

Partiella derivator

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Partiella derivator

- Defintion

$$\frac{\partial f}{\partial x} = \lim_{h \rightarrow 0} \frac{f(x+h, y) - f(x, y)}{h},$$
$$\frac{\partial f}{\partial y} = \lim_{h \rightarrow 0} \frac{f(x, y+h) - f(x, y)}{h}.$$

- Partiell deriviering: Derivera i en variabel med övriga variabler fixerade.
- Högre derivator

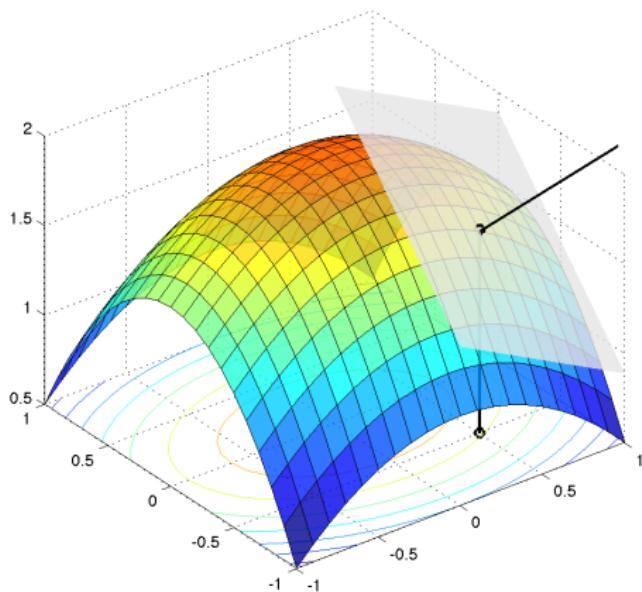
$$\frac{\partial^2 f}{\partial x^2}, \quad \frac{\partial^2 f}{\partial x \partial y}, \quad \frac{\partial^2 f}{\partial y \partial x}, \quad \frac{\partial^2 f}{\partial y^2}.$$

- Ordningen på blandade derivator kan kastas om (för "snälla" funktioner)

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}.$$

Normalvektorer och tangentplan

- Normalvektor: $n = (f_x(a, b), f_y(a, b), -1)$,
- Tangentplan: $z = f(a, b) + (x - a)f_x(a, b) + (y - b)f_y(a, b)$.



- $z = 2 - \frac{1}{2}x^2 - y^2$
- $(a, b) = \left(\frac{1}{2}, -\frac{1}{2}\right)$
- $n = \left(-\frac{1}{2}, 1, -1\right)$
- $z = \frac{19}{8} - \frac{1}{2}x + y$