Reglerteknik Ö6

Köp övningshäfte på kårbokhandeln

William Sandqvist william@kth.se

10.1 Stabilitet

Vilka processer är stabila?

$a)\quad \ddot{y} + \dot{y} - 6y = \dot{x} + x$

$b)\quad 2\ddot{y} + 6\dot{y} - 8y = 3\dot{x}$

$c)\quad \ddot{y} + 10\dot{y} + 21y = \dot{x} + 4x$

$d)\quad y''' + 3y'' - 2y' + y = u' + 5u$

$e)\quad y''' + 7y'' + y' + 4y = u'' + 3u$

$f)\quad y''' + 3y'' + y' + 6y = u' + 7u$

$g)\quad \dfrac{s+2}{s^2 - s - 6}$

$h)\quad \dfrac{s^2 + 3s + 2}{s+4}$

William Sandqvist william@kth.se

10.1 a lösning, Stabilitet

$$a) \quad \ddot{y} + \dot{y} - 6y = \dot{x} + x \quad \{L:\} \quad Ys^2 + sY - 6Y = Xs + X$$

$$\frac{Y}{X} = \frac{s+1}{s^2 + s - 6} \qquad s = -\frac{1}{2} \pm \sqrt{\tfrac{1}{4} + 6} = -\frac{1}{2} \pm \sqrt{\frac{25}{4}} = -\frac{1}{2} \pm \frac{5}{2}$$

$$\begin{cases} -3 \\ +2 \end{cases}$$

Pol i hhp
$\Rightarrow$ ostabilt

MATLAB

```
» pole(tf([1,1],[1,1,-6]))
ans =
    -3
     2
```

$b)$

10.1 b lösning, Stabilitet

$$2\ddot{y} + 6\dot{y} - 8y = 3\dot{x} \quad \{L:\} \quad 2Ys^2 + 6sY - 8Y = 3sX$$

$$\frac{Y}{X} = \frac{3s}{2s^2 + 6s - 8} \qquad s = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + 4} = -\frac{3}{2} \pm \frac{5}{3}$$

Pol i hhp, ostabilt

MATLAB

```
» pole(tf([3,0],[2,6,-8]))
ans =
    -4
     1
```

$c)$ **10.1 c** lösning, Stabilitet

$$\ddot{y} + 10\dot{y} + 21y = \dot{x} + 4x \quad \{L:\} \quad Ys^2 + 10Ys + 21Y = Xs + 4X$$

$$\frac{Y}{X} = \frac{s+4}{\boxed{s^2 + 10s + 21}} \qquad s = -5 \pm \sqrt{25 - 21} = -5 \pm 2$$

Polerna i vhp,
stabilt

MATLAB

```
» pole(tf([1,4],[1,10,21]))
ans =
    -7
    -3
```

$d)$ **10.1 d** lösning, Stabilitet

$$y''' + 3y'' - 2y' + y = u' + 5u \quad \{L:\}$$

$$Ys^3 + 3Ys^2 - 2Ys + Y = Us + 5U \qquad \frac{Y}{U} = \frac{s+5}{s^3 + 3s^2 - 2s + 1}$$

Routh

$$s^3 + 3s^2 - 2s + 1$$

1	-2	0
3	1	0
$-\frac{7}{3}$	0	0
1		

$$\frac{3\cdot(-2) - 1\cdot 1}{3} = = -\frac{7}{3}$$

MATLAB

```
» pole(tf([1,5],[1,3,-2,1]))
ans =
   -3.6274
   0.3137 + 0.4211i
   0.3137 - 0.4211i
```

