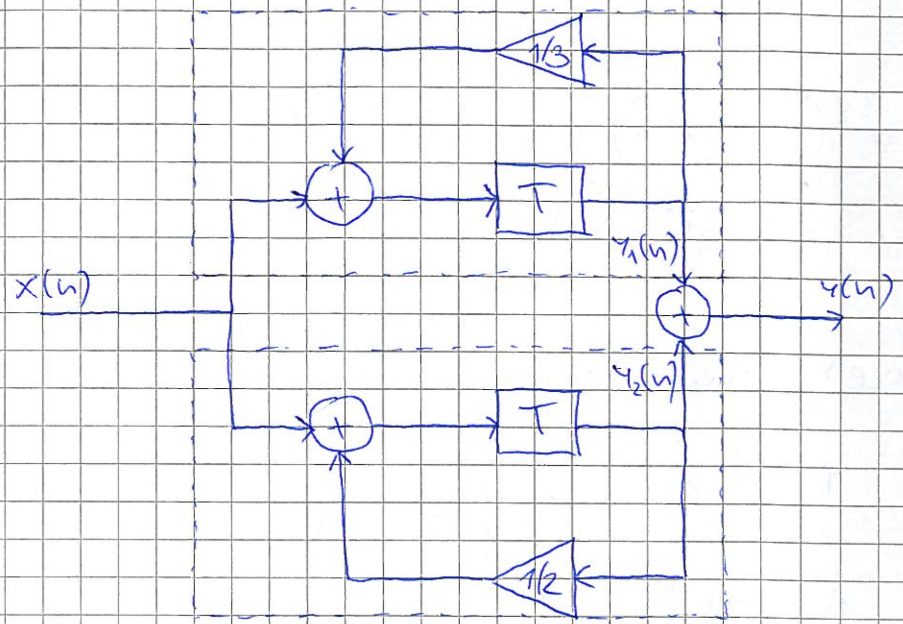
FILTERS

① DETERMINE THE TRANSFER FUNCTION $H(z)$ FOR THE FOLLOWING GRAPH AND FIND WHETHER THE SYSTEM IS STABLE.



$$y(n) = y_1(n) + y_2(n)$$

$$Y(z) = Y_1(z) + Y_2(z)$$

$$y_1(n) = x(n-1) + \frac{1}{3} y_1(n-1)$$

$$Y_1(z) = z^{-1} X(z) + \frac{1}{3} z^{-1} Y_1(z)$$

$$Y_1(z) = \frac{z^{-1}}{1 - \frac{1}{3} z^{-1}} X(z) \quad ①$$

$$y_2(n) = x(n-1) + \frac{1}{2} y_2(n-1)$$

$$Y_2(z) = z^{-1} X(z) + \frac{1}{2} z^{-1} Y_2(z)$$

$$Y_2(z) = \frac{z^{-1}}{1 - \frac{1}{2} z^{-1}} X(z) \quad ②$$

① AND ② $\Rightarrow Y(z) = \dfrac{z^{-1}}{1-\frac{1}{3}z^{-1}} X(z) + \dfrac{z^{-1}}{1-\frac{1}{2}z^{-1}} X(z)$

$$Y(z) = \underbrace{\left(\frac{z^{-1}}{1-\frac{1}{3}z^{-1}} + \frac{z^{-1}}{1-\frac{1}{2}z^{-1}} \right)}_{H(z)} X(z)$$

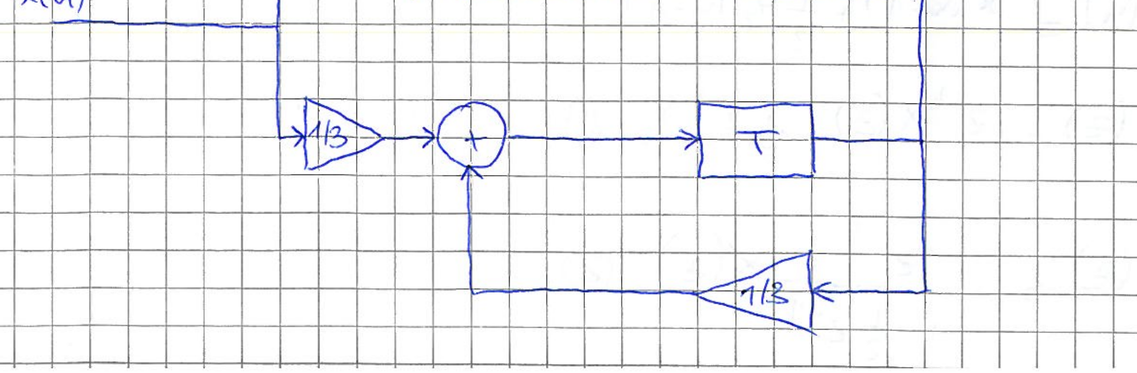
$$\Rightarrow H(z) = \frac{z^{-1}\left(1-\frac{1}{2}z^{-1}\right) + z^{-1}\left(1-\frac{1}{3}z^{-1}\right)}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)}$$

