ÖiDS
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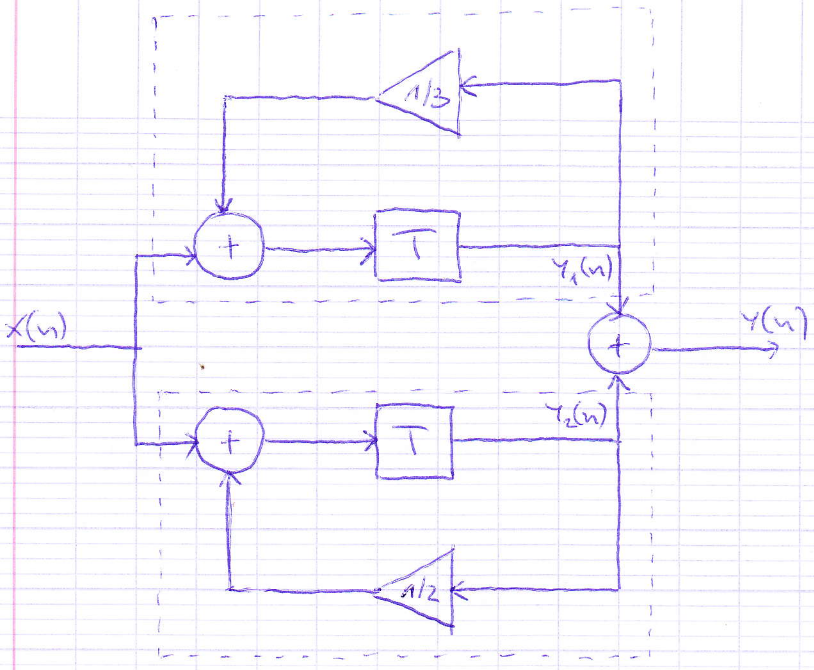

* Let's divide the circuit into 2 parts - this is represented by the dashed lines.

* Then the output $y(n)$ is the sum of the output of the first part ($y_1(n)$) and the second part ($y_2(n)$)

$$\boxed{\begin{aligned} y(n) &= y_1(n) + y_2(n) \\ Y(z) &= Y_1(z) + Y_2(z) \end{aligned}}$$

Then,

$$y_1(n) = x(n-1) + \frac{1}{3} y_1(n-1)$$

$$Y_1(z) = z^{-1} \cdot X(z) + \frac{1}{3} z^{-1} Y_1(z) \Rightarrow Y_1(z) = \frac{z^{-1}}{1 - \frac{1}{3} z^{-1}} \cdot X(z)$$

$$y_2(n) = x(n-1) + \frac{1}{2} y_2(n-1)$$

$$Y_2(z) = z^{-1} \cdot X(z) + \frac{1}{2} z^{-1} \cdot Y_2(z) \Rightarrow Y_2(z) = \frac{z^{-1}}{1 - \frac{1}{2} z^{-1}} \cdot X(z)$$

Then,

$$Y(z) = \frac{z^{-1}}{1-\frac{1}{3}z^{-1}} \cdot X(z) + \frac{z^{-1}}{1-\frac{1}{2}z^{-1}} \cdot X(z)$$

$$Y(z) = \underbrace{\left[\frac{z^{-1}}{1-\frac{1}{3}z^{-1}} + \frac{z^{-1}}{1-\frac{1}{2}z^{-1}}\right]}_{H(z)} \cdot X(z)$$

$$\Rightarrow H(z) = \frac{z^{-1}\left(1-\frac{1}{2}z^{-1}\right) + z^{-1}\left(1-\frac{1}{3}z^{-1}\right)}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)}$$

* The poles are the roots of the denominator.

$$\left.\begin{aligned} 1-\frac{1}{3}z^{-1} &= 0 \\ 1-\frac{1}{2}z^{-1} &= 0 \end{aligned}\right\} z_{p_1} = \frac{1}{3}, \quad z_{p_2} = \frac{1}{2}$$

$\Rightarrow$ it is stable!

* Simplest form of $H(z)$

$$H(z) = \frac{z^{-1} - \frac{1}{2}z^{-2} + z^{-1} - \frac{1}{3}z^{-2}}{1 - \frac{1}{2}z^{-1} - \frac{1}{3}z^{-1} + \frac{1}{6}z^{-2}}$$

$$H(z) = \frac{2z^{-1} - \frac{5}{6}z^{-2}}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}}$$

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