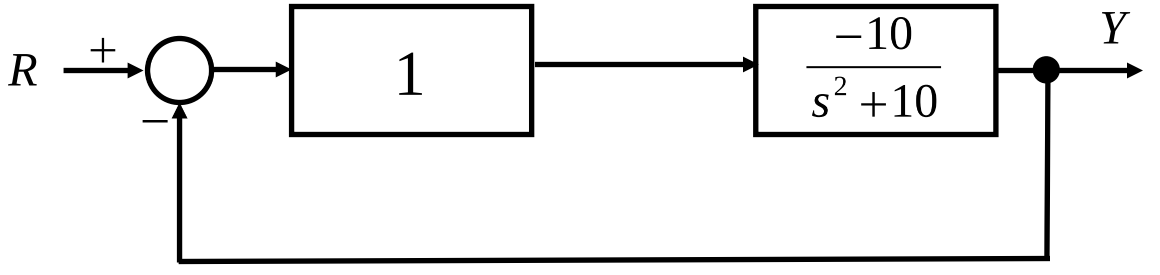
Teckenbyte, pol i hhp, ostabilt

poler i hhp, ostabilt

William Sandqvist william@kth.se

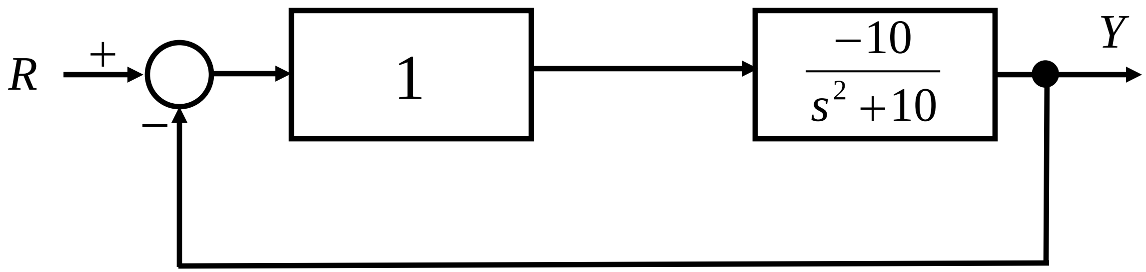
10.2 Stabilitet

Är detta slutna system stabilt?



10.2 lösning, Stabilitet

Är detta slutna system stabilt?

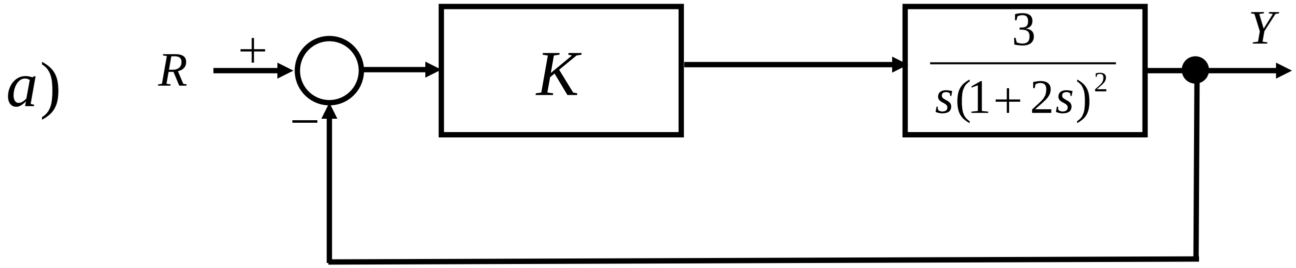


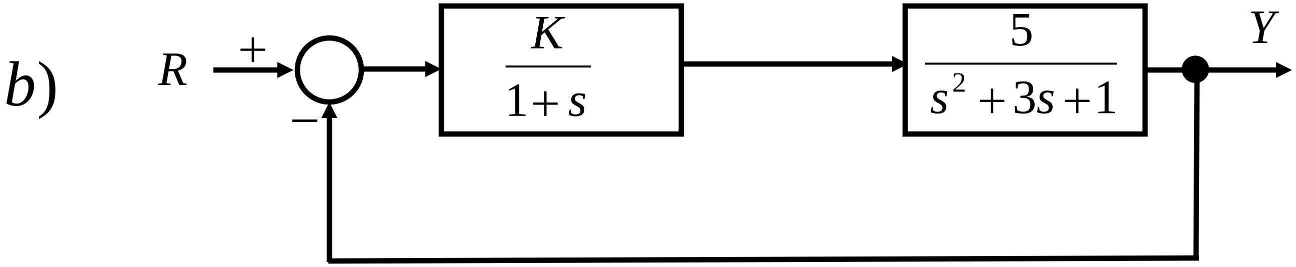
$$G(s) = \frac{1 \cdot \frac{-10}{s^2-10}}{1 + 1 \cdot \frac{-10}{s^2-10}} = \frac{-10}{s^2-10-10} = \frac{-10}{s^2-20} \qquad s = \pm\sqrt{20}$$

