Reglerteknik **3**

Kapitel 7

Köp bok och övningshäfte på kårbokhandeln

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Lektion 3 kap 7

- Modellering
- Identifiering

Teoretisk modellering

Man använder grundläggande fysikaliska naturlagar och deras ekvationer för att härleda överföringsfunktionen. Tex

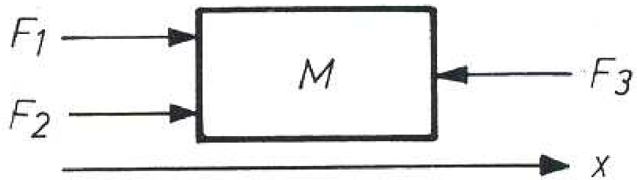
- Kraftekvationen
- Kirchhoffs lagar
- Energibalansprincipen
- Massbalansprincipen
- Volymbalansprincipen

Teoretisk modellering används när enkla modeller som kan ställas upp med måttlig möda är tillräckliga.

Genomgång av Processtyper

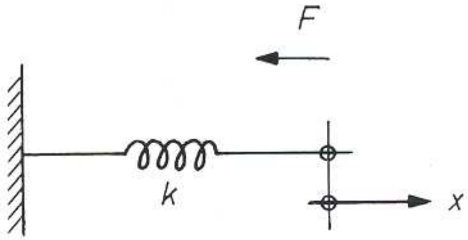
- Mekaniska system
- Elektriska system
- Temperatursystem
- Nivåsystem

Mekaniska system, grundelement



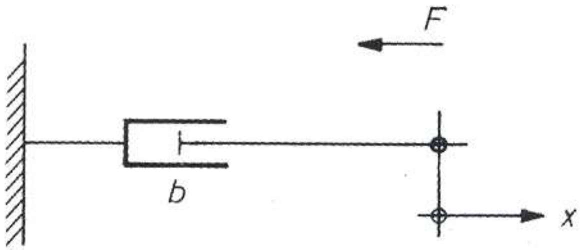
$$\sum F = M \cdot \ddot{x}$$

Newtons
kraftekvation.



$$F = k \cdot x$$

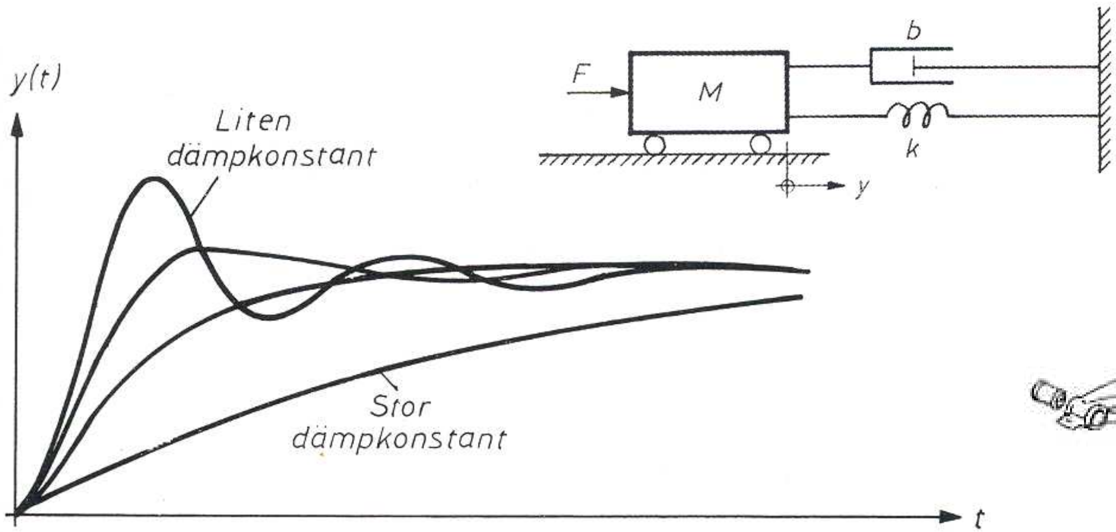
Hook's lag.
F proportionell mot
fjäderns förlängning.

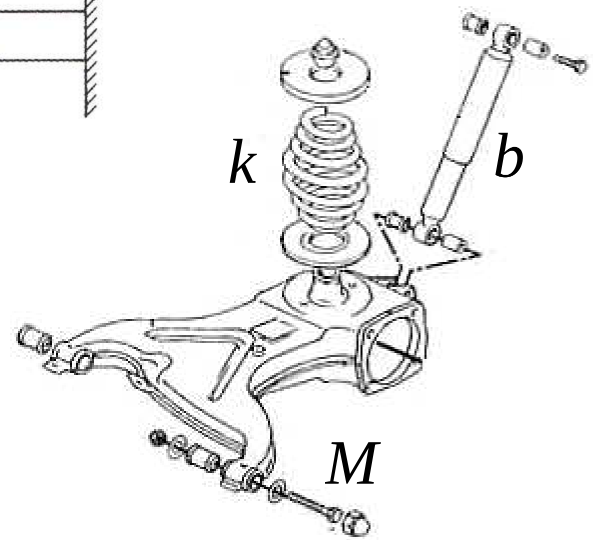


$$F = b \cdot \dot{x}$$

Hastighets-
dämpare.

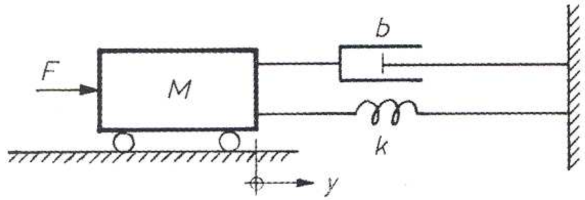
Sammansatt system





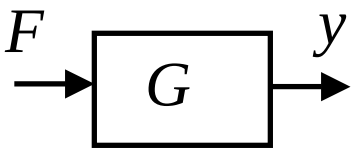
Man måste välja *k b* och *M* för bästa köregenskaper …

Sammansatt system

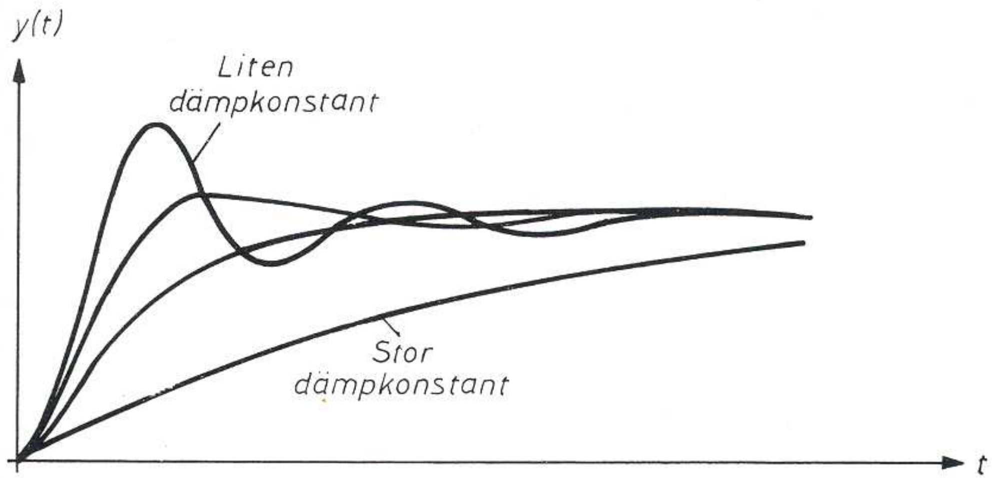


$$M \cdot \ddot{y} = F - ky - b\dot{y} \quad \Rightarrow \quad M \cdot \ddot{y} + b\dot{y} + ky = F$$

$$\overset{L:}{\Leftrightarrow} \quad Y(Ms^2 + bs + k) = F \quad \Rightarrow \quad G(s) = \frac{Y}{F} = \frac{1}{Ms^2 + bs + k}$$

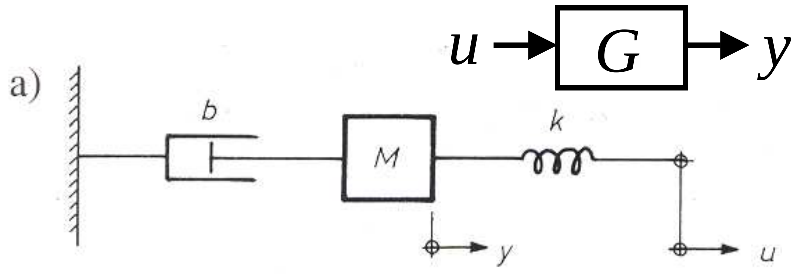


Stegsvar då man ”trycker” på massan med en konstant kraft, och följer med rörelsen.



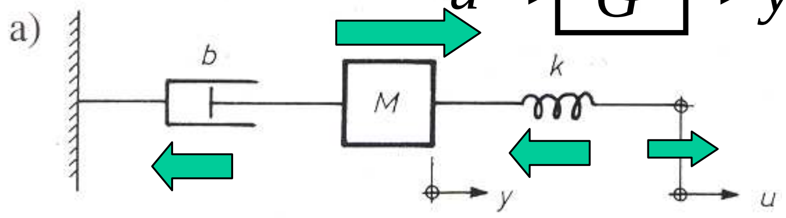
7.1 **a** Mekanisk modell

Ställ upp differentialekvation
och överföringsfunktion.