* THE POLES ARE THE ROOTS OF THE DENOMINATOR

$$\left.\begin{array}{l} 1-\frac{1}{3}z^{-1} = 0 \\ 1-\frac{1}{2}z^{-1} = 0 \end{array}\right\} z_1 = \frac{1}{2}, \quad z_2 = \frac{1}{3}$$

$\Rightarrow$ IT IS STABLE $\left(|z_{1,2}| < 1 \right)$

② A DISCRETE-TIME SYSTEM IS GIVEN BY THE FOLLOWING GRAPH. DETERMINE THE VALUE OF THE REAL CONSTANT a, FOR WHICH THE SYSTEM IS STABLE



$$y_1(n) = \frac{1}{3}x(n-1) + \frac{1}{3}y_1(n-1)$$

$$y_1(n) - \frac{1}{3}y_1(n-1) = \frac{1}{3}x(n-1)$$

$$Y_1(z)\left(1 - \frac{1}{3}z^{-1}\right) = \frac{1}{3}z^{-1}X(z) \quad (1)$$

$$y(n) = x(n-1) + y_1(n-1) + a\,y(n-1)$$

$$y(n) - a\,y(n-1) = x(n-1) + y_1(n-1)$$

$$Y(z)\left(1 - az^{-1}\right) = z^{-1}X(z) + z^{-1}Y_1(z) \quad (2)$$

(1) AND (2) $\Rightarrow$ $Y(z)(1-az^{-1}) = z^{-1}X(z) + \dfrac{\frac{1}{3}z^{-1}}{1-\frac{1}{3}z^{-1}} \cdot z^{-1} \cdot X(z)$

$$Y(z)(1-az^{-1}) = X(z)\left(\frac{z^{-1} \cancel{- \frac{1}{3}z^{-2}} \cancel{+ \frac{1}{3}z^{-2}}}{1-\frac{1}{3}z^{-1}}\right)$$

$$Y(z) = X(z)\underbrace{\left(\frac{z^{-1}}{(1-az^{-1})(1-\frac{1}{3}z^{-1})}\right)}_{H(z)}$$

* THE POLES ARE THE ROOTS OF THE DENOMINATOR

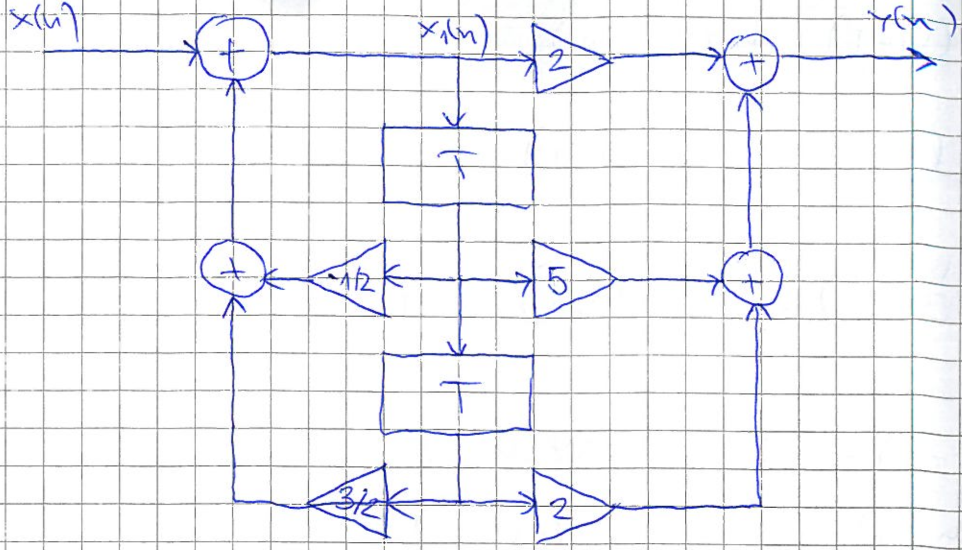
$$\left.\begin{aligned} 1 - az^{-1} &= 0 \\ 1 - \frac{1}{3}z^{-1} &= 0 \end{aligned}\right\} z_1 = \frac{1}{3}, \quad z_2 = a$$

$\Rightarrow$ FOR $|a| < 1$, IT IS STABLE

(3) A DISCRETE-TIME SYSTEM IS GIVEN BY THE GRAPH.