pol i hhp, ostabilt

10.3 Stabilitet

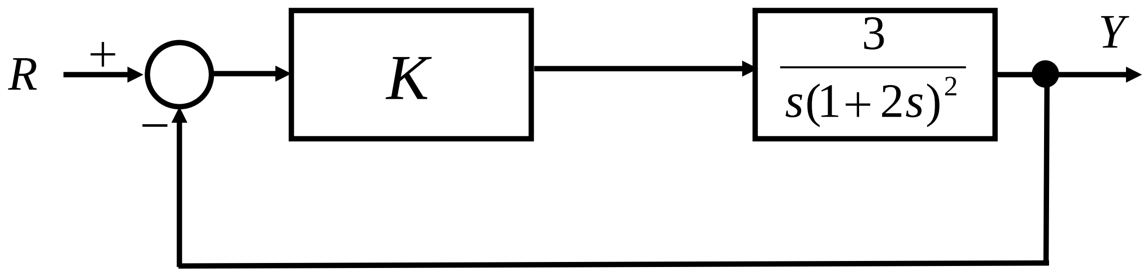
Bestäm maximalt värde på K för stabilt system.





10.3 a lösning, Stabilitet

Bestäm maximalt värde på *K* för stabilt system.



Vi använder det **öppna systemet** och **Bodeanalys**:

$$G(s) = \frac{3K}{s(1+2s)^2} \quad \underline{G}(j\omega) = \frac{3K}{j\omega(1+2j\omega)^2} \quad G(\omega) = \frac{3K}{\omega\cdot(1+4\omega^2)}$$

$$\angle\underline{G}(j\omega) = -\frac{\pi}{2} - \arctan 2\omega - \arctan 2\omega = -\frac{\pi}{2} - 2\arctan 2\omega$$

William Sandqvist william@kth.se

10.3 a lösning, Stabilitet

$$\angle \underline{G}(j\omega) = -\frac{\pi}{2} - 2\arctan 2\omega$$

- *Fasvilkor*: $-\pi = -\frac{\pi}{2} - 2\arctan 2\omega_\pi \quad \Rightarrow \quad -2\arctan 2\omega_\pi = -\frac{\pi}{2}$

$$\arctan 2\omega_\pi = \frac{\pi}{4} \quad \Rightarrow \quad 2\omega_\pi = 1 \quad \Rightarrow \quad \omega_\pi = \frac{1}{2}$$

$$G(\omega) = \frac{3K}{\omega \cdot (1 + 4\omega^2)}$$

$$G(\omega = \tfrac{1}{2}) = \frac{3K}{0{,}5 \cdot (1 + 4 \cdot 0{,}5^2)} = \frac{3K}{1}$$

- *Amplitudvilkor*:

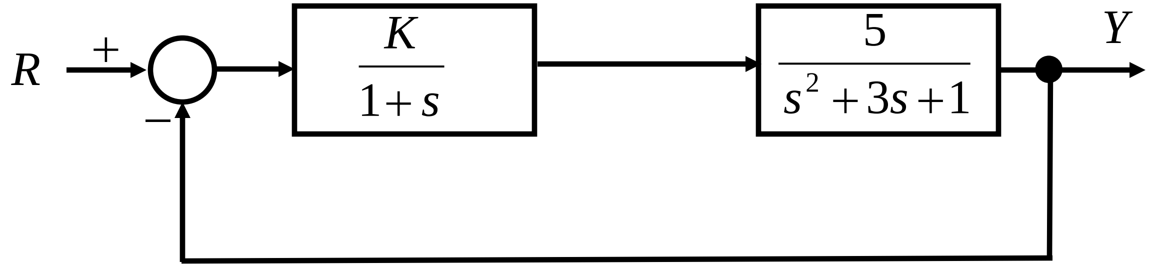
$$3K < 1 \quad \Rightarrow \quad K < \tfrac{1}{3}$$

