Homework 1

Analytical solutions of the Navier–Stokes equations

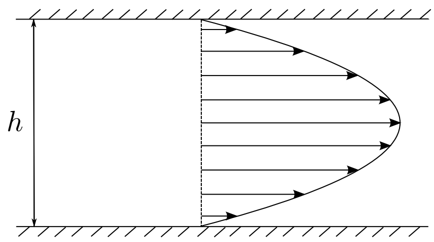
due January 26, 2015

Task 1

An aeroplane is flying with a trajectory given by $x_i = r_i(t, X_0)$. It measures the temperature field $T(t, x_1, x_2, x_3)$ in the surrounding air with a thermometer fixed to the wing. Find an expression for the rate of change of the measured temperature. Explain the physical meaning of the terms and compare the expression with that of the material derivative.

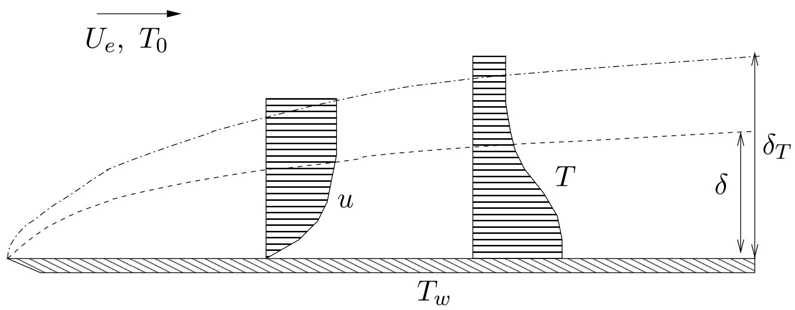
Task 2

Consider the fluid flow between two fixed infinite parallel plates, where a pressure gradient $\frac{\partial p}{\partial x} = C$ is imposed in a direction parallel to the plates. Find a steady solution to the incompressible Navier–Stokes equations for this flow case, i.e. so-called plane Poiseuille flow.



Hint: Assume that the flow is fully developed, meaning $\frac{\partial}{\partial x} = 0$ for all velocity components.

Task 3



Consider the incompressible flow with free-stream velocity U_e over a semi-infinite flat plate. Let the incoming flow have a temperature T_0 while the plate is kept at a higher temperature T_w. The temperature field around the plate obeys the equation

$$\rho c_p \frac{\mathrm{D}T}{\mathrm{D}t} = \kappa \nabla^2 T \ ,$$

where κ is the thermal conductivity, ρ the density and c_p the specific heat at constant pressure. Assume that the Péclet number, $Pe = RePr$, is large so that a thin *thermal* boundary layer develops on the plate.

Derive the appropriate boundary-layer approximation of the temperature equation in the limit of large Péclet number and estimate the thermal boundary-layer thickness δ_T.

Hint: Assume steady flow and $\delta/\delta_T \sim \mathcal{O}(1)$.