7.1 **a** lösning Mekanisk modell

$$u \rightarrow \boxed{G} \rightarrow y$$



Fjäder: $\vec{F} = k(\vec{u} - \vec{y})$

Dämpare: $\vec{F} = -b\dot{\vec{y}}$

Kraftekvationen: $\sum \vec{F} = M \cdot \ddot{\vec{y}}$

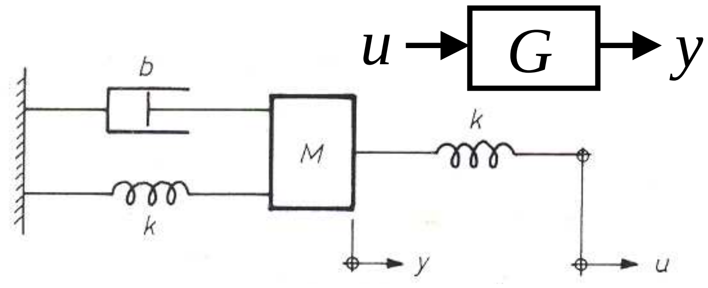
$$M \cdot \ddot{y} = k(u-y) - b\dot{y} \quad \overset{L:}{\Leftrightarrow} \quad Ms^2Y = kU - kY - bsY$$

$$G(s) = \frac{Y}{U} = \boxed{\frac{k}{Ms^2 + bs + k}}$$

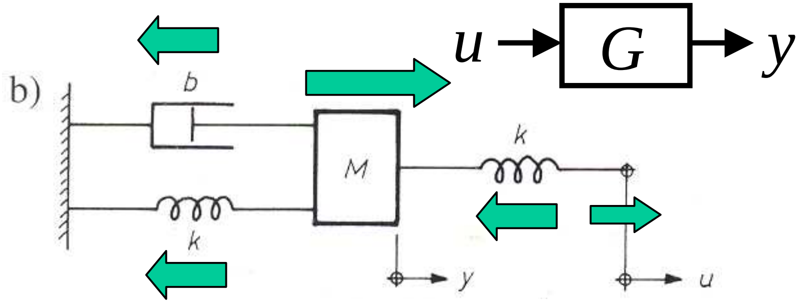
7.1 **b** Mekanisk modell

Ställ upp differentialekvation
och överföringsfunktion.

b)



7.1 **b** lösning Mekanisk modell



Fjäder 1: $\vec{F} = -k\vec{y}$

Fjäder 2: $\vec{F} = k(\vec{u} - \vec{y})$

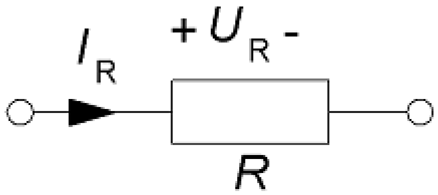
Dämpare: $\vec{F} = -b\dot{\vec{y}}$

Kraftekvationen: $\sum \vec{F} = M \cdot \ddot{\vec{y}}$

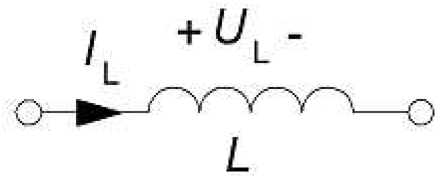
$$M \cdot \ddot{y} = k(u - y) - ky - b\dot{y} \quad \overset{L:}{\Leftrightarrow} \quad Ms^2Y = kU - 2kY - bsY$$

$$G(s) = \frac{Y}{U} = \boxed{\frac{k}{Ms^2 + bs + 2k}}$$

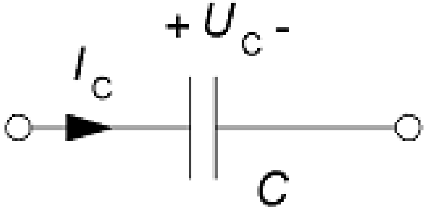
Elektriska system, grundelement



$$u_R = i_R \cdot R \quad U_R = I_R \cdot R$$



$$u_L = L \cdot \frac{d}{dt} i_L \quad U_L = Ls \cdot I_L$$



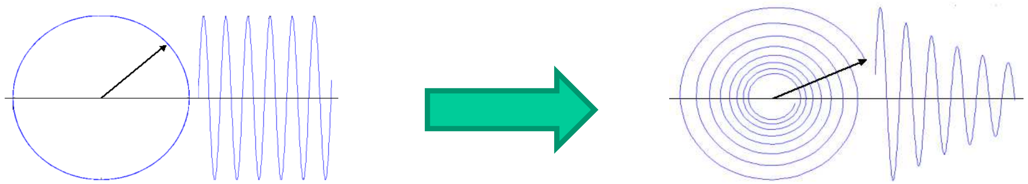
$$u_C = \frac{Q_C}{C} = \frac{\int i_C dt}{C} \quad U_C = \frac{1}{C \cdot s} \cdot I_C$$

Elektriska nät, jω-metoden

$$s = \sigma + j\omega \quad im[s] = \omega$$

Du som redan kan jω-metoden ställer upp de elektriska överföringsfunktionerna direkt, och **byter** sedan jω mot *s*.

$$j\omega \qquad\qquad s = \sigma + j\omega$$



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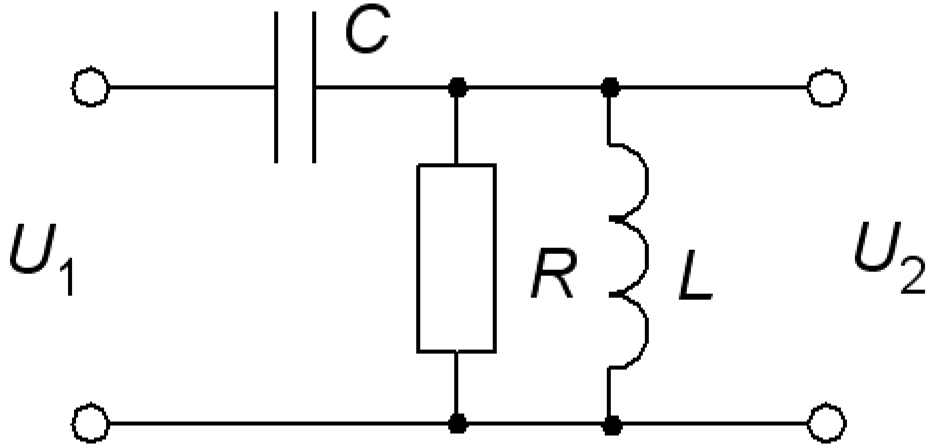
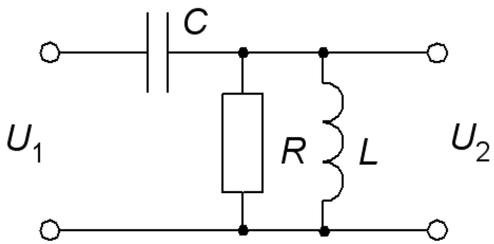
Sammansatt elektriskt system



$$R \parallel L = \frac{j\omega L \cdot R}{R + j\omega L}$$

$$\frac{\underline{U}_2}{\underline{U}_1} = \frac{\dfrac{j\omega L \cdot R}{R + j\omega L}}{\dfrac{1}{j\omega C} + \dfrac{j\omega L \cdot R}{R + j\omega L}} \quad \Rightarrow \quad G(s) = \frac{\dfrac{sL \cdot R}{R + sL}}{\dfrac{1}{sC} + \dfrac{sL \cdot R}{R + sL}}$$

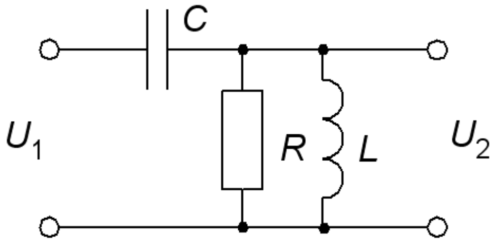
Sammansatt elektriskt system



$$G(s) = \frac{\dfrac{sL \cdot R}{R + sL}}{\dfrac{1}{sC} + \dfrac{sL \cdot R}{R + sL}}$$

$$G(s) = \frac{\dfrac{sL \cdot R}{R + sL}}{\dfrac{1}{sC} + \dfrac{sL \cdot R}{R + sL}} = \frac{LRs \cdot sC}{(R + Ls) + LRs \cdot sC} = \boxed{\frac{LRCs^2}{LRCs^2 + Ls + R}}$$

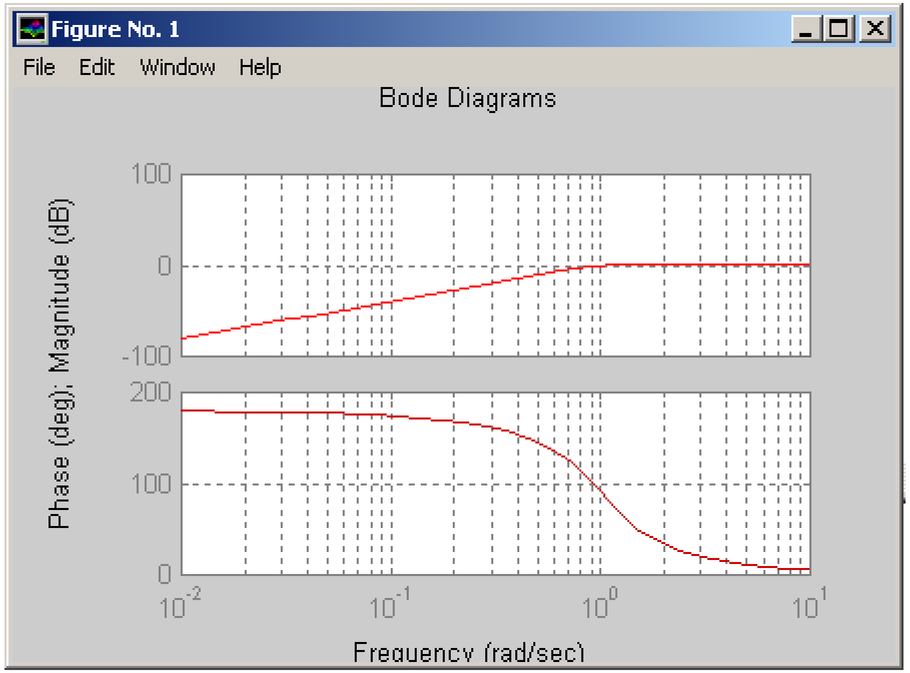
Sammansatt elektriskt system MATLAB



$$G(s) = \frac{LRCs^2}{LRCs^2 + Ls + R}$$

För enkelhets skull $L = R = C = 1$

```
G=tf([1,0,0],[1,1,1])
bode(G)
```



Från ellärans laborationer är Du van vid **Bode**-diagrammet.

Temperatursamband

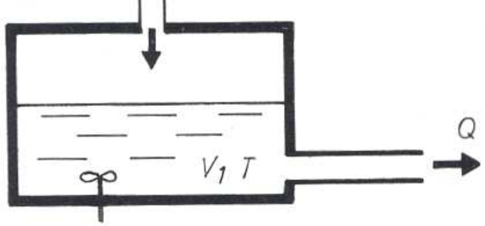
Energibalans: $\Delta P = P_{in} - P_{out}$ $\quad \dfrac{dE}{dt} = P_{in} - P_{out}$

Termisk energi: $E = T \cdot V \cdot c \cdot \rho$ $\quad$ T temperatur, V volym 4 m^3, c värmekapacitet, ρ densitet

Utflöde Q_{out} och inflöde Q_{in} är **samma** Q = 0,1 m^3/s. Sökt, över-föringsfunktion mellan T uttemperatur och T_{i} intemperatur.

