



KTH Electrical Engineering

Examination in **POWER ELECTRONICS** (EJ2300)

Saturday, 26<sup>th</sup> of October 2013, 9:00 – 14:00 (Questions 10:00, 11:30, and 13:00)

Exam responsible: Hans-Peter Nee, 070 – 695 34 70

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Permitted aid: Calculator, mathematical handbook, and three-phase paper.

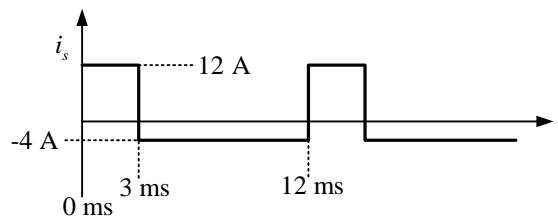
The requirement to pass is 15 points. Problems 1–5 give 3 points each and problems 6–8 give 5 points each.

**For full marks, the solutions have to be readable and *the line of thought has to be recognizable!***

**Do not forget to scale axes of graphs!**

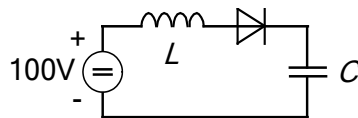
**Only one problem per paper! (Because different persons will mark different problems!)**

1. In the figure below the waveform of a current is given. What is the RMS value of the current and what is the frequency of the fundamental component?



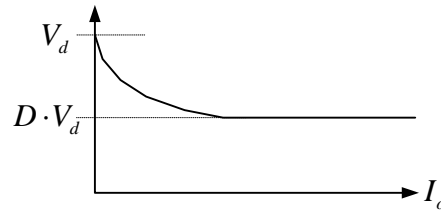
2. An ideal Buck-Boost converter has an input voltage of 540 V and an output voltage of 2000 V. What is the minimum voltage rating of the switch if everything is ideal and if no overvoltages are considered.

3. For the circuit in the figure below, draw the waveform of the diode current for  $t > 0$ . Initial values:  $I_{L0} = 0$ ;  $V_{C0} = 0$ .  $L = 1$  mH;  $C = 1$   $\mu$ F

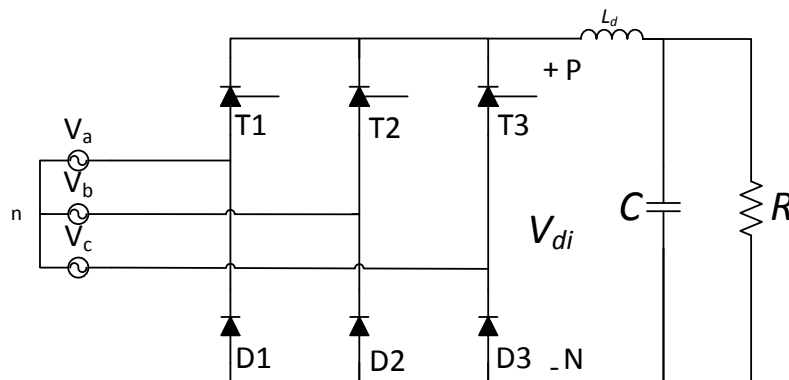


4. A single-phase thyristor rectifier is connected to an ac source  $V_s$ , and is driving a dc motor, whose armature inductance  $L_a$ , is considered to be relatively small. The dc-side current is thus a discontinuous current, with a zero interval equal to the half of the firing angle  $\alpha$ . Estimate the voltage at the motor terminals during the time the dc-side current is zero, as a function of  $V_s$  and  $\alpha$ . Assume that  $R_a = 0$ . (3p)

5. A step-down dc-dc converter operates with a constant duty ratio at a high output current. The current decreases and suddenly the output voltage increases. It is found that the output voltage has an output-current dependence as shown in the figure below. Explain what is actually happening at low values of the output current.

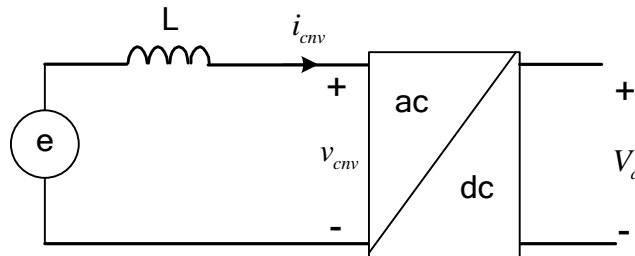


6. The three-phase converter shown in the figure below is supplied from a balanced three-phase having an rms line-to-neutral voltage of 230 V at 50 Hz. The upper branch of this converter consists of three thyristors which are controlled by the firing angle  $\alpha$ , while normal diodes are employed in the lower branch. Consider that the inductor,  $L_d$ , connected on the output is very large, and that the output filter capacitor is also very large. The commutation inductance is assumed to be negligible.

