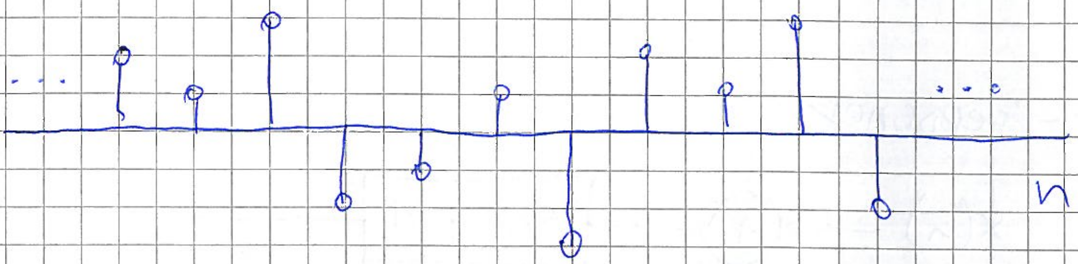
LINEAR TIME-INVARIANT SYSTEMS (LTI)

* SIGNALS

$x(t)$ - CONTINUOUS TIME SIGNAL

$x(n)$ - DISCRETE TIME SIGNAL



* SYSTEMS

IN GENERAL, A SYSTEM, TRANSFORMS INPUT SIGNAL TO OUTPUT SIGNAL

$$x(n) \longrightarrow \boxed{\text{SYSTEM}} \longrightarrow y(n)$$

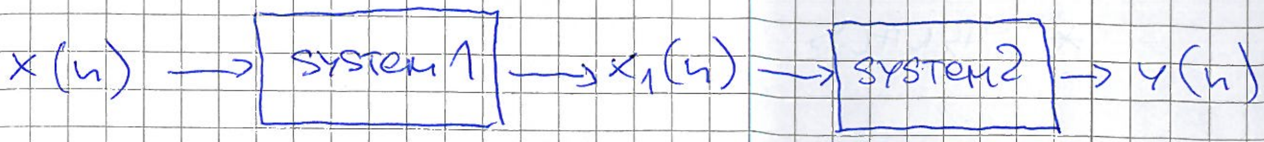
$$x(n) \rightarrow y(n)$$

$x(n)$ - INPUT SIGNAL

$y(n)$ - OUTPUT SIGNAL

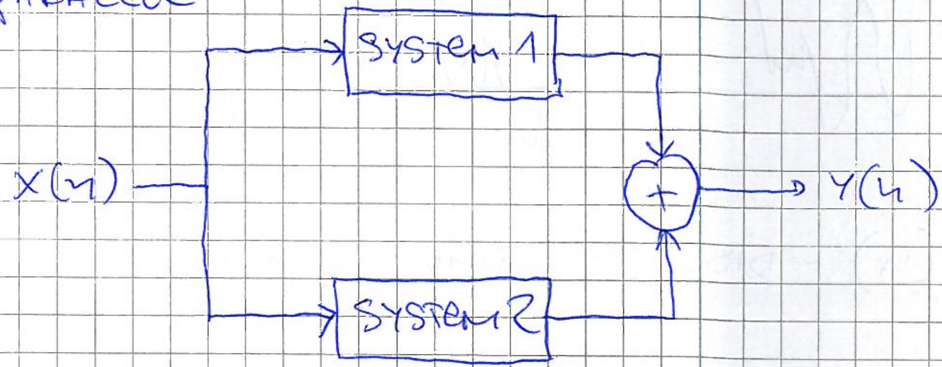
* INTERCONNECTIONS OF SIGNALS

- SERIES

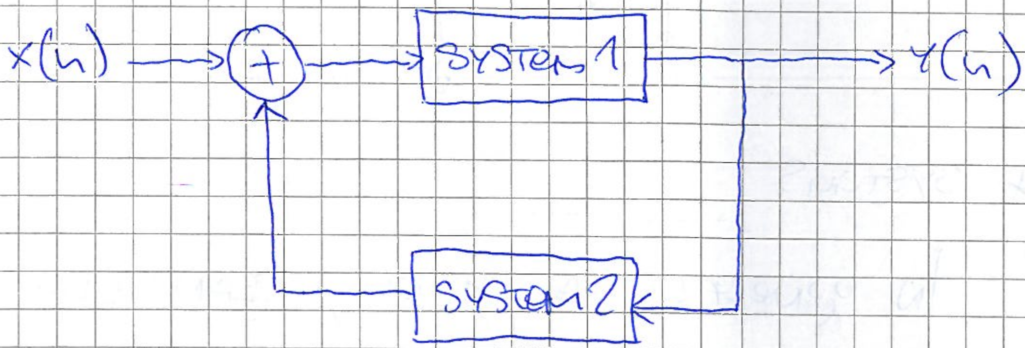


ORDER IS IMPORTANT !