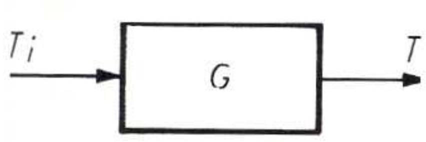
T_{i} intemperatur
T tanktemperatur och uttemperatur

varierande inloppstemperatur



inflöde = utflöde

Temperatursamband



Energibalans:

$$\frac{dE}{dt} = P_{in} - P_{out}$$

$E = T \cdot V \cdot c \cdot \rho$ Värmeenergi i tanken

$P_{in} = T_i \cdot Q \cdot c \cdot \rho$ Tillförd effekt

$P_{out} = T \cdot Q \cdot c \cdot \rho$ Bortförd effekt

$$\frac{d}{dt}(TVc\rho) = T_i Qc\rho - TQc\rho \quad \Rightarrow \quad V\cancel{c\rho}\frac{dT}{dt} = T_i Q\cancel{c\rho} - TQ\cancel{c\rho} \quad \Rightarrow$$

$$V\frac{dT}{dt} + QT = QT_i \quad \overset{L:}{\Leftrightarrow} \quad V \cdot T(s) \cdot s + Q \cdot T(s) = Q \cdot T_i(s)$$

$$\frac{T(s)}{T_i(s)} = G(s) = \frac{Q}{Vs+Q} = \frac{1}{1+\frac{V}{Q}s} \qquad \boxed{G = \frac{1}{1+40s}}$$

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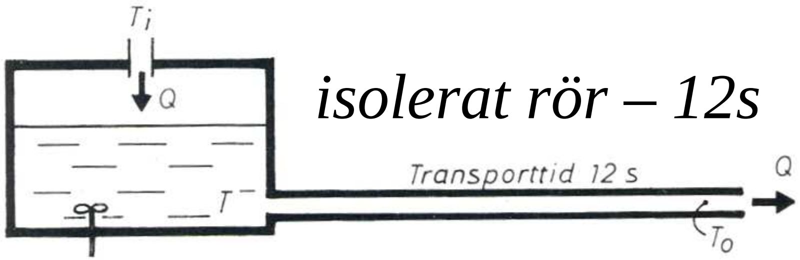
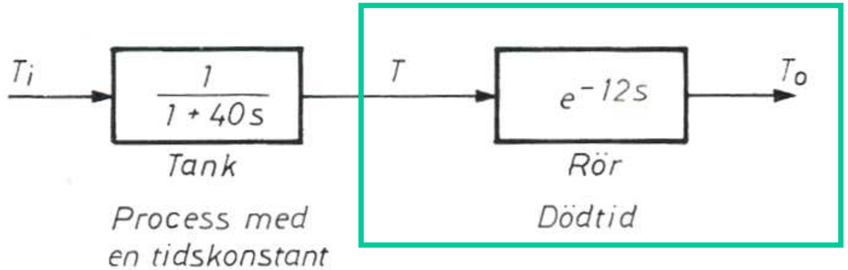
Temperatursamband **med dödtid**

isolerat rör – 12s



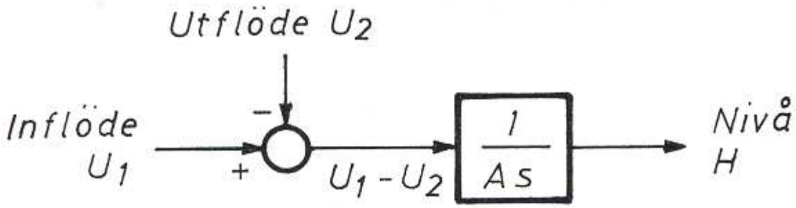


Isolerat rör. Överföringsfunktion mellan T_0 och T.

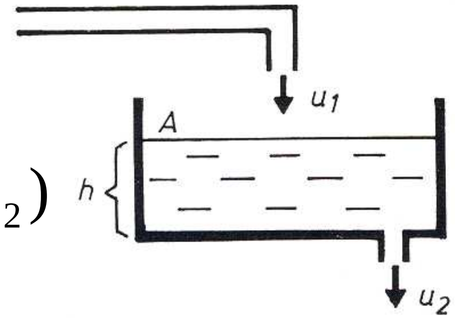
$$T_0(t) = T(t-12) \quad G(s) = e^{-12s} \quad G = G_1 \cdot G_2 = \frac{e^{-12s}}{1+40s}$$

```
G=tf([1],[40 1],'InputDelay',12)
Transfer function:
                1
exp(-12*s) * --------
             40 s + 1
```

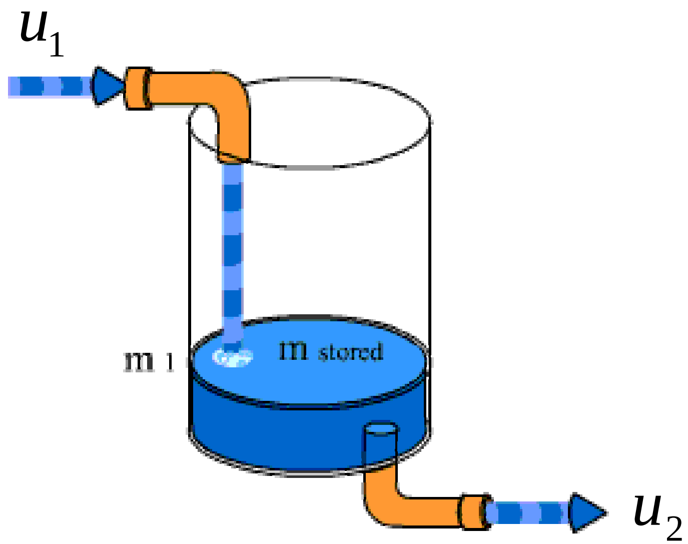
Nivåsamband



Balansekvation:



$$\frac{dV}{dt} = A\frac{dh}{dt} = (u_1 - u_2)$$

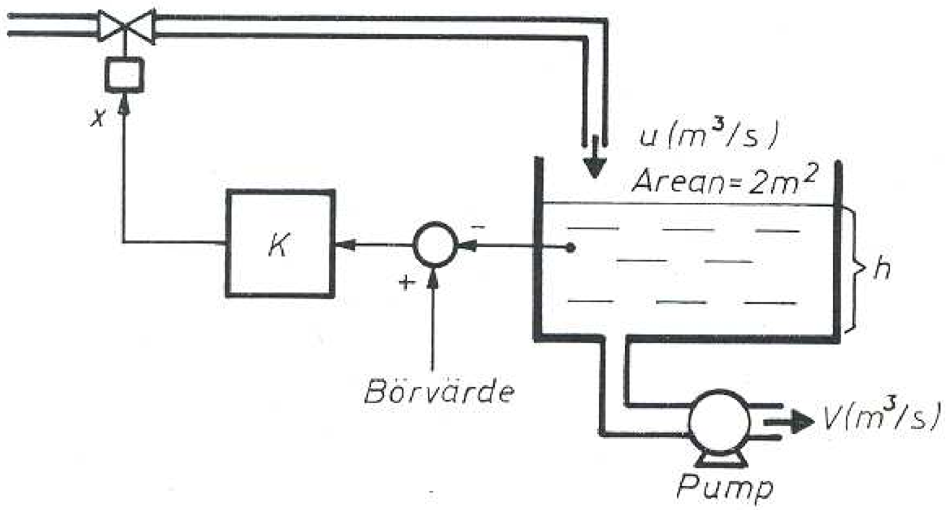


$u_1\ u_2$	In- och utflöde
$A\ h$	Area och nivå
$V = A \cdot h$	Vätskevolym i tank

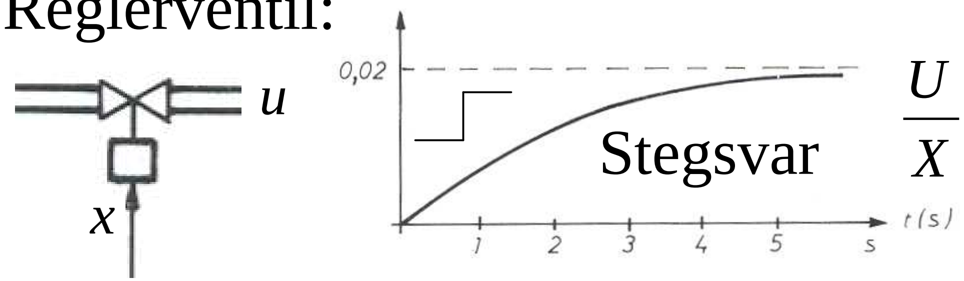
$$\frac{dV}{dt} = A\frac{dh}{dt} = (u_1 - u_2) \quad \overset{\mathcal{L}:}{\Leftrightarrow} \quad AHs = (U_1 - U_2) \quad \Rightarrow$$

$$\frac{H}{(U_1 - U_2)} = G(s) = \boxed{\frac{1}{As}}$$

7.11 Nivåregulator för tank



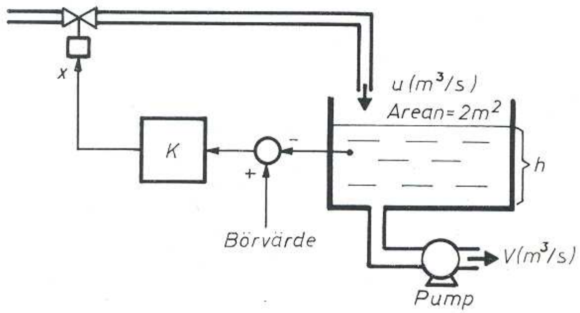
Reglerventil:

