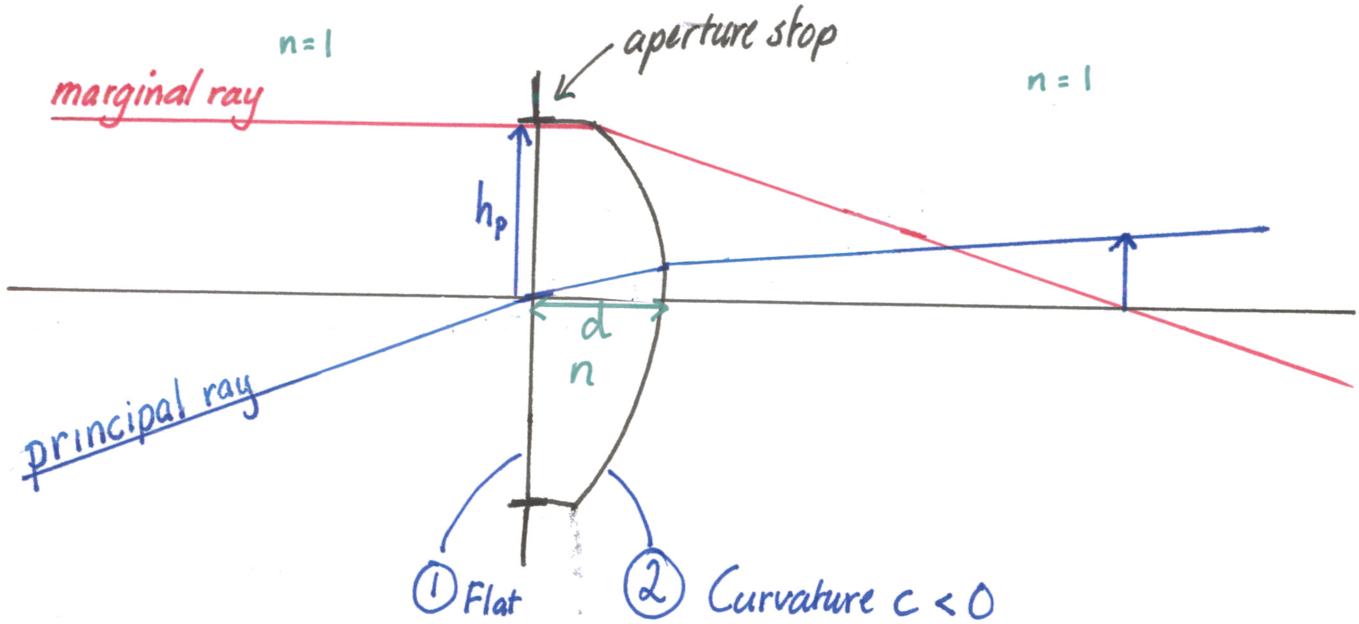
Solution to task 2

Exam optical design SK2300, 2015-01-14



Step 1: Trace the marginal and principal rays paraxially through the system.

Paraxial raytracing equations;

refraction: $h_i' = h_i$

$$n_i' u_i' = n_i u_i - h_i c_i (n_i' - n_i)$$

translation: $n_i' u_i' = n_{i+1} u_{i+1}$

$$h_{i+1} = h_i + u_i' d_{i+1}$$

Marginal ray

Entering the system: $u_0 = 0$

$h_0 = h_p$

Surface 1: $u_1 = u_1' = 0$ (no refraction) $\Rightarrow A_1 = 0$

$h_1 = h_p$

Surface 2: $u_2' = n \cdot 0 - h_p c (1-n) = h_p c (n-1) \Rightarrow A_2 = n(0 + h_p c) =$