- PARALLEL



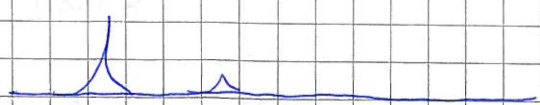
- FEEDBACK



* DOMAINS

TIME

FREQUENCY



x(t), x(n)

FOURIER TRANSFORM

Z - TRANSFORM

* CONTINUOUS TIME SINUSOIDAL SIGNALS

$x(t) = A\cos(w_0 t + \varphi)$

A - AMPLITUDE

w_0 - FREQUENCY

φ - PHASE

T - PERIOD

- PERIODIC

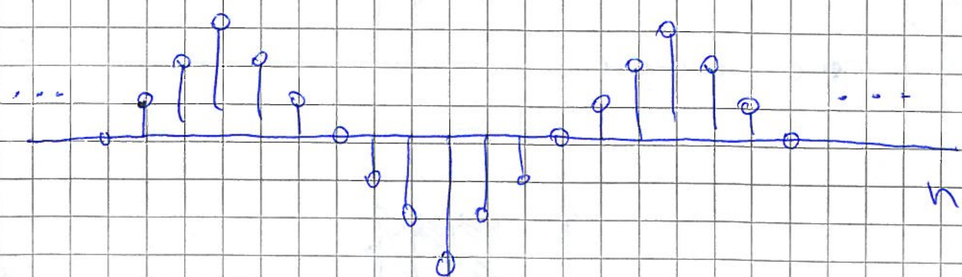
$$x(t) = x(t+T)$$

- SYMMETRY

$$x(t) = x(-t) \quad , \text{cos is even}$$

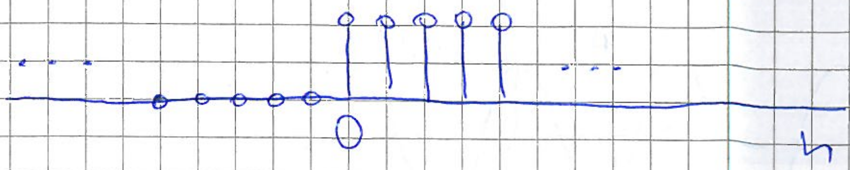
$$x(t) = -x(-t) \quad , \text{sin is ODD}$$

* DISCRETE TIME SINUSOIDAL SIGNALS



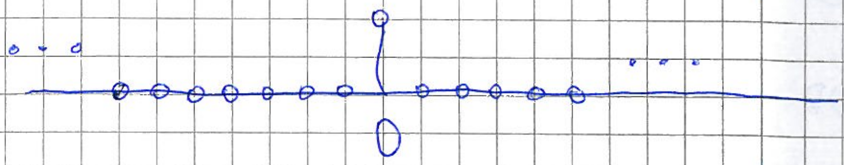
$x(n) = A\cos(w_0 n + \varphi)$

* DISCRETE UNIT STEP FUNCTION



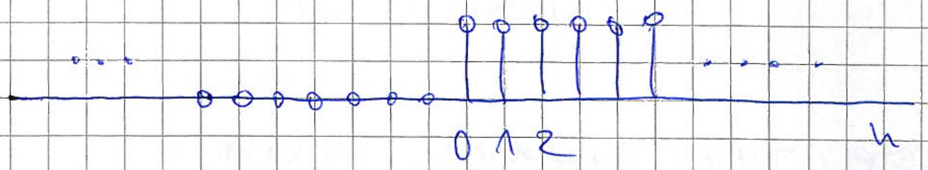
$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