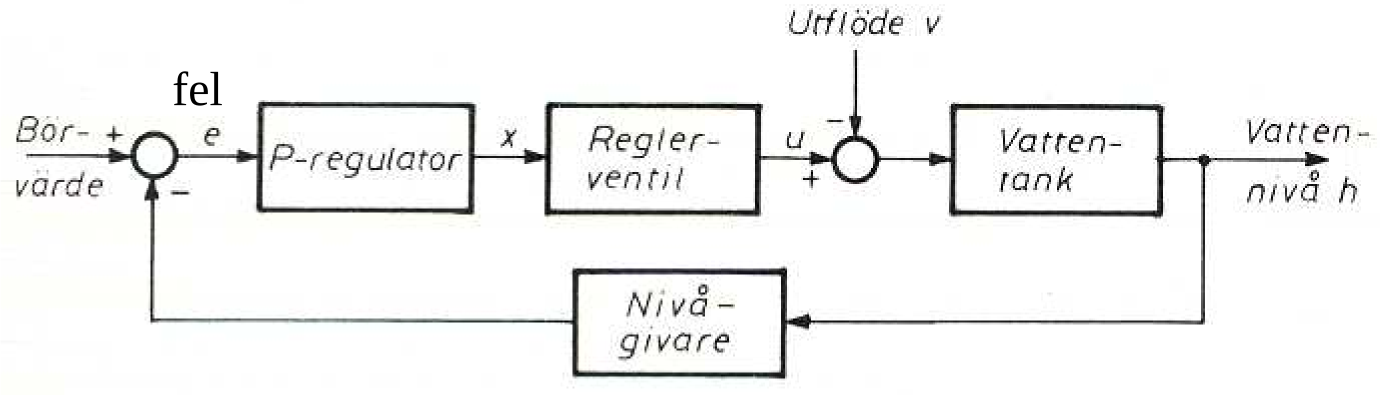


$$\frac{U}{X}$$

Vi har mätt upp reglerventilens stegsvar

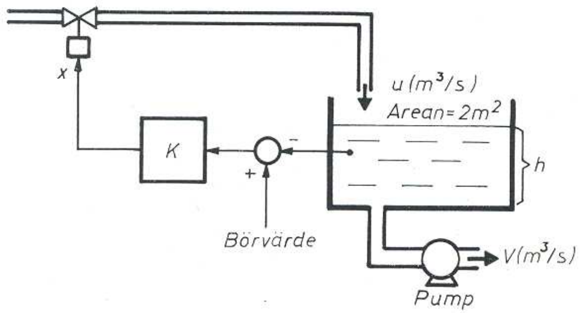
7.11 **a** Nivåregulator blockschema



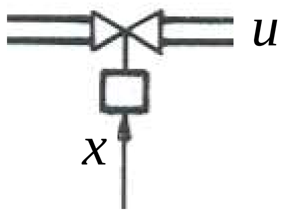


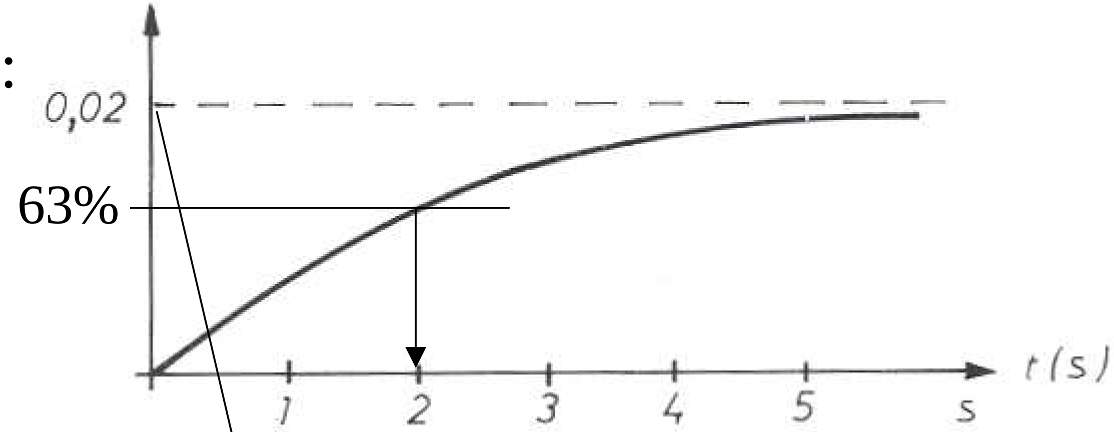
7.11 **b** Nivåregulator

$\boxed{\text{reglerventil}}$



Reglerventil:





$T = 2\ \text{s}$

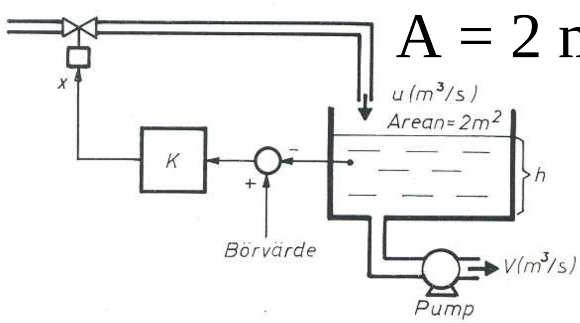
$$G(s) = \frac{U}{X} = \frac{0{,}02}{1+2s}$$

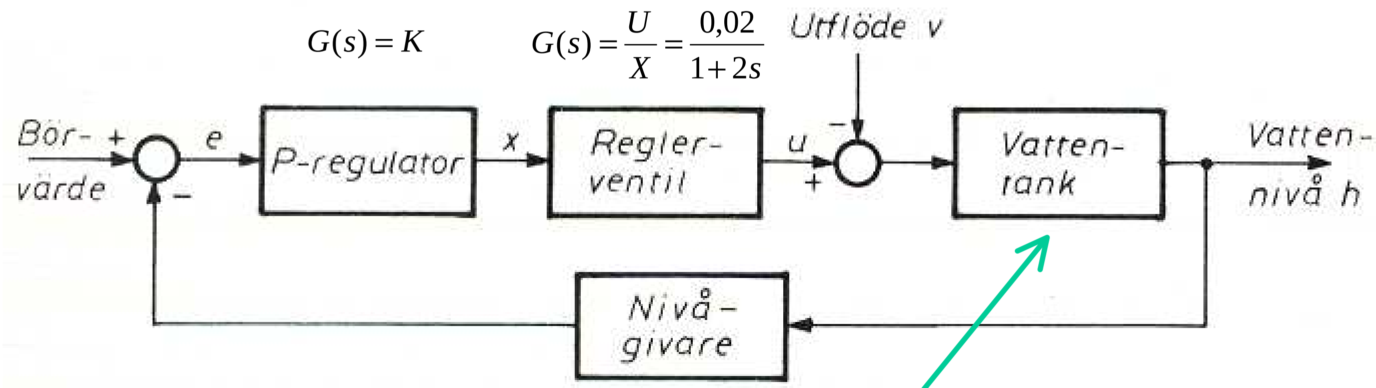
Överföringsfunktion

$$X \rightarrow \boxed{G} \rightarrow U$$

7.11 **c** Nivåregulator $\boxed{\text{vattentank}}$

A = 2 m^2





Tank:

$$\frac{dV}{dt} = A\frac{dh}{dt} = (u - v) \quad \{L:\} \quad AsH = (U - V)$$

$$\frac{H}{(U-V)} = G(s) = \frac{1}{As} = \frac{1}{2s}$$

Överföringsfunktion

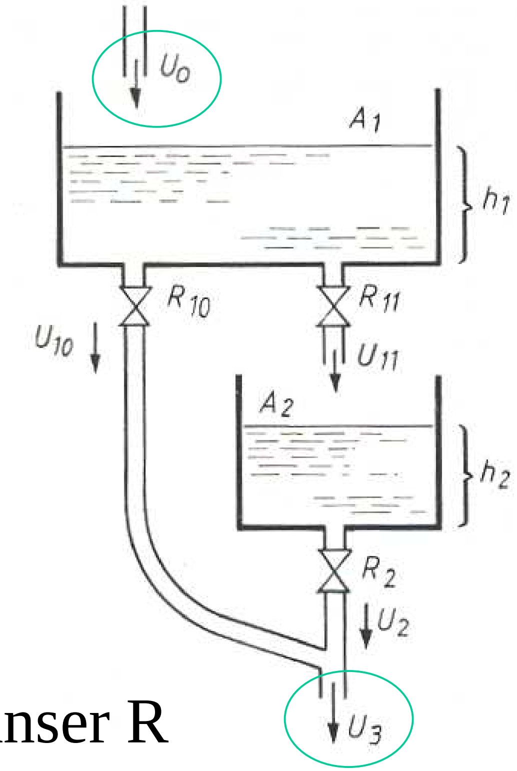
$$(U - V) \rightarrow \boxed{G} \rightarrow H$$

Har ni sett vattentankarna på labbet?

Uppgift 7.12 kommer ni säkert studera noggrant!

(**7.12** Kopplade tankar)

U_0 = inflöde tank 1
U_{10}, U_{11} = utflöden tank 1
U_2 = utflöde tank 2
A_1 = area tank 1
A_2 = area tank 2
h_1 = nivå tank 1
h_2 = nivå tank 2



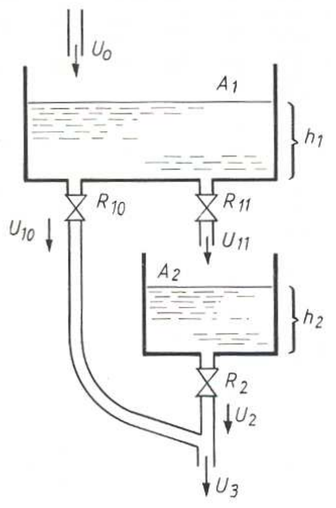
Sökt överföringsfunktion

$$U_0 \rightarrow \boxed{G} \rightarrow U_3$$

Ventilresistanser R

$$U_{10} = \frac{h_1}{R_{10}} \quad U_{11} = \frac{h_1}{R_{11}} \quad U_2 = \frac{h_2}{R_2}$$

(7.12 lösning Kopplade tankar)



Tank 1:

$$A_1 \frac{dh_1}{dt} = u_0 - u_{10} - u_{11} = u_0 - \frac{h_1}{R_{10}} - \frac{h_1}{R_{11}} \quad \{L:\}$$

$$\left(A_1 s + \frac{1}{R_{10}} + \frac{1}{R_{11}} \right) H_1 = U_0 \qquad (1)$$

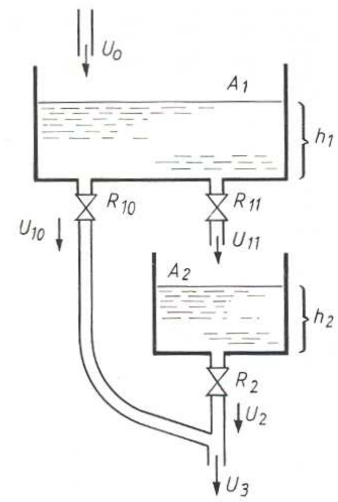
Tank 2:

$$A_2 \frac{dh_2}{dt} = u_{11} - u_2 = \frac{h_1}{R_{11}} - \frac{h_2}{R_2} \quad \{L:\}$$

$$\left(A_2 s + \frac{1}{R_2} \right) H_2 = \frac{1}{R_{11}} H_1 \qquad (2)$$

Summa: $\{L:\} \quad U_3 = U_{10} + U_2 = \frac{1}{R_{10}} H_1 + \frac{1}{R_2} H_2 \qquad (3)$

(7.12 lösning Kopplade tankar)



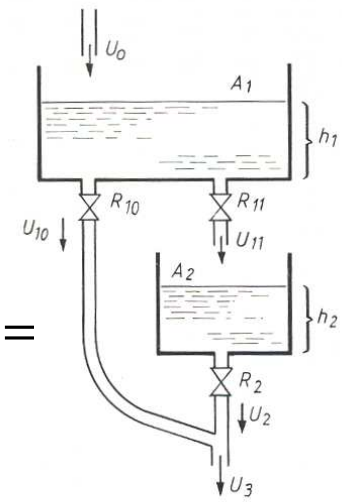
$(1),(2)\ in\ (3)$

$$U_3 = \frac{1}{R_{10}} \cdot \frac{U_0}{A_1 s + \frac{1}{R_{10}} + \frac{1}{R_{11}}} + \frac{1}{R_2} \cdot \frac{\frac{1}{R_{11}} \cdot H_1}{A_2 s + \frac{1}{R_2}} \qquad (4)$$

$(1)\ in\ (4)$

$$U_3 = \frac{U_0}{A_1 R_{10} s + 1 + \frac{R_{10}}{R_{11}}} + \frac{\frac{1}{R_{11}}}{A_2 R_2 s + 1} \cdot \frac{U_0}{A_1 s + \frac{1}{R_{10}} + \frac{1}{R_{11}}} \quad \Rightarrow$$

