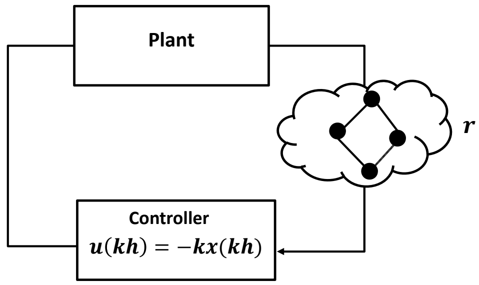

Figure 3: Closed loop system over a WSN.

Consider the Wireless Sensor Network Control System (WSN-CS) in Fig. 3. The system consists of a continuous plant

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1a}$$
$$y(t) = Cx(t)\,, \tag{1b}$$

where $A = a$, $B = b$, $C = c$, with a, b, and c being scalars. The system is sampled with sampling time h, and the discrete controller is given by

$$u(kh) = -Kx(kh), \qquad k = 0, 1, 2, \dots\,,$$

where K is a constant.

(a) [3p] Suppose that the sensor network has a medium access control and routing protocols that introduce a delay $\tau \leq h$. Derive a sampled system corresponding to Eq.(1) with a zero-order-hold.

(b) [2p] Under the same assumption above that the sensor network introduces a delay $\tau \leq h$, give an augmented state-space description of the closed loop system so to account for such a delay.

(c) [2p] Under the same assumption above that the sensor network introduces a delay $\tau \leq h$, characterize the conditions for which the closed loop system becomes unstable [Hint: no need of computing numbers, equations will be enough]

(d) [3p] Now, suppose that the network does not induce any delay, but unfortunately introduces packet losses with probability p. Let $r = 1 - p$ be the probability of successful packet reception. Give and discuss sufficient conditions for which the closed loop system is stable. If these conditions are not satisfied, discuss what can be done at the network level (protocol parameters) or at the controller level so to still ensure closed loop stability.