William Sandqvist william@kth.se

10.3 b lösning, Stabilitet

Bestäm maximalt värde på *K* för stabilt system.

b)



Vi använder det **slutna systemet** och **Routh-tabell**

$$G(s) = \frac{\frac{K}{1+s}\cdot\frac{5}{s^2+3s+1}}{1+\frac{K}{1+s}\cdot\frac{5}{s^2+3s+1}} = \frac{5K}{(1+s)\cdot(s^2+3s+1)+5K} =$$

$$= \frac{5K}{s^3+4s^2+4s+(1+5K)}$$

William Sandqvist william@kth.se

10.3 b lösning, Stabilitet

Routh-tabell

$$s^3 + 4s^2 + 4s + (1+5K)$$

1	4	0
4	$1+5K$	0
$\frac{4\cdot 4-(1+5K)}{4}$	*Vi stoppar här.*	

$$\frac{4\cdot 4-(1+5K)}{4} > 0$$

$$16-1-5K > 0$$

$$\boxed{K < 3}$$

William Sandqvist william@kth.se

10.4 Stabilt område parametrar

Bestäm vilka värden (områden) *a* och *b* får ha för att ge stabila överföringsfunktioner med nämnarpolynomen.

$$a) \quad s^3 + as^2 + s + b$$

$$b) \quad as^3 + bs^2 + s + 1$$

10.4 a lösning, Stabilt område

Routh-tabell

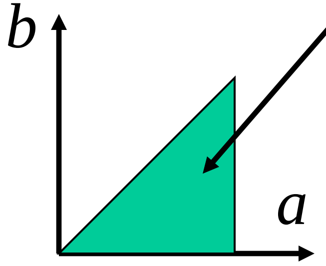
$$s^3 + as^2 + s + b$$

1	1	0
a	b	0
$\frac{a-b}{a}$	0	0
b	0	0

$$a > 0$$

$$\frac{a-b}{a} > 0 \quad \Rightarrow \quad a - b > 0 \quad \Rightarrow \quad a > b$$

$$b > 0$$

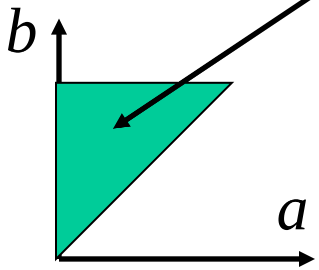


10.4 b lösning, Stabilt område

Routh-tabell

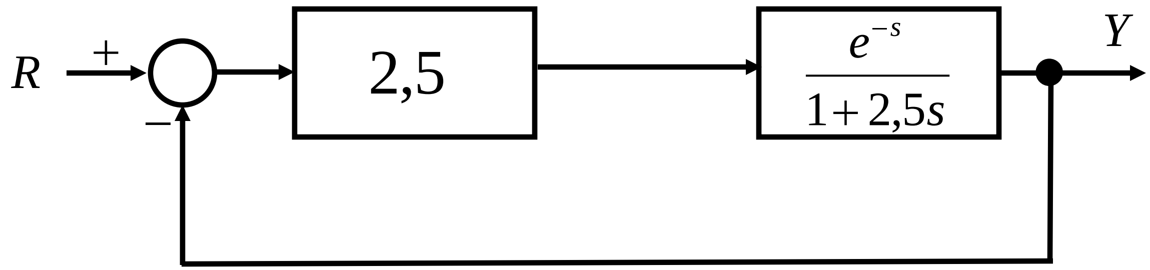
$$as^3 + bs^2 + s + 1$$

a	1	0	$a > 0$
b	1	0	$b > 0$
$\frac{b-a}{b}$	0	0	$\frac{b-a}{a} > 0 \Rightarrow b - a > 0 \Rightarrow b > a$
1	0	0	