* DISCRETE UNIT IMPULSE FUNCTION

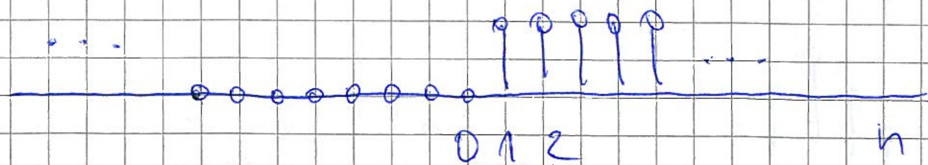


$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

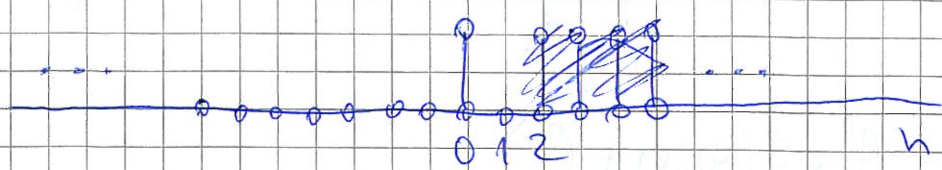
$$\boxed{\delta(n) = u(n) - u(n-1)}$$



$u(n)$

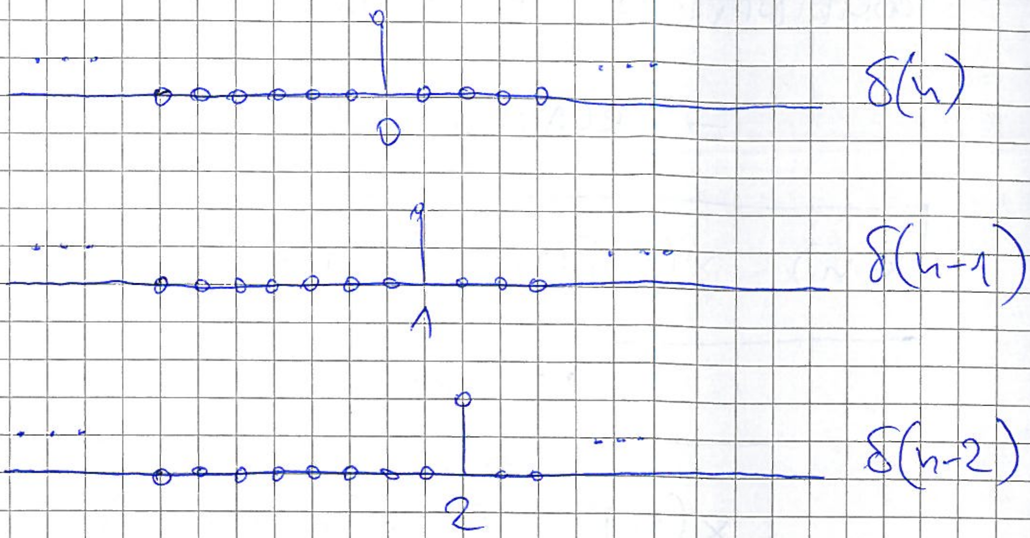


$u(n-1)$



$u(n) - u(n-1)$

$$\boxed{u(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \dots}$$



* LTI

- LINEARITY

$$x_1(n) \rightarrow y_1(n)$$

$$x_2(n) \rightarrow y_2(n)$$

$$\boxed{a \cdot x_1(n) + b \cdot x_2(n) \rightarrow a \cdot y_1(n) + b \cdot y_2(n)}$$

- TIME INVARIANC

$$x(n) \rightarrow y(n)$$

$$\boxed{x(n-T) \rightarrow y(n-T)}$$

* STRATEGY

DECOMPOSE THE INPUT SIGNAL INTO LINEAR COMBINATION OF BASIC SIGNALS

- DELAYED IMPULSES → CONVOLUTION

$$\boxed{x(n) = x(0)\delta(n) + x(1)\delta(n-1) + \dots + x(k)\delta(n-k)}$$

DECOMPOSITION INTO DELAYED IMPULSOS

$$\boxed{x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)}$$

LINEAR COMBINATION OF BASIC SIGNALS

- FOR LTI

$$\boxed{\begin{gathered} y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \\ y(n) = x(n) * h(n) \end{gathered}}$$

IF WE KNOW THE SYSTEM RESPONSE ($h(n)$) FOR $\delta(n)$, WE CAN FIND THE OUTPUT BY CONVOLUTION.

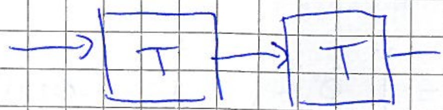
1) DRAW THE BLOCK DIAGRAM OF A SYSTEM DESCRIBED BY THE FOLLOWING EQUATION.

