C: $-\frac{1}{2\sigma^2}(X\omega)^T(X\omega) - \frac{1}{2}\omega^T\Sigma^{-1}\omega =$

$= \left\{(AB)^T = B^TA^T\right\} = -\frac{1}{2\sigma^2}\omega^T X^T X\omega - \frac{1}{2}\omega^T\Sigma^{-1}\omega =$

$= -\frac{1}{2}\omega^T\left(\frac{1}{\sigma^2}X^TX\right)\omega - \frac{1}{2}\omega^T\Sigma^{-1}\omega =$

$= -\frac{1}{2}\omega^T\left(\frac{1}{\sigma^2}X^TX + \Sigma^{-1}\right)\omega$

Identify: $S^{-1} = \frac{1}{\sigma^2}X^TX + \Sigma^{-1}$

B: $\frac{1}{\sigma^2}y^T(X\omega) = \left\{(AB)^T = B^TA^T\right\} = \frac{1}{\sigma^2}\omega^TX^Ty$

Where does the mean pop up?

– In the mixed term

$x^T\Sigma^{-1}\mu \Rightarrow \omega^TS^{-1}\mu = \omega^T\left(\frac{1}{\sigma^2}X^TX + \Sigma^{-1}\right)\mu = \frac{1}{\sigma^2}\omega^TX^Ty$

Solve for μ:

$$\cancel{\omega^T}\left(\frac{1}{\sigma^2}X^TX + \Sigma^{-1}\right)\mu = \frac{1}{\sigma^2}\cancel{\omega^T}X^Ty$$

$$\left(\frac{1}{\sigma^2}X^TX + \Sigma^{-1}\right)\mu = \frac{1}{\sigma^2}X^Ty$$

$$\Rightarrow \mu = \frac{1}{\sigma^2}\left(\frac{1}{\sigma^2}X^TX + \Sigma^{-1}\right)X^Ty$$

$p(\omega|Y,X) \propto N\left(\frac{1}{\sigma^2}\left(\frac{1}{\sigma^2}X^TX + \Sigma^{-1}\right)X^Ty, \frac{1}{\sigma^2}X^TX + \Sigma^{-1}\right)$

This procedure is called "Completing the Square"