Look at exponent.

$$-\frac{1}{2}\begin{pmatrix} x_1-\mu_1 \\ x_2-\mu_2 \end{pmatrix}^T \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}^{-1} \begin{bmatrix} x_1-\mu_1 \\ x_2-\mu_2 \end{bmatrix} =$$

$$= -\frac{1}{2}\begin{bmatrix} x_1-\mu_1 \\ x_2-\mu_2 \end{bmatrix}^T \begin{bmatrix} I & 0 \\ -\Sigma_{22}^{-1}\Sigma_{21} & I \end{bmatrix} \begin{bmatrix} (\Sigma/\Sigma_{22})^{-1} & 0 \\ 0 & \Sigma_{22}^{-1} \end{bmatrix} \begin{bmatrix} I & -\Sigma_{12}\Sigma_{22}^{-1} \\ 0 & I \end{bmatrix} \begin{bmatrix} x_1-\mu_1 \\ x_2-\mu_2 \end{bmatrix} =$$

$$= -\frac{1}{2}\begin{bmatrix} x_1-\mu_1 \\ x_2-\mu_2 \end{bmatrix}^T \begin{bmatrix} I & 0 \\ -\Sigma_{22}^{-1}\Sigma_{21} & I \end{bmatrix} \begin{bmatrix} (\Sigma/\Sigma_{22})^{-1} & -(\Sigma/\Sigma_{22})^{-1}\Sigma_{12}\Sigma_{22}^{-1} \\ 0 & \Sigma_{22}^{-1} \end{bmatrix} \begin{bmatrix} x_1-\mu_1 \\ x_2-\mu_2 \end{bmatrix} =$$

$$= -\frac{1}{2}\begin{bmatrix} x_1-\mu_1 \\ x_2-\mu_2 \end{bmatrix}^T \begin{bmatrix} (\Sigma/\Sigma_{22})^{-1} & -(\Sigma/\Sigma_{22})^{-1}\Sigma_{12}\Sigma_{22}^{-1} \\ -\Sigma_{21}\Sigma_{22}^{-1}(\Sigma/\Sigma_{22})^{-1} & \Sigma_{22}^{-1} \end{bmatrix} \begin{bmatrix} x_1-\mu_1 \\ x_2-\mu_2 \end{bmatrix}$$

$$= -\frac{1}{2}\left(x_1-\mu_1-\Sigma_{21}\Sigma_{22}^{-1}(x_2-\mu_2)\right)^T(\Sigma/\Sigma_{22})^{-1}\left(x-\mu_1-\Sigma_{12}\Sigma_{22}^{-1}(x_2-\mu)\right) - \underbrace{-\frac{1}{2}(x_2-\mu_2)^T\Sigma_{22}^{-1}(x_2-\mu_2)}_{p(x_2)}$$

$$\Rightarrow p(x_1|x_2) \propto \exp\left(-\frac{1}{2}(x_1-\mu_1-\Sigma_{21}\Sigma_{22}^{-1}(x_2-\mu_2))^T(\Sigma/\Sigma_{22})^{-1}(x-\mu_1-\Sigma_{12}\Sigma_{22}^{-1}(x_2-\mu))\right)$$

$\Rightarrow$ mean: $\mu_{1|2} = \mu_1 + \Sigma_{21}\Sigma_{22}^{-1}(x_2-\mu_2)$

variance: $\Sigma_{1|2} = \Sigma/\Sigma_{22} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$