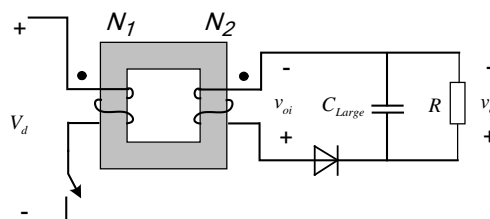


- Draw the waveforms for the output voltage  $V_{di}$ , as this is shown in the circuit diagram above, as well as, the inverse voltage across thyristor T1 when  $\alpha=30^\circ$ . (2 p)
- Calculate the average output voltage which appears across the resistive load if  $\alpha=30^\circ$ . (2 p)
- Instead of the resistive load, a dc generator is connected on the dc-side. Can this converter supply negative power (i.e. power supplied for the dc-side to the ac-side)? Motivate your answer. (1 p)

7. The figure below shows a single-phase full-bridge switch-mode converter. The converter is PWM-modulated and the switching frequency is high. The converter is connected to a strong 50 Hz-grid with  $E = 230\text{ V}$ , via an inductor  $L = 15\text{ mH}$ , which is not considered to be part of the grid. The direct voltage  $V_d = 400\text{ V}$ . The converter is controlled in such a way that  $I_{cnv(1)} = 10\text{ A}$  and that the grid does not have to supply any reactive power. Find the amplitude modulation ratio  $m_a$ .



8. A flyback converter operating in the *discontinuous* conduction mode is supplied from  $V_d = 100\text{ V}$ . The switching frequency is  $100\text{ kHz}$  and the switch duty ratio  $D$  is  $0,3$ . The transformer's magnetizing inductance is  $L_m = 100\text{ }\mu\text{H}$  and the turns ratio is  $N_1 / N_2 = 1/1$ . Assume that the load condition is such that the output voltage  $V_o = 100\text{ V}$ . (Apart from the magnetizing inductance of the transformer, all components are assumed to be ideal.)  
 Draw the waveform of the internal output voltage  $v_{oi}(t)$ .  
 Draw the waveforms of the primary and secondary current.  
 How much energy is transferred from the input to the output side every cycle?  
 Find the output power.  
 Find the numerical value of the load resistor  $R$ .  
 Find the peak voltage over the switch.



## Solutions

$$1. I_{RMS} = \sqrt{\frac{1}{T} \cdot \left( \int_0^{T_+} I_+^2 dt + \int_{T_+}^T I_-^2 dt \right)} = \sqrt{\frac{1}{12 \cdot 10^{-3}} \cdot \left( \int_0^{3 \cdot 10^{-3}} 144 dt + \int_{3 \cdot 10^{-3}}^{12 \cdot 10^{-3}} 16 dt \right)} = \underline{6,9 \text{ A}}$$

$$f_1 = \frac{1}{12 \cdot 10^{-3}} = \underline{83 \text{ Hz}}$$

2. The voltage rating of the switch must be at least the input voltage plus the output voltage. When the switch is off and the diode conducts, the emitter of the switch is connected to the negative output voltage, and the collector of the switch is connected to the positive input voltage.

The minimum voltage rating is thus 2540 V.

3.

$100V$

$I_L=0$   
 $V_C=0$

$$\begin{cases} 100 = L \frac{di_L}{dt} + V_C \\ i_L = C \frac{dV_C}{dt} \end{cases} \Rightarrow 0 = LC \frac{d^2 i_L}{dt^2} + i_L$$

$$\Rightarrow i_L = \hat{I}_L \sin \frac{1}{\sqrt{LC}} \cdot t$$

$$T/2 = \frac{1}{2} \cdot 2\pi \sqrt{LC} = \underline{99 \mu s}$$

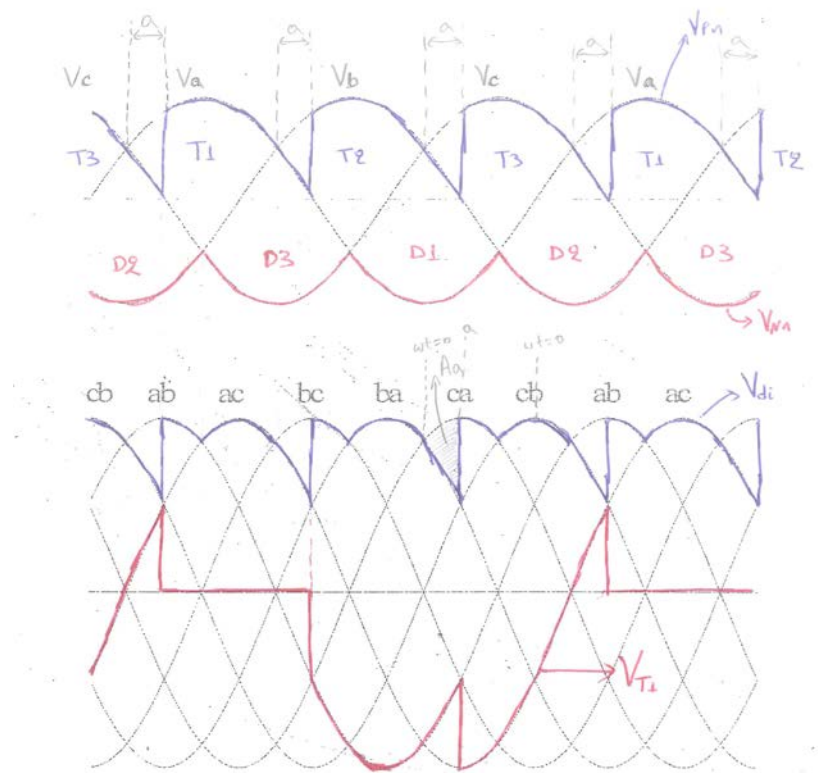
$$Q = \int_0^{T/2} i_L dt ; Q = C \cdot 2V_d \Rightarrow \hat{I}_L = \underline{3,2 A}$$