- GENERAL EQUATION

$$y(n) = x(n) + a\,x(n-k)$$

- DELAY

$x(n-1)$ 

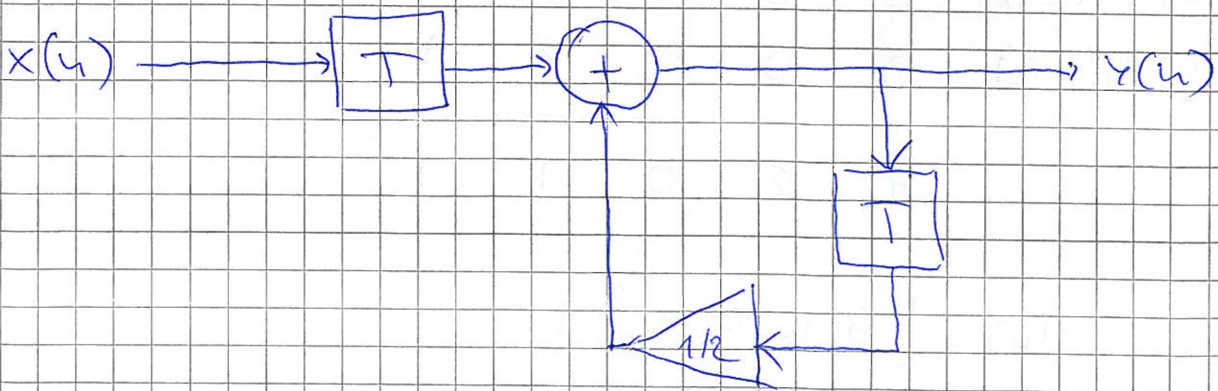
$x(n-2)$ 

- ADDITION

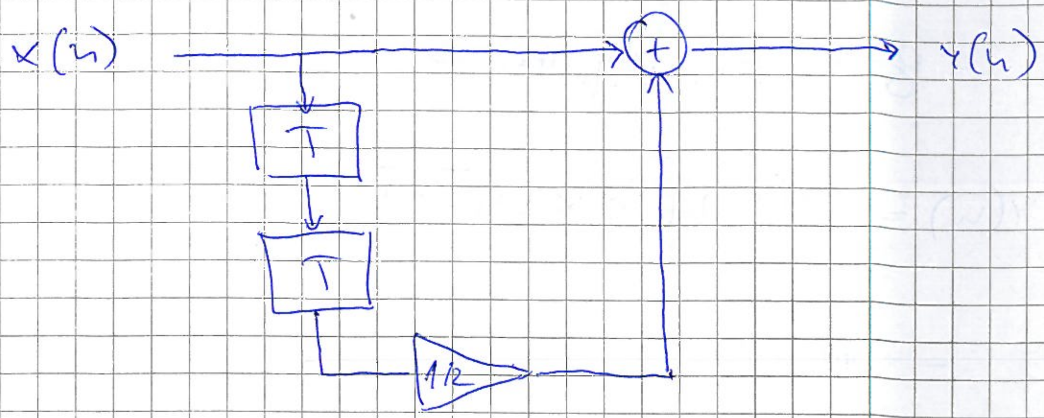
- MULTIPLICATION

a) $y(n) - \frac{1}{2}y(n-1) = x(n-1)$

$$y(n) = x(n-1) + \frac{1}{2}y(n-1)$$



b) $y(n) = x(n) + \frac{1}{2} x(n-2)$



② A SYSTEM HAS THE FOLLOWING IMPULSE RESPONSE $h(n) = [3, 2, 1]$. COMPUTE THE OUTPUT OF THE SYSTEM IF THE INPUT IS,

a) $x(n) = [3, 2, 1]$

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$

n	0	1	2	3	4	5
$x(n)$	3	2	1			
$h(n)$	3	2	1			
$x(0) \cdot h(n-0)$	9	6	3			
$x(1) \cdot h(n-1)$	0	6	4	2		
$x(2) \cdot h(n-2)$	0	0	3	2	1	
	9	12	10	4	1	