(7.12 lösning Kopplade tankar)



$$\Rightarrow \quad \frac{U_3}{U_0} = \frac{1}{A_1 R_{10} + 1 + \dfrac{R_{10}}{R_{11}}} + \frac{1}{(A_2 R_2 s + 1)\left(A_1 R_{11} s + \dfrac{R_{11}}{R_{10}} + 1 \right)} =$$

$$= \frac{R_{11}}{A_1 R_{10} R_{11} s + R_{11} + R_{10}} + \frac{R_{10}}{(A_2 R_2 s + 1)(A_1 R_{10} R_{11} s + R_{11} + R_{10})} =$$

$$= \frac{A_2 R_2 R_{11} s + R_{11} + R_{10}}{(A_2 R_2 s + 1)(A_1 R_{10} R_{11} s + R_{11} + R_{10})} =$$

$$\boxed{= \frac{A_2 R_2 R_{11} s + (R_{10} + R_{11})}{A_1 A_2 R_{10} R_{11} R_2 \, s^2 + \left(A_1 R_{10} R_{11} + A_2 R_2 (R_{10} + R_{11}) \right) s + (R_{10} + R_{11})}}$$

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Experimentella analyser

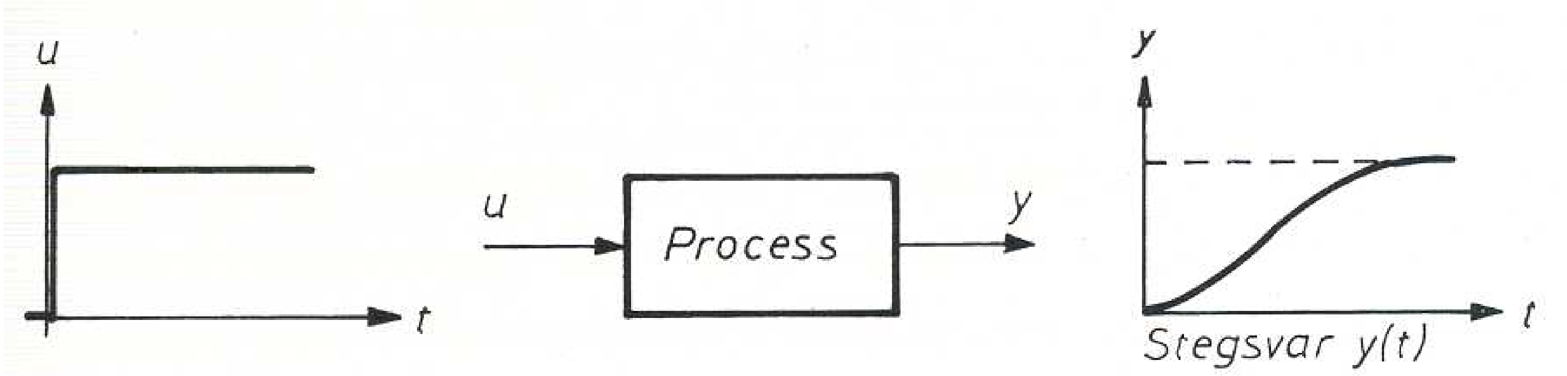
Man går in med ”testsignaler” på processen och analyserar svaret på utgången.

- **Stegsvarsanalys**
  Stegsvarstabell i Formelsamlingen

- **Frekvensanalys**
  Testa med sinus-generator och rita Bodediagram.

- **Parametrisk identifiering**
  Identifiera parametrar med minsta kvadratmetoden.

Stegsvar

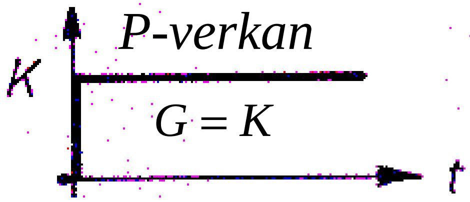


- Enhetssteget är den vanligaste ”testsignalen”.

Stegsvarstabell

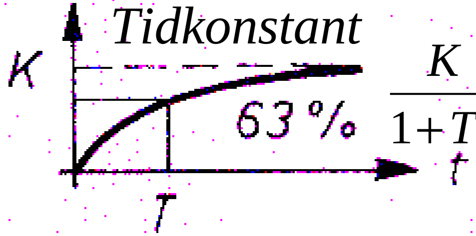
P-verkan

$G = K$



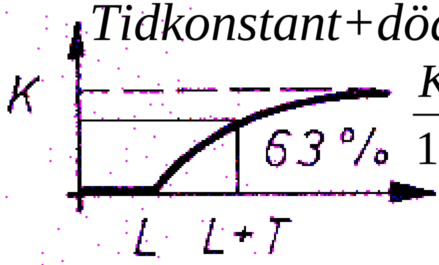
Tidkonstant

$$\frac{K}{1+Ts}$$



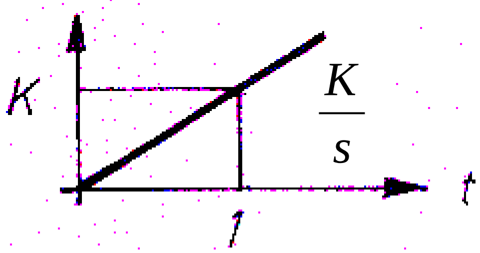
Tidkonstant+dödtid

$$\frac{Ke^{-Ls}}{1+Ts}$$



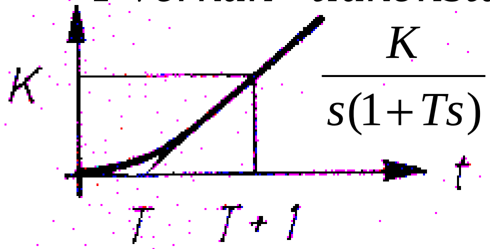
I-verkan

$$\frac{K}{s}$$



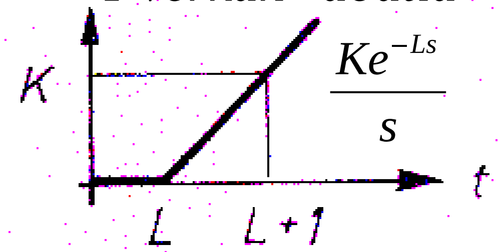
I-verkan+tidkonstant

$$\frac{K}{s(1+Ts)}$$



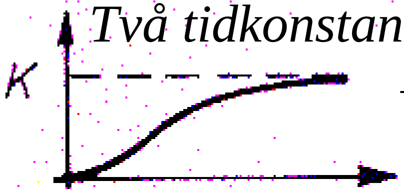
I-verkan+dödtid

$$\frac{Ke^{-Ls}}{s}$$



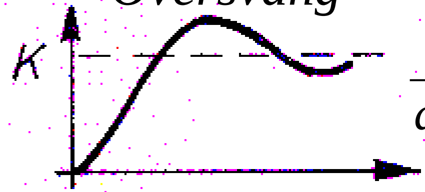
Två tidkonstanter

$$\frac{K}{(1+T_1s)(1+T_2s)}$$



Översväng

$$\frac{K}{as^2+bs+1}$$



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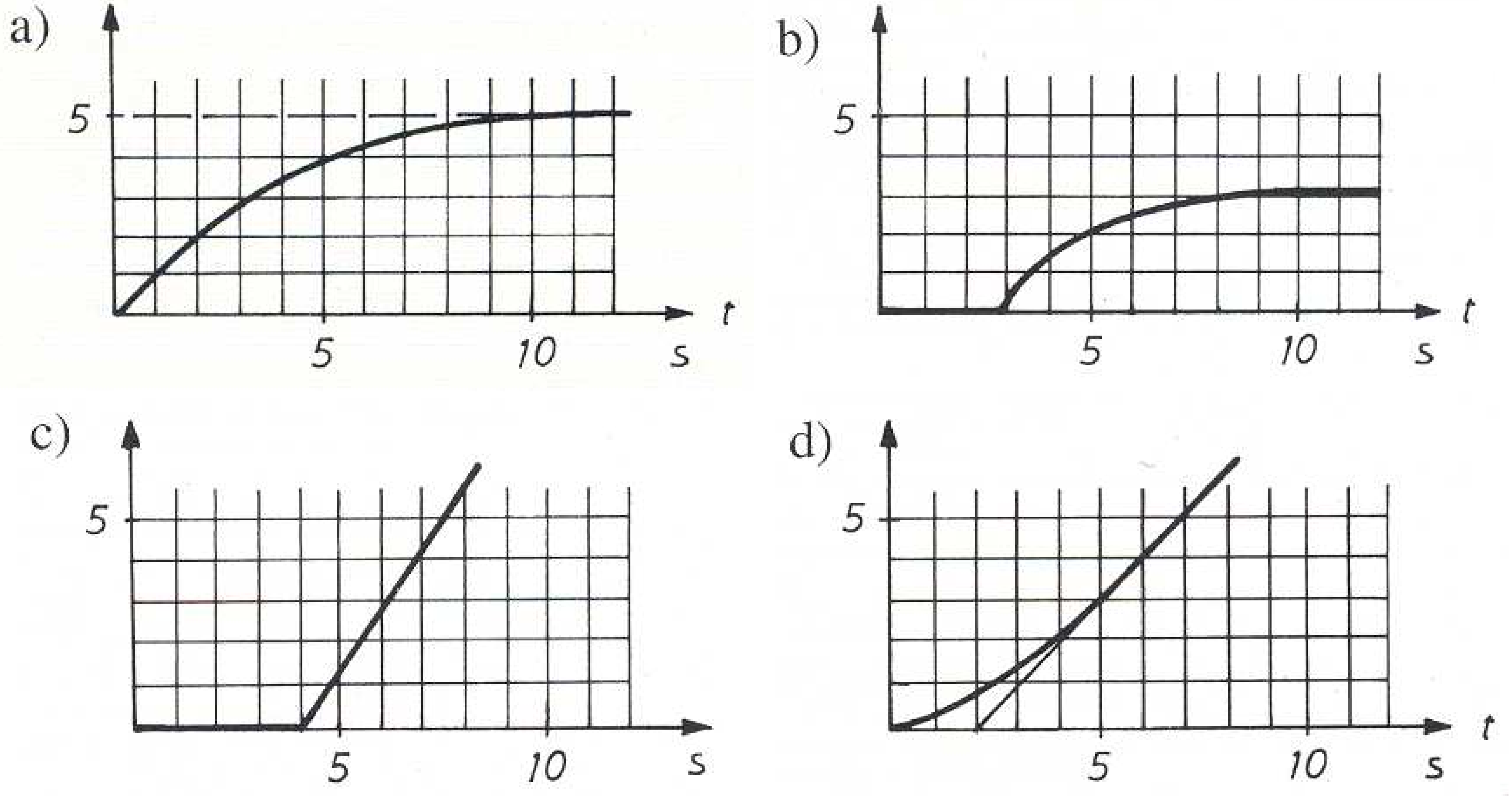
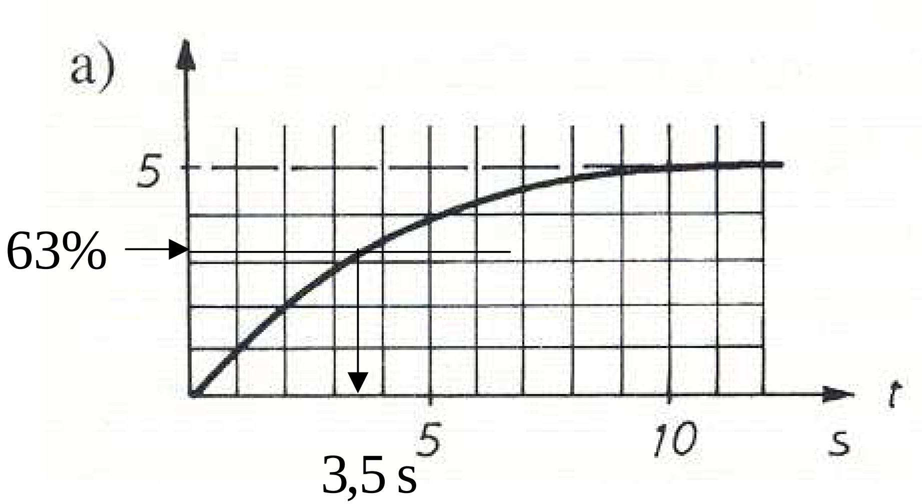
7.16 Stegsvarsanalys