William Sandqvist william@kth.se

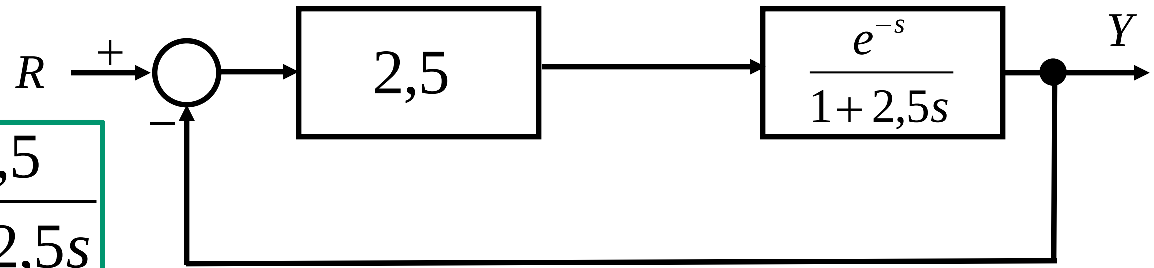
10.7 Marginaler ur Bodediagram



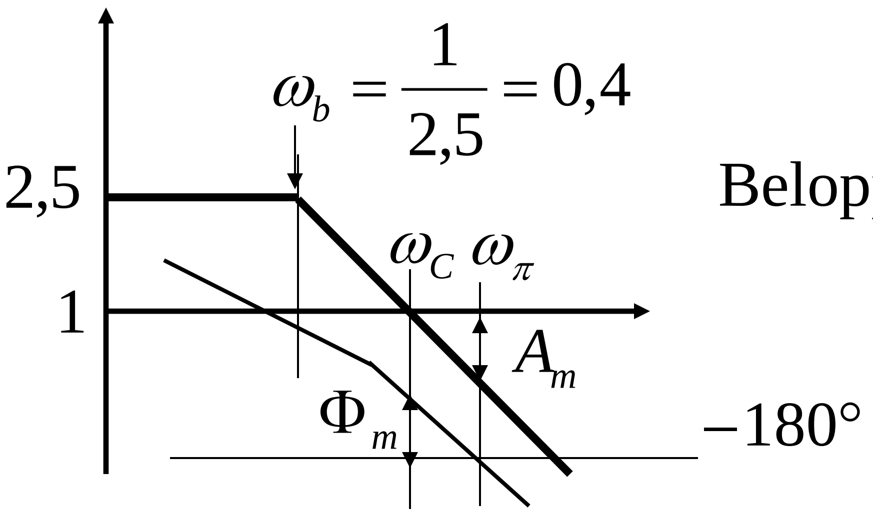
William Sandqvist william@kth.se

10.7 Lösning, Marginaler

openloop :



$$G(s) = e^{-s} \cdot \frac{2,5}{1+2,5s}$$



Fasfunktion:

$$\phi(\omega) = -\omega - \arctan(2,5\omega)$$

Beloppsfunktion:

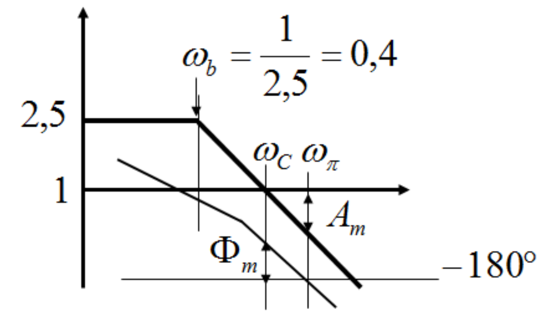
$$A(\omega) = 1 \cdot \frac{2,5}{\sqrt{1+(2,5\omega)^2}}$$

