Posterior Distribution

General normal:

$$N(\mu, \Sigma) \propto e^{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)} =$$

$$= e^{-\frac{1}{2}x^T\Sigma^{-1}x} \cdot e^{x^T\Sigma^{-1}\mu} \cdot \underbrace{e^{-\frac{1}{2}\mu^T\Sigma^{-1}\mu}}_{\text{const.}} \qquad (*)$$

$$p(y|w, x) = N(wx, \sigma^2 I)$$

$$p(w) = N(0, \Sigma^{-1})$$

$$p(w|y, x) \propto p(y|w, x)p(w) \propto$$

$$\propto e^{-\frac{1}{2\sigma^2}(y - Xw)^T(y - Xw)} \cdot e^{-\frac{1}{2}w^T\Sigma^{-1}w}$$

Lets look at the exponent

$$\Rightarrow -\frac{1}{2\sigma^2}(y - Xw)^T(y - Xw) - \frac{1}{2}w^T\Sigma^{-1}w = \{(*)\} =$$

$$= \underbrace{-\frac{1}{2\sigma^2}y^Ty}_{A} + \underbrace{\frac{1}{\sigma^2}y^T(Xw)}_{B} \underbrace{- \frac{1}{2\sigma^2}(Xw)^T(Xw) - \frac{1}{2}w^T\Sigma^{-1}w}_{C}$$

Identify: A - our new constant term
B - our new mixed term
C - the new term with quadratic in parameters.