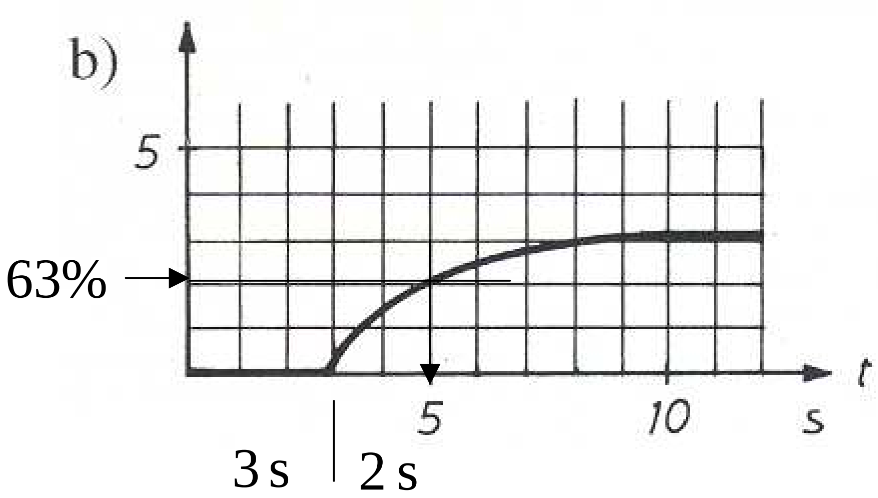
7.16 **a** lösning Stegsvarsanalys



$$\frac{K}{1+Ts} \Rightarrow \frac{5}{1+3{,}5s}$$

- Enkel tidkonstant

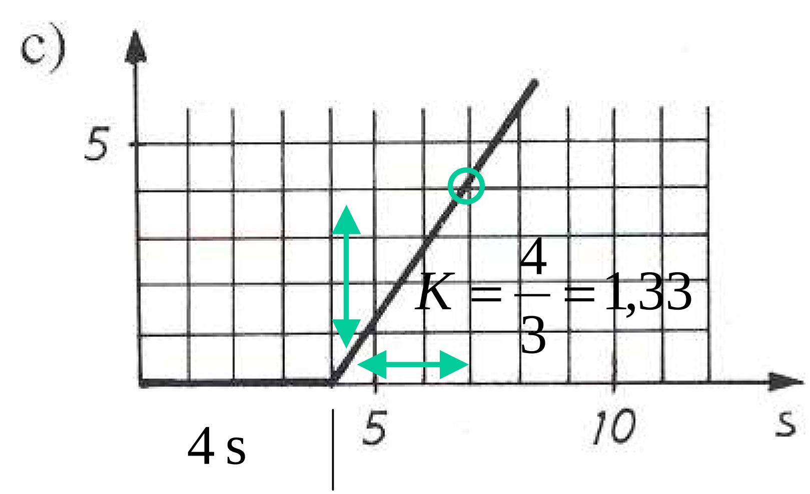
7.16 **b** lösning Stegsvarsanalys



$$\frac{Ke^{-Ls}}{1+Ts} \Rightarrow \frac{3\cdot e^{-3s}}{1+2s}$$

- Dödtid
- Enkel tidkonstant

7.16 **c** lösning Stegsvarsanalys



$$K = \frac{4}{3} = 1{,}33$$

4 s

$$\frac{Ke^{-Ls}}{s} \Rightarrow \frac{1{,}33 \cdot e^{-4s}}{s}$$

- Dödtid
- Integrering

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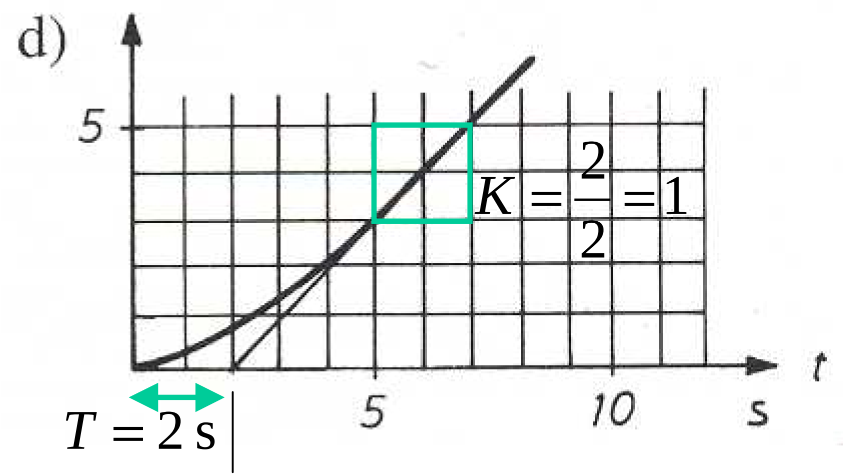
7.16 **d** lösning Stegsvarsanalys

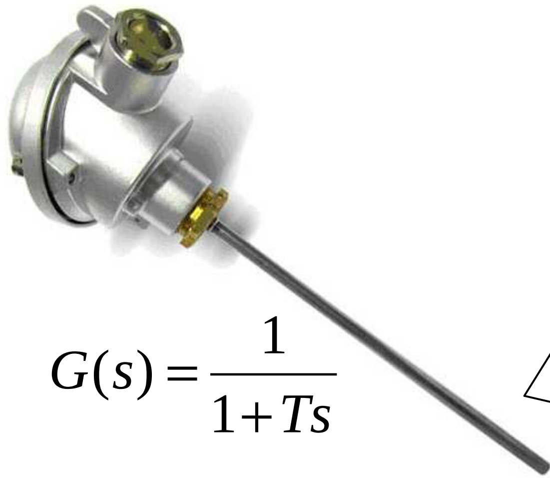


$$\frac{K}{s(1+Ts)} \Rightarrow \frac{1}{s(1+2s)}$$

- Integrering
- Tidkonstant

7.9 Temperaturgivare, tidkonstant

Från 0° till 80° efter 20 s visar givaren 65° Bestäm tidkonstanten T.



$$G(s) = \frac{1}{1+Ts}$$

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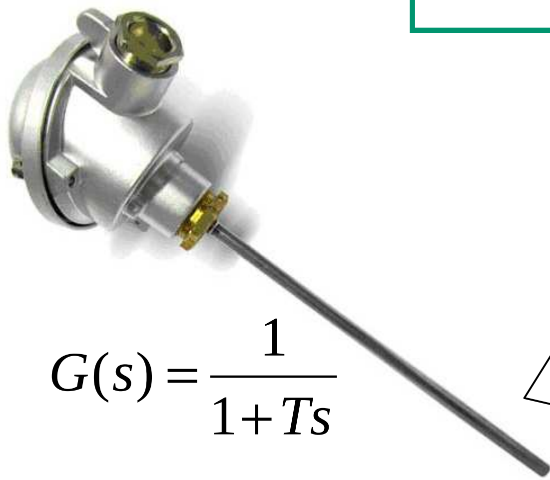
7.9 lösning, tidkonstant

$$\frac{1}{s(1+Ts)} \quad \{L^{-1}\} \quad 1-e^{-\frac{t}{T}}$$

Från 0° till 80° efter 20 s visar givaren 65° Bestäm tidkonstanten T.



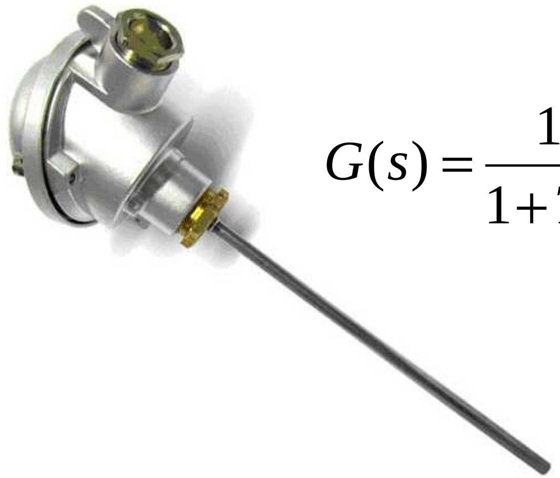
$$G(s)=\frac{1}{1+Ts}$$

Temperatursteg: $\frac{80}{s} \quad \Rightarrow \frac{80}{s}\cdot\frac{1}{1+Ts}$

$$\{L^{-1}:\} \quad x(t)=80(1-e^{-\frac{t}{T}}) \quad \Rightarrow \quad 65=80(1-e^{-\frac{20}{T}})$$

$$-\frac{20}{T}=\ln\left(\frac{80-65}{80}\right) \quad \Rightarrow \quad T=-\frac{20}{\ln\left(\frac{80-65}{80}\right)}\approx 12\text{ s}$$

7.9 lösning, ”snabbformeln”



$$G(s)=\frac{1}{1+Ts}$$

Från 0° till 80° efter 20 s visar givaren 65° Bestäm tidkonstanten T.

