PUT IT ALL TOGETHER

$$\underbrace{\begin{bmatrix} I & -FH^{-1} \\ O & I \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} E & F \\ G & H \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} I & O \\ -H^{-1}G & I \end{bmatrix}}_{B} = \underbrace{\begin{bmatrix} E-FH^{-1}G & O \\ O & H \end{bmatrix}}_{C}$$

$$(AMB)^{-1} = C^{-1}$$

$$(AMB)^{-1} = B^{-1}M^{-1}A^{-1}$$

$$B^{-1}M^{-1}A^{-1} = C^{-1}$$

$$\underbrace{BB^{-1}}_{I}M^{-1}A^{-1} = BC^{-1} \Rightarrow M^{-1}A^{-1} = BC^{-1}$$

$$M^{-1}A^{-1}A = BC^{-1}A \Rightarrow M^{-1} = BC^{-1}A$$

$$M^{-1} = \begin{bmatrix} I & O \\ -HG^{-1} & I \end{bmatrix} \begin{bmatrix} (E-FH^{-1}G)^{-1} & O \\ O & H^{-1} \end{bmatrix} \begin{bmatrix} I & -FH^{-1} \\ O & I \end{bmatrix} =$$

$$= \begin{bmatrix} (E-FH^{-1}G)^{-1} & -(E-FH^{-1}G)^{-1}FH^{-1} \\ -H^{-1}G(E-FH^{-1}G)^{-1} & H^{-1}+H^{-1}G(E-FH^{-1}G)^{-1}FH^{-1} \end{bmatrix}$$

Now we have our inverse co-varians.

The term $E-FH^{-1}G$ is the Schur complement of M with respect to H (M/H).