Solutions SK2330 Optical Design 2015-01-14 14-19 FD41

Grading limits: 0-10p F, 10p or over Fx, 12p or over D, 15p or over C, 19p or over B, 22–24p A

1. a) The on-axis plots show that there is spherical aberration and defocus. As the off-axis plots are different from the on-axis, there must be at least one more aberration. The tangential and saggital plots are identical, excluding coma, astigmatism, and distortion. In conclusion, there must be spherical aberration, defocus, and field curvature.
   b) In the on-axis plots, the 4th order spherical aberration curve is clearly modified by defocus. But in the off-axis plots, it looks like a pure spherical aberration curve. So at relative image height 1, defocus and field curvature must cancel each other (which they can, as they are both second-order in x and y). Hence I can use the off-axis plots to estimate $W_{040} = 4\lambda$. From the on-axis plots, I then have $W_{020} = -2\lambda$ (found at $y = \pm 1$). And the field curvature coefficient must be the opposite, $W_{220} = 2\lambda$.

2. See extra file. Took me too long to get it into the document!

3. The diameter of the blurry circle in the middle is estimated at $1.8\,\mathrm{cm}$ in the image, and the diameter of the sector star to $9.7\,\mathrm{cm}$. (These values are estimated, variations may occur.) Hence the diameter of the blurry circle in the object is $d = 1.8 \cdot \frac{10}{9.7} = 1.86\,\mathrm{cm}$, and its circumference is $\pi \cdot 18.8\,\mathrm{mm} = 58.3\,\mathrm{mm}$. As the sectorstar contains 119 black lines, the spatial frequency is $\frac{119}{58.3} \approx 2\,\mathrm{lines/mm}$. The diffraction limit is as the object-space spatial frequency $s = \frac{D}{\lambda l} = 3{\cdot}10^{-3}/(550{\cdot}10^{-9}{\cdot}250{\cdot}10^{-3})\,\mathrm{m}^{-1} \approx 22\,\mathrm{mm}^{-1}$. So the camera is far from diffraction limited.

4. An expanded, collimated laser beam means the object is on-axis, leaving only spherical aberration and chromatic aberration. Laser light is monochromatic, so the only relevant aberration is sperical aberration. The wavefront aberration coefficient for a thin lens is

$$W_{040} = \frac{1}{32} h^4 K^3 \left(AX^2 + BXY + CY^2 + D \right) \quad (1)$$

   where the height of the marginal ray at the stop h and the refractive power K is the same for the two cases. Hence the only difference lies in the structural coefficient $\sigma_I = AX^2 + BXY + CY^2 + D$. The object is at infinity, so the conjugate factor is $Y = -1$. To minimize spherical aberraion the curved side of the lens should face the object, so the bending factor is $X = 1$. Inserting the values of n into the expressions for A, B, C and D gives $\sigma_I = 9.33$ for $n = 1.5$ and $\sigma_I = 4.7$. So the aberrations are reduced by 50% by changing form a low refractive index (1.5) to a high refractive index (1.8).

5. As the lens is at the stop, there is no distorion, so the only remaining aberrations are field curvature and astigmatism. The longitudinal aberration for field curvature is given by

$$LA_x = LA_y = -\frac{2l'^2}{h_p^2} W_{222} \eta^2 \quad (2)$$

and for astigmatism by

$$LA_y = -\frac{2l'^2}{h_p^2} W_{222} \eta^2 \ , \ LA_x = 0 \, . \tag{3}$$

(The expression in your notes may be wrong - if you have used that instead, you still get full points for the solution.) The plot shows that for a relative image height of $\eta = 1$ and a defocus of -2.85 mm, the focus becomes a vertical line. (I assume that the full image height is that for an object angle of 10 degrees, as indicated in the plot.) That implies a radial focus, formed by rays in the sagittal plane, which are only affected by field curvature. So for sagittal rays we can conclude that $LA_x = -2.85$ mm, $\frac{l'}{2h_p} = 5$, $\eta = 1$, and hence that $2.85\,\mathrm{mm} = 5^2 \cdot 8 \cdot W_{220}$ giving $W_{220} = 0.01425\,\mathrm{mm}$. At a relative image height of $\eta = 0.67$ and a defocus of $-2.85\,\mathrm{mm}$, the focal line is instead horisontal, or tangential, and is affected by both astigmatism and field curvature. Hence $-2.85\,\mathrm{mm} = LA_y = -5^2(W_{220} + W_{222}) \cdot 8 \cdot 0.67^2$ and $W_{220} + W_{222} = 0.0317\,\mathrm{mm}$, so $W_{220} = 0.0175\,\mathrm{mm}$. As we can't know if the tangential and radial focii were exactly at those distances, the values are approximate.