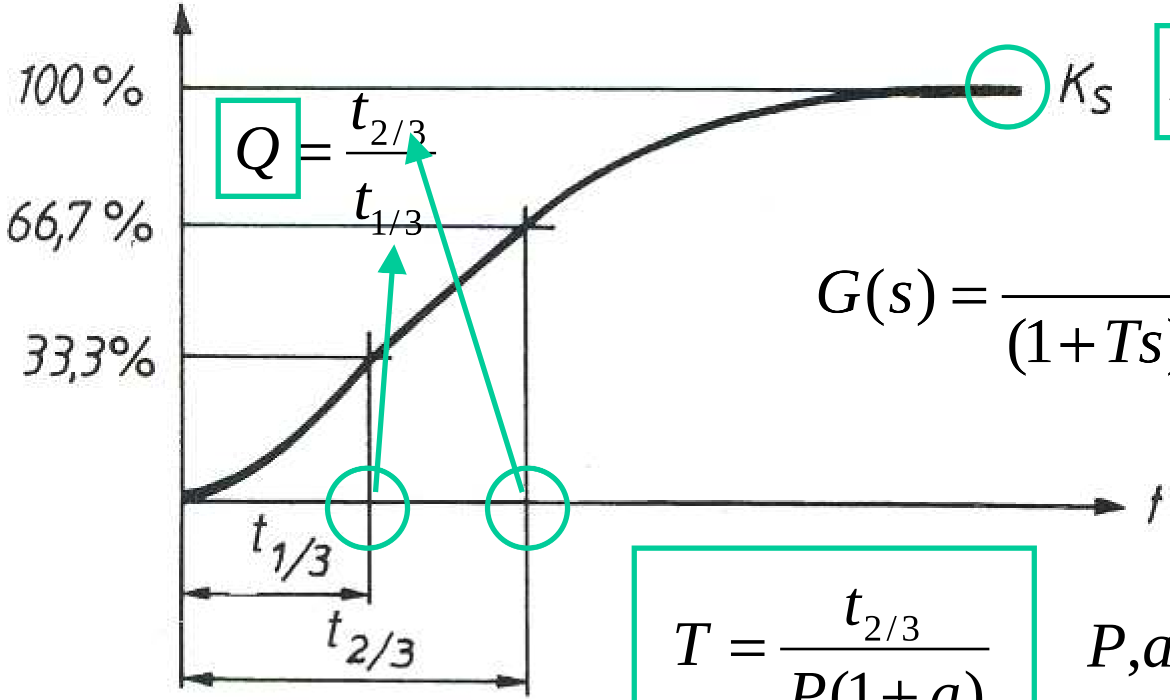
0°

80°

$$t=T\ln\left(\frac{hela}{resten}\right)\quad\Rightarrow\quad 20=T\ln\left(\frac{80-0}{80-65}\right)$$

$$T=\frac{20}{\ln\left(\frac{80}{80-65}\right)}\approx 12\text{ s}$$

Processer med två tidkonstanter



$$\boxed{Q} = \frac{t_{2/3}}{t_{1/3}}$$

$$\boxed{K} = \frac{K_S}{h} \leftarrow steget$$

$$G(s) = \frac{K}{(1+Ts)(1+aTs)}$$

$$\boxed{T = \frac{t_{2/3}}{P(1+a)}}$$

P,a-diagram, n = 2

$$1{,}92 < Q < 2{,}71$$

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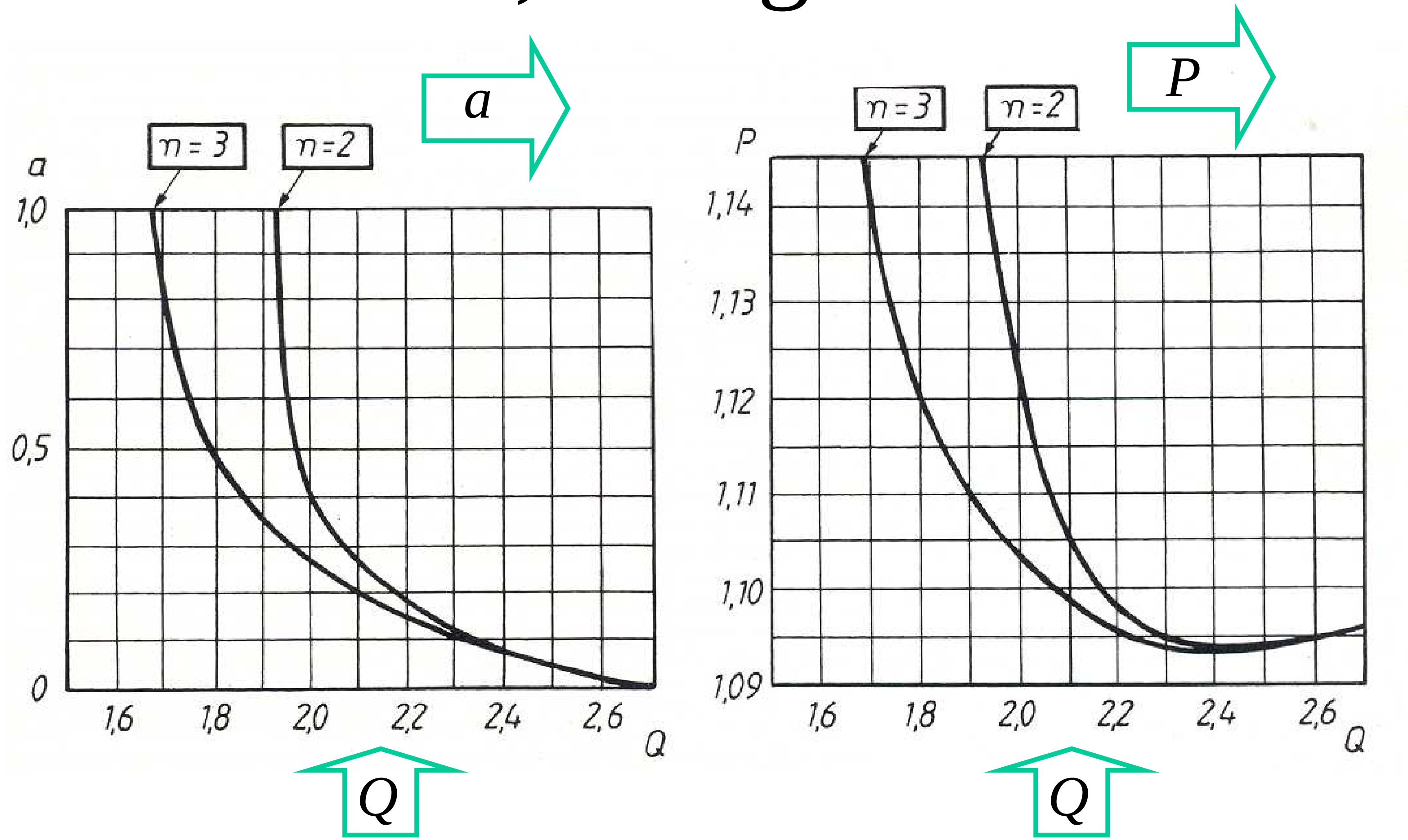
P,*a*-diagram

$$Q = \frac{t_{2/3}}{t_{1/3}}$$

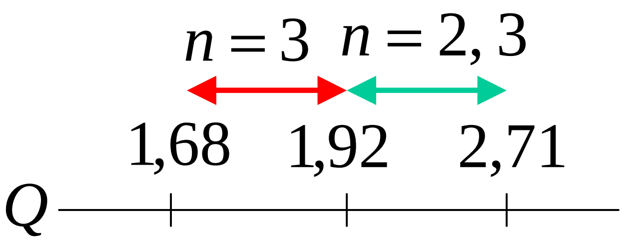


Processer med tre tidkonstanter

$$G(s)=\frac{K}{(1+Ts)(1+aTs)(1+a^2Ts)}$$

$$1{,}68<Q<2{,}71$$

$$\boxed{T=\frac{t_{2/3}}{P(1+a+a^2)}}$$ *P*,*a*-diagram, n = 3



- Om $\boxed{1{,}68<Q<1{,}92}$ vet man att det är tre tidkonstanter.
- Om 1,92<Q2,71 kan man välja mellan 2 eller 3 tidkonstanter i modellen.

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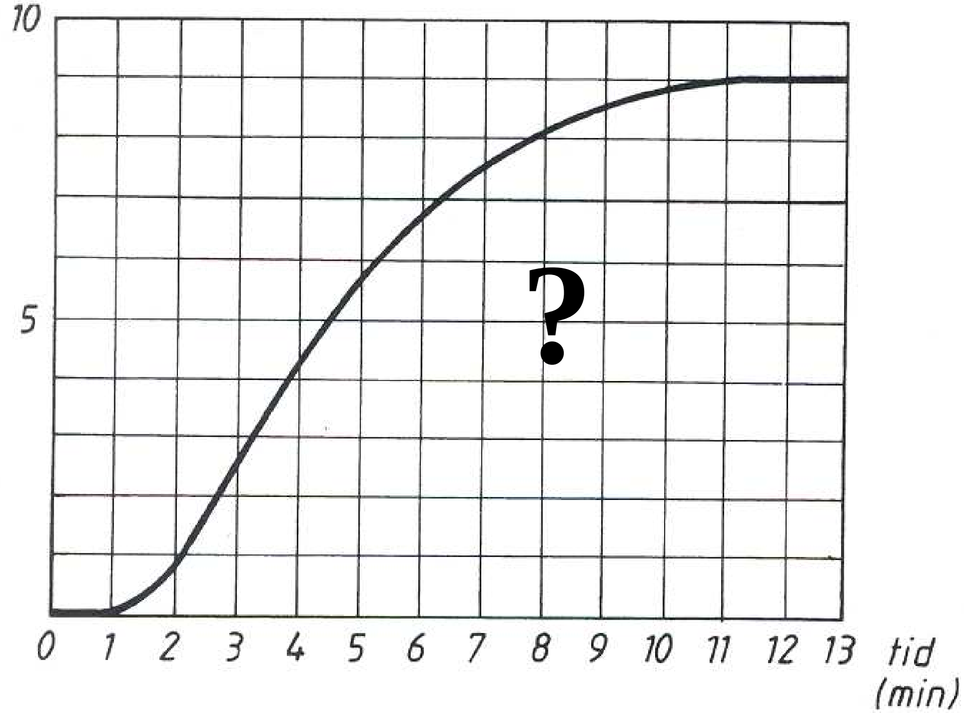
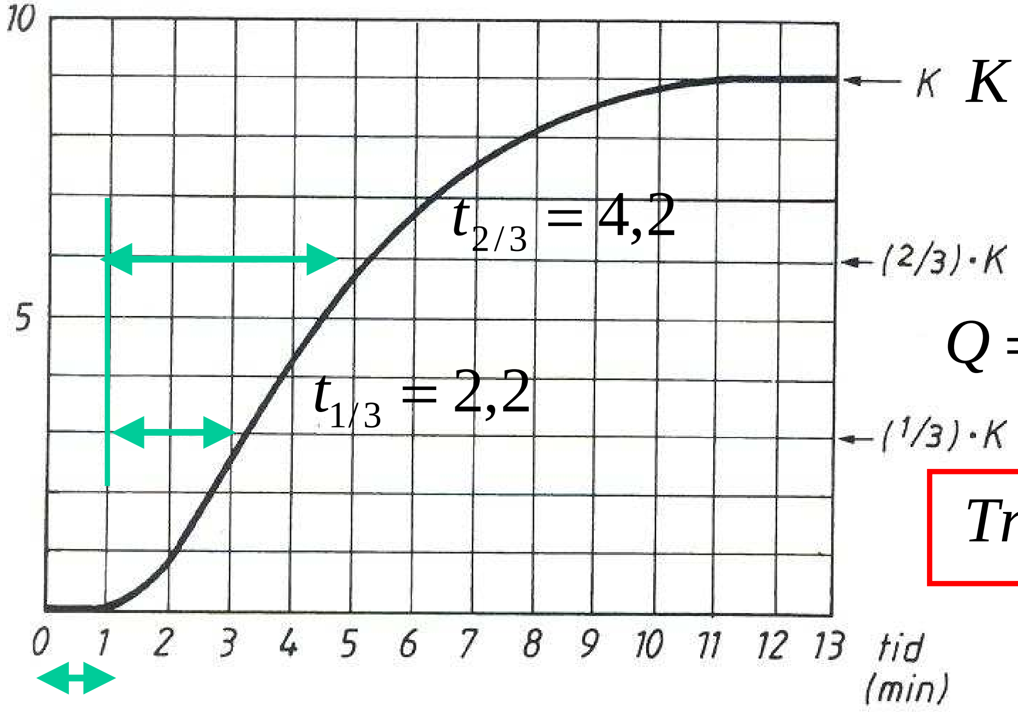
Ex. Identifiera Process?