$\Rightarrow y(n) = [9, 12, 10, 4, 1]$

b)

$$x(n) = [1, 2, 3]$$

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$

n	0	1	2	3	4	5
$x(n)$	1	2	3			
$h(n)$	3	2	1			
$x(0) \cdot h(n-0)$	3	2	1			
$x(1) \cdot h(n-1)$	0	6	4	2		
$x(2) \cdot h(n-2)$	0	0	9	6	3	
	3	8	14	8	3	

$$\Rightarrow y(n) = [3, 8, 14, 8, 3]$$

3) A SYSTEM IS DESCRIBED BY THE FOLLOWING EQUATION,

$$y(n) = \frac{1}{2} x(n) + \frac{1}{2} x(n-2)$$

THE SYSTEM IS FED WITH INPUT SIGNAL,

$$x(n) = \cos(\omega_0 n)$$

DETERMINE THE OUTPUT SIGNAL WHEN,

a) $\omega_0 = 0$

b) $\omega_0 = \pi/4$

c) $\omega_0 = \pi/2$

d) $\omega_0 = 3\pi/4$

e) $\omega_0 = \pi$

$$\Rightarrow y(n) = \frac{1}{2}\cos(\omega_0 n) + \frac{1}{2}\cos(\omega_0(n-2))$$

$$\boxed{\cos\varphi = \frac{e^{j\varphi} + e^{-j\varphi}}{2}}$$

$$\Rightarrow y(n) = \frac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{4} + \frac{e^{j\omega_0(n-2)} + e^{-j\omega_0(n-2)}}{4}$$

$$\Rightarrow y(n) = \frac{1}{4}\left(e^{j\omega_0 n} + e^{-j\omega_0 n} + e^{j\omega_0 n} \cdot e^{-2j\omega_0} + e^{-j\omega_0 n} \cdot e^{2j\omega_0}\right)$$

$$\Rightarrow y(n) = \frac{1}{4}\left(e^{j\omega_0 n}\left(1 + e^{-2j\omega_0}\right) + e^{-j\omega_0 n}\left(1 + e^{2j\omega_0}\right)\right)$$

$$\boxed{\begin{aligned} e^{j\omega_0 - j\omega_0} &= e^{-j\omega_0 + j\omega_0} = e^{0} = 1 \\ e^{-2j\omega_0} &= e^{-j\omega_0} \cdot e^{-j\omega_0} \\ e^{2j\omega_0} &= e^{j\omega_0} \cdot e^{j\omega_0} \end{aligned}}$$

$$\Rightarrow y(n) = \frac{1}{4}\left(e^{j\omega_0 n}\left(e^{-j\omega_0 + j\omega_0} + e^{-2j\omega_0}\right) + e^{-j\omega_0 n}\left(e^{j\omega_0 - j\omega_0} + e^{2j\omega_0}\right)\right)$$

$$\boxed{e^{-j\omega_0 + j\omega_0} = e^{-j\omega_0} \cdot e^{j\omega_0}}$$

$$\Rightarrow y(n) = \frac{1}{4}\left(e^{j\omega_0 n}\left(e^{-j\omega_0} \cdot e^{j\omega_0} + e^{-j\omega_0} \cdot e^{-j\omega_0}\right) + e^{-j\omega_0 n}\left(e^{j\omega_0} \cdot e^{-j\omega_0} + e^{j\omega_0} \cdot e^{j\omega_0}\right)\right)$$

$$\Rightarrow y(n) = \frac{1}{4}\left(e^{j\omega_0 n} \cdot e^{-j\omega_0}\left(e^{j\omega_0} + e^{-j\omega_0}\right) + e^{-j\omega_0 n} \cdot e^{j\omega_0}\left(e^{j\omega_0} + e^{-j\omega_0}\right)\right)$$

$$\Rightarrow y(n) = \frac{1}{4}\left(\underbrace{\left(e^{j\omega_0} + e^{-j\omega_0}\right)}_{= 2\cos\omega_0}\underbrace{\left(e^{j\omega_0(n-1)} + e^{-j\omega_0(n-1)}\right)}_{= 2\cos(\omega_0(n-1))}\right)$$

$$\Rightarrow y(n) = \cos\omega_0 \cdot \cos(\omega_0(n-1))$$

a) $y(n) = \cos 0 \cdot \cos 0 = 1$

b) $y(n) = \cos\frac{\pi}{4} \cdot \cos\left(\frac{\pi}{4}(n-1)\right) = \frac{\sqrt{2}}{2}\cos\left(\frac{\pi}{4}(n-1)\right)$

c) $y(n) = \cos\frac{\pi}{2} \cdot \cos\left(\frac{\pi}{2}(n-1)\right) = 0$

d) $y(n) = \cos\frac{3\pi}{4} \cdot \cos\left(\frac{3\pi}{4}(n-1)\right) = \frac{\sqrt{2}}{2}\cos\left(\frac{3\pi}{4}(n-1)\right)$

e) $y(n) = \cos\pi \cos(\pi(n-1)) = \cos^2\pi \cdot \cos(\pi n) = \cos\pi n$