a) DETERMINE THE TRANSFER FUNCTION $H(z)$

b) IS IT STABLE ?



$$y(n) = 2x_1(n) + 5x_1(n-1) + 2x_1(n-2) \qquad (1)$$

$$x_1(n) = x(n) - \frac{1}{2}x_1(n-1) + \frac{3}{2}x_1(n-2)$$

$$X_1(z) = X(z) - \frac{1}{2}z^{-1}X_1(z) + \frac{3}{2}z^{-2}X_1(z)$$

$$X_1(z)\left(1 + \frac{1}{2}z^{-1} - \frac{3}{2}z^{-2}\right) = X(z)$$

$$X_1(z) = \frac{X(z)}{\left(1 + \frac{1}{2}z^{-1} - \frac{3}{2}z^{-2}\right)} \qquad (2)$$

(1) AND (2) $\Rightarrow$ $Y(z) = 2X_1(z) + 5X_1(z)z^{-1} + 2X_1(z)z^{-2}$

$$Y(z) = \underbrace{\left(\frac{2 + 5z^{-1} + 2z^{-2}}{1 + \frac{1}{2}z^{-1} - \frac{3}{2}z^{-2}}\right)}_{H(z)} X(z)$$

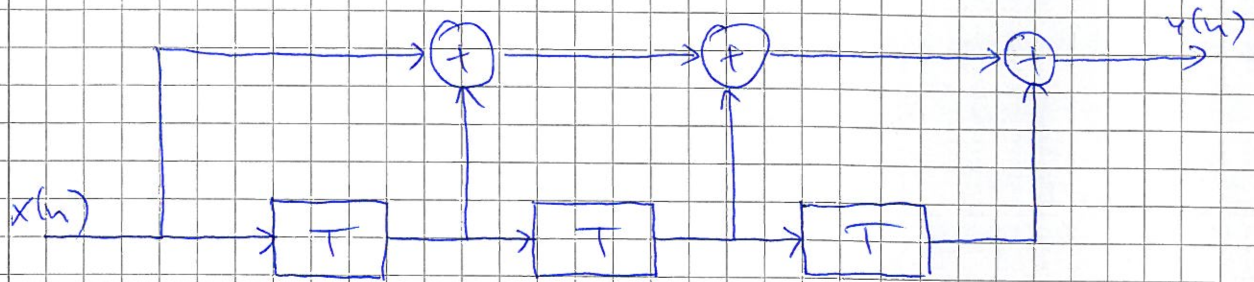
$$\Rightarrow H(z) = \frac{2z^2 + 5z + 2}{z^2 + \frac{1}{2}z - \frac{3}{2}}$$

$$z^2 + \frac{1}{2}z - \frac{3}{2} = 0$$

$$(z-1)\left(z+\frac{3}{2}\right) = 0$$

$$\Rightarrow z_1 = 1 \ , \ z_2 = -\frac{3}{2}$$ – IT IS NOT STABLE

(4) GIVEN THE FILTER DIAGRAM, FIND THE POLES AND THE ZEROS OF $H(z)$. FIND THE MAGNITUDE OF THE FREQUENCY RESPONCE $|H(e^{j\omega})|$.



$$y(n) = x(n) + x(n-1) + x(n-2) + x(n-3)$$

$$Y(z) = X(z) + X(z)z^{-1} + X(z)z^{-2} + X(z)z^{-3}$$

$$Y(z) = X(z)\underbrace{\left(1 + z^{-1} + z^{-2} + z^{-3}\right)}_{H(z)}$$

$$\Rightarrow H(z) = 1 + z^{-1} + z^{-2} + z^{-3}$$

* POLES $z_{1,2,3} = 0$

* ZEROS $z_1 = -1 \ , \ z_{2,3} = \pm j$

$$H(e^{j\omega}) = e^{-0j\omega} + e^{-1j\omega} + e^{-2j\omega} + e^{-3j\omega} = e^{-1,5j\omega}\left(e^{1,5j\omega} + e^{0,5j\omega} + e^{-0,5j\omega} + e^{-1,5j\omega}\right) =$$

$$H(e^{j\omega}) = e^{-1,5j\omega}\left(2\cos(1,5\omega) + 2\cos(0,5\omega)\right)$$

$$\left|H(e^{j\omega})\right| = 2\underbrace{\left|e^{-1,5j\omega}\right|}_{1}\left|\cos\frac{3\omega}{2} + \cos\frac{\omega}{2}\right|$$

$$\Rightarrow \left|H(e^{j\omega})\right| = 2\left|\cos\frac{3\omega}{2} + \cos\frac{\omega}{2}\right|$$