Ex. Identifiera Process?



$K = 9$

$$Q = \frac{t_{2/3}}{t_{1/3}} = \frac{4{,}2}{2{,}2} = \boxed{1{,}91}$$

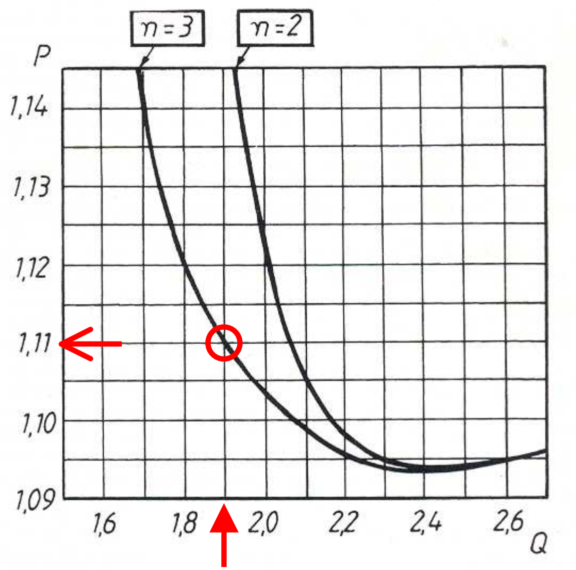
Tre tidkonstanter!

1 s dödtid

$G_1(s) = e^{-s}$

P,a-diagrammet

$$Q = 1{,}91 \quad \Rightarrow$$



$$\boxed{T = \frac{t_{2/3}}{P(1 + a + a^2)}}$$

$$\Rightarrow a = 0{,}33 \qquad\qquad \Rightarrow P = 1{,}11$$

$$T = \frac{4{,}2}{1{,}11(1 + 0{,}33 + 0{,}33^2)} = 2{,}63$$

$$aT = 0{,}33 \cdot 2{,}63 = 0{,}87$$

$$a^2T = 0{,}33^2 \cdot 2{,}63 = 0{,}29$$

Approximativ överföringsfunktion

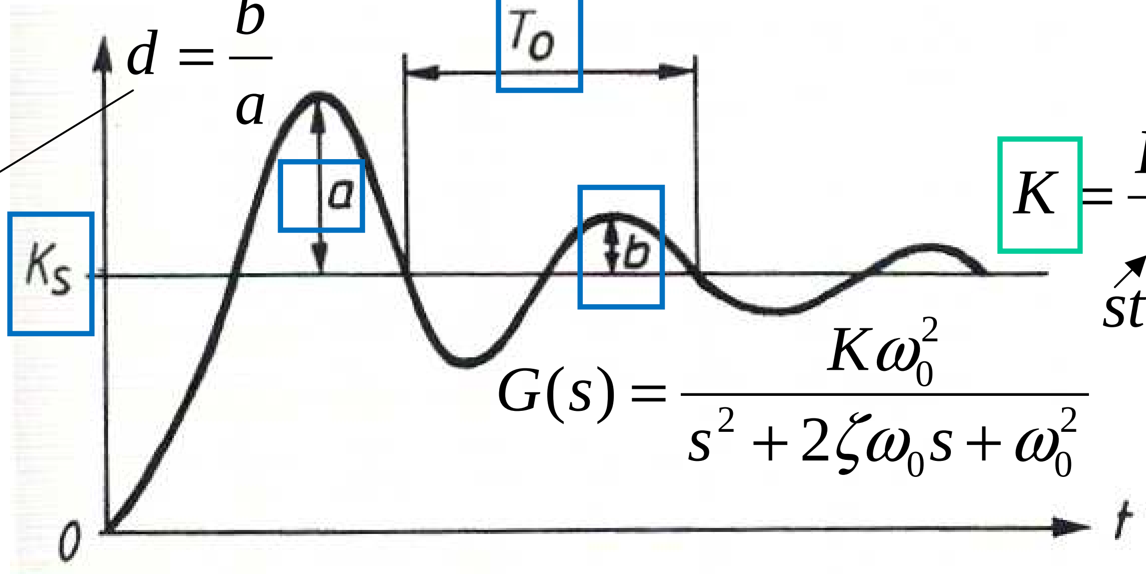
$$G(s) = e^{-s} \cdot \frac{K}{(1+Ts)(1+aTs)(1+a^2Ts)} =$$

$$= \frac{9 \cdot e^{-s}}{(1+2{,}63s)(1+0{,}87s)(1+0{,}29s)}$$

Processer med översväng

$$\zeta = \frac{1}{\sqrt{\left(\frac{2\pi}{\ln d}\right)^2 + 1}}$$

$$d = \frac{b}{a}$$



$$K = \frac{K_S}{h}$$

steget

$$G(s) = \frac{K\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$$\omega_0 = \frac{2\pi}{T_0\sqrt{1-\zeta^2}}$$

- Parametrar som kan identifieras efter ett stegsvarsexperiment.

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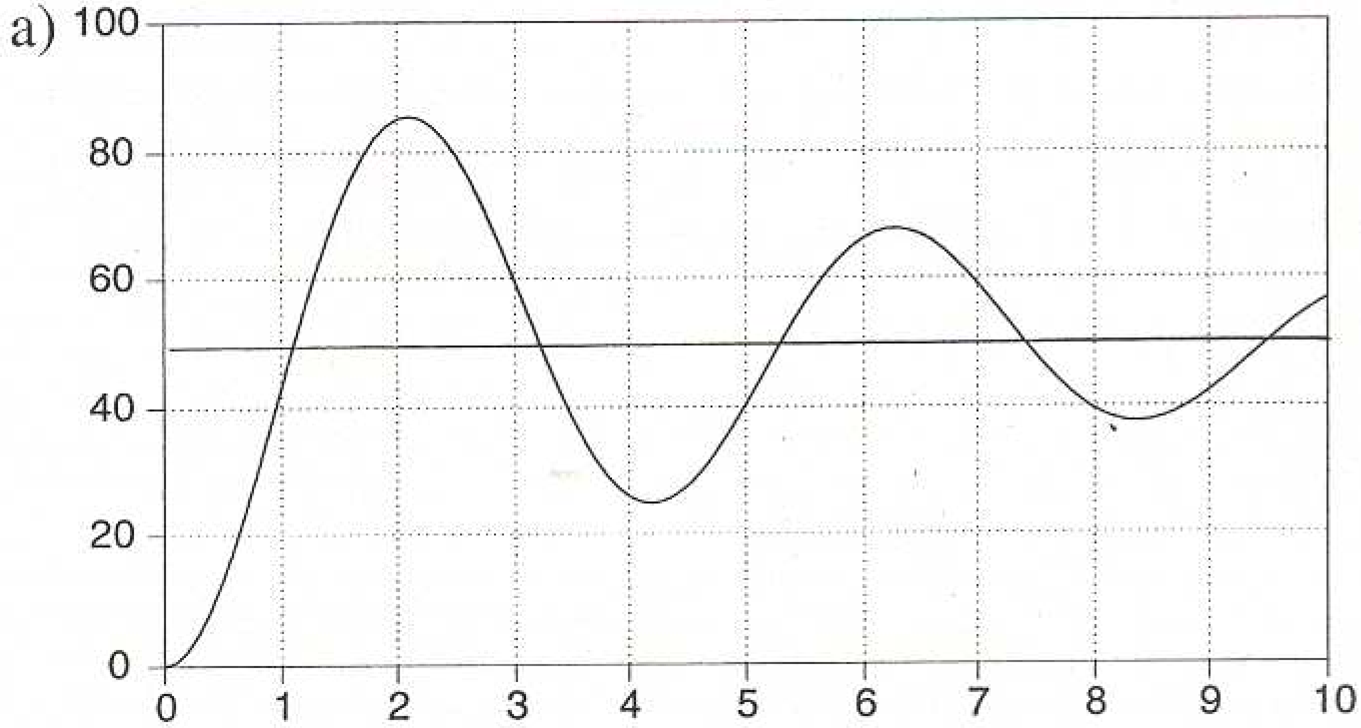
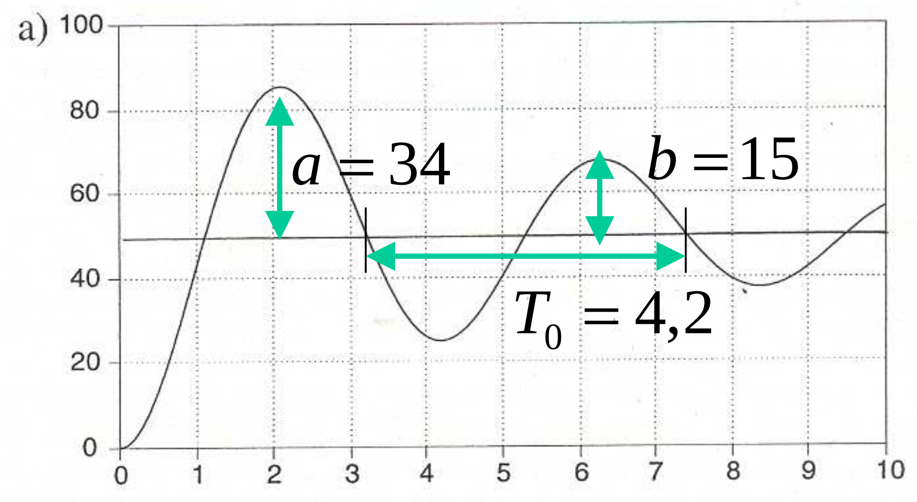
7.17 a Stegsvarsanalys



7.17 **a** lösning Stegsvarsanalys



$$d = \frac{b}{a} = \frac{15}{34} = 0{,}44$$

$$K = \frac{K_S}{h} = \frac{50}{1} = 50$$

$$\omega_0 = \frac{2\pi}{T_0\sqrt{1-\zeta^2}} =$$

$$= \frac{2\pi}{4{,}2\sqrt{1-0{,}13^2}} \approx 1{,}51$$

$$\zeta = \frac{1}{\sqrt{\left(\frac{2\pi}{\ln d}\right)^2 + 1}} = \frac{1}{\sqrt{\left(\frac{2\pi}{\ln 0{,}44}\right)^2 + 1}} \approx 0{,}13$$

$$G(s) = \frac{K\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} = \frac{50\cdot 1{,}51^2}{s^2 + 2\cdot 0{,}13\cdot 1{,}51 s + 1{,}51^2} = \frac{9{,}58}{s^2 + 0{,}39 s + 2{,}28}$$