FOURIER ANALYSIS

* CONTINUOUS - TIME FOURIER SERIES

- PERIODIC

$$x(t) = x(t+T_0)$$

- SYMMETRY

$$x(t) = x(-t) \quad \text{- even}$$
$$x(t) = -x(-t) \quad \text{- odd}$$

T_0 - period

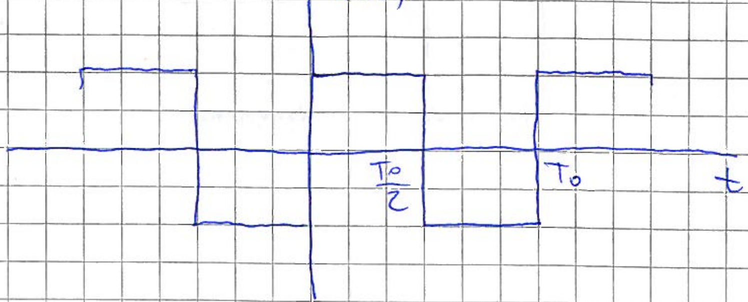
$w_0 = \frac{2\pi}{T_0}$ - FUNDAMENTAL FREQUENCY

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jkw_0 t}} \quad \text{SYNTHESIS}$$

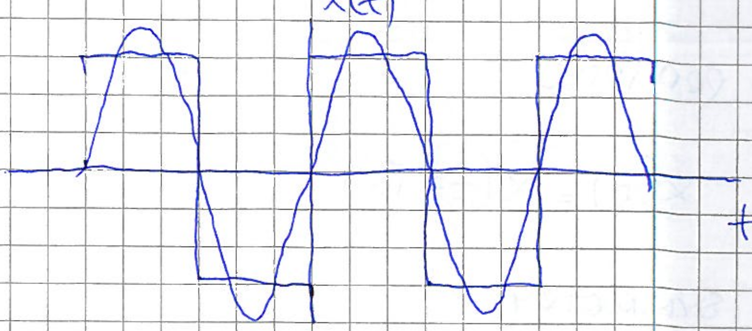
PERIODIC SIGNALS CAN BE REPRESENTED AS A LINEAR COMBINATION OF COMPLEX EXPONENTIALS

$$\boxed{c_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jkw_0 t} \, dt} \quad \text{ANALYSIS}$$

EXAMPLE - ASSYMETRIC PERIODIC SQUARE WAVE



$$c_k = \begin{cases} -\dfrac{2j}{k\pi} & , \; k = 1, 3, 5, \dots \\ 0 & , \; k = 0, 2, 4, \dots \end{cases}$$

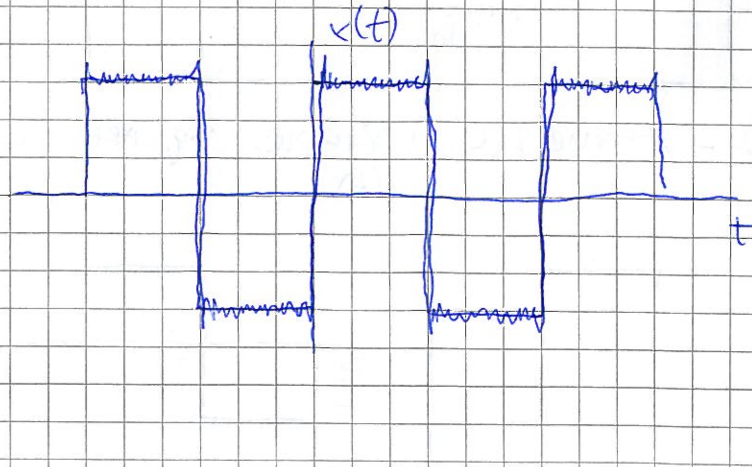


$k = 1$

$k = 3$

$k = 5$

$\vdots$



$k = 101$

* DISCRETE-TIME FOURIER TRANSFORM (DTFT)

$$\boxed{X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}}$$

SPECIAL CASE OF Z-TRANSFORM

$$z = e^{j\omega}$$

* DISCRETE FOURIER TRANSFORM (DFT)

$$\boxed{\begin{aligned} X(k) &= \sum_{n=0}^{N-1} x(n) e^{-jnk\frac{2\pi}{N}} \\ x(n) &= \frac{1}{N} \sum_{n=0}^{N-1} X(k) e^{jnk\frac{2\pi}{N}} \end{aligned}}$$

$x(n)$ - AMPLITUDE OF INPUT SIGNAL

$X(k)$ - SPECTRUM AT FREQUENCY $\frac{2\pi k}{N}$

IMPLEMENTED WITH FFT

1. GIVEN THE SIGNAL

$$x(n) = u(n) - u(n-6)$$

DETERMINE $|X(k)|$, WHERE $X(k)$ IS THE FIRST N POINTS OF THE DFT OF $x(n)$.

a) $N = 6$

b) $N = 12$

$$X(k) = \sum_{n=0}^{N-1} \left(u(n) - u(n-6)\right) \cdot e^{-jnk\frac{2\pi}{N}}$$

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$u(n-6) = \begin{cases} 1, & n \geq 6 \\ 0, & n < 6 \end{cases}$$

a) $$X(k) = \sum_{n=0}^{5} e^{-jnk\frac{\pi}{3}}$$

$$\boxed{\sum_{k=0}^{n} a^n = \frac{a^{n+1} - 1}{a - 1}}$$ geometric series

$$\Rightarrow \quad X(k) = \frac{e^{-jk2\pi} - 1}{e^{-jk\frac{\pi}{3}} - 1}$$