$h_2' = h_p$ $\qquad = n h_p c$

Principal ray

Entering the system: $\bar{u}_0$ is the principal ray angle

Surface 1:

$$\bar{u}_1 = \bar{u}_0 \; ; \; \bar{u}_1' = \frac{\bar{u}_0}{n} \qquad \bar{A}_1 = \bar{u}_0$$

$$\bar{h}_1 = \bar{h}_1' = 0$$

Surface 2:

$$\bar{h}_2' = \bar{h}_2 = 0 + \bar{u}_1' d = \frac{\bar{u}_0 d}{n}$$

$$\bar{u}_2' = n \cdot \frac{\bar{u}_0}{n} - \bar{h}_2 c (1-n) = \bar{u}_0 + \bar{u}_0 dc \frac{(n-1)}{n} =$$

$$= \bar{u}_0 \left(1 + dc \frac{n-1}{n}\right)$$

$$\Rightarrow \bar{A}_2 = n\left(\frac{\bar{u}_0}{n} + \bar{h}_2 c\right) = \bar{u}_0 + \frac{n \bar{u}_0 d}{n} c = \bar{u}_0 (1 + dc)$$

The Lagrange invariant is $H = -1 \cdot \bar{u}_0 \cdot h_p = -\bar{u}_0 h_p$

$$\Delta_1 \left\{\frac{u}{n}\right\} = \frac{u_1'}{n} - \frac{u_1}{1} = 0$$

$$\Delta_2 \left\{\frac{u}{n}\right\} = \frac{u_2'}{1} - \frac{u_2}{n} = u_2' = h_p c (n-1) \qquad (<0, \text{ as } c<0)$$

$$P_1 = c_1 \Delta_1 \left\{\frac{1}{n}\right\} = 0$$

$$P_2 = c \Delta_2 \left\{\frac{1}{n}\right\} = c\left(\frac{1}{1} - \frac{1}{n}\right) = c\left(1 - \frac{1}{n}\right) = c \cdot \frac{n-1}{n}$$

The Seidel coefficients:

$$S_I = -\sum_i A_i^2 h_i \Delta_i\left\{\frac{u}{n}\right\} = \underbrace{-\overset{=0}{A_1^2} h_1 \overset{=0}{\Delta_1\left\{\frac{u}{n}\right\}}}_{0} - A_2^2 h_2 \Delta_2\left\{\frac{u}{n}\right\} =$$

$$= -(n h_p c)^2 h_p \cdot h_p c(n-1) = -h_p^4 c^3 n^2 (n-1)$$

$$S_{II} = -\sum_i A_i \bar{A}_i h_i \Delta_i\left\{\frac{u}{n}\right\} = \underbrace{-A_1 \bar{A}_1 h_1 \Delta_1\left\{\frac{u}{n}\right\}}_{=0} - A_2 \bar{A}_2 h_2 \Delta_2\left\{\frac{u}{n}\right\} =$$

$$= -n h_p c \cdot \bar{u}_0 (1+dc) h_p \cdot h_p c(n-1) = -h_p^3 c^2 (1+dc)\bar{u}_0 n(n-1)$$

$$S_{III} = -\sum_i \bar{A}_i^2 h_i \Delta_i\left\{\frac{u}{n}\right\} = \underbrace{-\bar{A}_1^2 h_i \Delta_1 \overset{=0}{\left\{\frac{u}{n}\right\}}}_{=0} - \bar{A}_2^2 h_2 \Delta_2\left\{\frac{u}{n}\right\} =$$

$$= -[\bar{u}_0(1+dc)]^2 h_p \cdot h_p c(n-1) = -h_p^2 c(1+dc)^2 \bar{u}_0^2 (n-1)$$

$$S_{IV} = -\sum_i H^2 P_i = \underbrace{-H^2 P_1}_{=0} - H^2 P_2 = -(-\bar{u}_0 h_p)^2 \cdot c\,\frac{n-1}{n} =$$

$$= -h_p^2 c\, \bar{u}_0^2 \frac{n-1}{n}$$

This kind of task is easy to get wrong, but we can do some tests.

The unit of $S_I - S_{IV}$ is the same as for the wavefront aberration coefficients, i.e., length. This is correct.

S_I does not depend on $\bar{u}_0$ - good, should be the same across the image field.

—— // —— d - good, since rays parallell to optical axis refract only at ②.

S_{IV} does not depend on d - good, since it should only depend on P_i.

If $h_p = 0$, all aberrations are 0 - good.

If $\bar{u}_0 = 0$, only S_I remains - good.

If $c = 0$, no aberrations - agrees with home task 5.