MATEMATISKA INSTITUTIONEN
KTH
Teachers: Petter Brändén and Svante Linusson
HT2014

Second set of hand-in-problems for SF 2741, Enumerative Combinatorics.

The problems are due **November 18** at 10.15. You may discuss the problems with other students in the class, but with no one else. You may not copy solutions you have found elsewhere. If you discuss with other student(s) in the class you should mention the name(s) for each problem. Each and every one should write down your own solution in your own words.

Write your name in the upper right corner of each paper. Staple in the upper left corner.

Maximal credit will be given only to complete and clear solutions.

1. Problem 2.14a, you should prove the formulas for at least $A_1(n), A_2(n)$ and $A_3(n)$.
2. Problem 2.25 in the book. There are two typos in the left-hand-side of the equation displayed in 2.25b. The equation should read:
$$\sum_{m,n\geq 0}\sum_{i\geq 0} f_i(m,n)t^i\frac{x^m y^n}{m!n!} = e^{-x-y}\sum_{i\geq 0}\sum_{j\geq 0}(1+t)^{ij}\frac{x^i y^j}{i!j!}.$$
3. The number $\beta_n(S)$ (=the number of permutations in $\mathfrak{S}_n$ with descent set S) can be written in terms of a determinant of a certain matrix, as seen in class and equation (2.17) in the book. Given n and S, state a problem on non-intersecting lattice paths having the determinant of the same matrix as the answer. Can you give a direct bijection between permutations with a certain descent set S and some set of k-tuples of non-intersecting lattice paths?
4. Problem 3.57a in the book.
5. Problem 3.70a and b in the book.
6. Problem 3.90 in the book.
7. A function $f : \mathbb{Z}_n \to \mathbb{Z}_k$ is *periodic* of period r, where $1 \leq r \leq n-1$, if
$$f(m+r) = f(m), \qquad \text{for all } m \in \mathbb{Z}_n.$$
Otherwise f is *non-periodic*. For $n, k \geq 1$, let $N(n,k)$ be the number of non-periodic functions $f : \mathbb{Z}_n \to \mathbb{Z}_k$ and prove
$$N(n,k) = \sum_{d|n}\mu(d,n)k^d,$$
where μ is the Möbius function of the division lattice D_n.
8. Problem 3.129 in the book. (Hence $\sigma : P \to P$ is an order preserving bijection such that $\sigma(x) \neq x$ and $\sigma^p(x) = x$ for all $x \in P$.)

Lycka till!

Petter and Svante