$$4. V_{avg} = \frac{1}{\pi} \left[ V_{d1} \left( \pi + \frac{\alpha}{2} - \alpha \right) + V_{avg} \frac{\alpha}{2} \right] \Rightarrow V_{avg} = V_{d1}.$$

$$V_{d1} = \frac{1}{\pi - \alpha/2} \int_{\alpha}^{\pi + \alpha/2} \sqrt{2} V_s \sin(\omega t) d\omega t = \frac{2\sqrt{2} V_s}{2\pi - \alpha} \left( \cos \alpha + \cos \frac{\alpha}{2} \right)$$

5. At low values the converter operates in the discontinuous conduction mode. In this mode the output voltage depends on the output current.

6. a)



b) The average output voltage can be calculated in a similar way as for the three-phase diode rectifier, but subtracting the area  $A_a$  shown in the waveforms above. It must also be noted that the time period which is considered for estimating the average output voltage equals  $120^\circ$  instead of  $60^\circ$ .

Thus:

$$A_a = \int_0^\alpha \sqrt{2} \cdot V_{LL} \cdot \sin(\omega t) \cdot d(\omega t) = \sqrt{2} \cdot V_{LL} \cdot [-\cos \alpha + \cos 0]$$

$$= \sqrt{2} \cdot V_{LL} \cdot [1 - \cos \alpha]$$

And then the average output voltage can be calculated as:

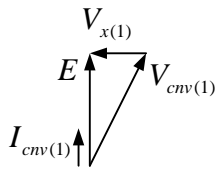
$$V_d = \frac{1}{2\pi} \cdot \left[ 2 \cdot \int_{\frac{\pi}{6}}^{\frac{\pi}{6} + \alpha} \sqrt{2} \cdot V_{LL} \cdot \sin(\omega t) \cdot d(\omega t) - A_a \right] \Rightarrow V_d$$

$$= \frac{3\sqrt{2}V_{LL}}{2\pi} \cdot [1 + \cos \alpha]$$

$$\Rightarrow V_d = \frac{3\sqrt{2}(\sqrt{3} \cdot 230)}{2\pi} \cdot [1 + \cos(30^\circ)] \Rightarrow V_d = 502 \text{ V}$$

- c) If a dc generator is connected on the dc-side the converter cannot supply power to the ac-side, because the converter cannot generate a negative direct voltage. See the waveforms in 5a), where it is obvious that the positive output terminal always has a higher or equal voltage compared to the negative terminal.

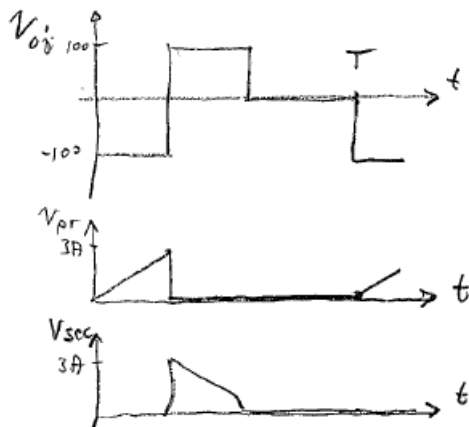
7. If the grid should not supply any reactive power,  $I_{cnv(1)}$  has to be in phase with  $E$ . The following phasor diagram is, thus, obtained.



where  $V_{x(1)} = 15 \cdot 10^{-3} \cdot 2\pi \cdot 50 \cdot 10 = 47,1239 \text{ V}$

$$V_{cn(1)} = \sqrt{230^2 + 47,1239^2} = 234,78 \text{ V}; m_a = \frac{V_{cn(1)} \cdot \sqrt{2}}{V_d} = \frac{234,78 \cdot \sqrt{2}}{400} = \underline{0,83}$$

- 8.



$$E = \frac{1}{2} L_m \hat{i}_{pr}^2 = \underline{4,5 \cdot 10^{-4} \text{ J}}$$

$$P = \frac{E}{T} = \underline{45 \text{ W}}$$

$$R = \frac{V_0^2}{P} = \underline{222 \Omega}$$

$$\hat{V}_T = 100 + 100 = \underline{200 \text{ V}}$$