10.7 Lösning, Marginaler Φ_m

$$A(\omega) = 1 \cdot \frac{2{,}5}{\sqrt{1+(2{,}5\omega)^2}}$$

$$\Rightarrow \quad A(\omega_C) = 1 = \frac{2{,}5}{\sqrt{1+(2{,}5\omega_C)^2}} \quad \Leftrightarrow \quad 2{,}5^2 = 1 + (2{,}5\omega_C)^2 \quad \Rightarrow$$

$$\Rightarrow \quad \omega_C = \frac{\sqrt{6{,}25-1}}{2{,}5} = 0.92$$



$$\varphi(\omega) = -\omega - \arctan(2{,}5\omega)$$

$$\Rightarrow \quad \varphi(\omega_C) = -0{,}92 - \arctan(2{,}5 \cdot 0{,}92) = -2{,}08 \cdot \frac{180°}{\pi} = -119°$$

$$\Phi_m = \left|(-180°) - \varphi(\omega_C)\right| = 180° - 119° = 61°$$

William Sandqvist william@kth.se

10.7 Lösning, Marginaler ω_π

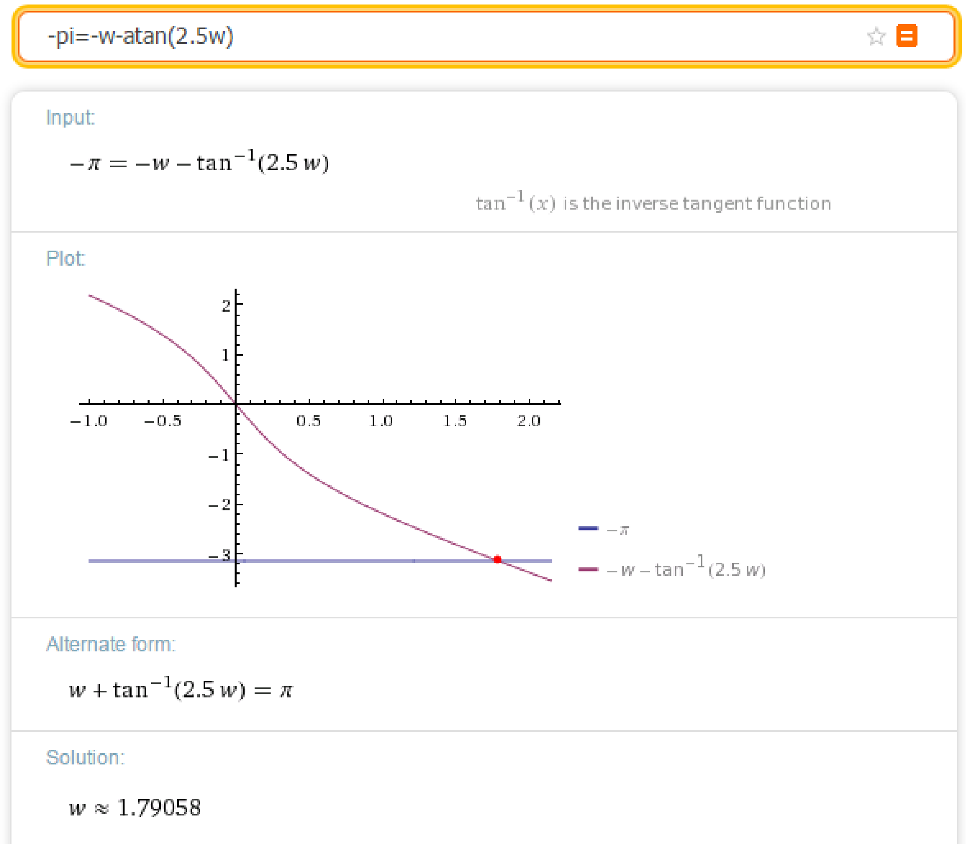
$$\phi(\omega) = -\omega - \arctan(2,5\omega)$$

Svårt att lösa ut ω_π ?   för $\phi = -\pi$

Jag tar hjälp av **WolframAlpha** på webben

$$\omega_\pi = 1,79$$

WolframAlpha computational knowledge engine



10.7 Lösning, Marginaler A_m

$$A(\omega) = 1 \cdot \frac{2{,}5}{\sqrt{1+(2{,}5\omega)^2}}$$

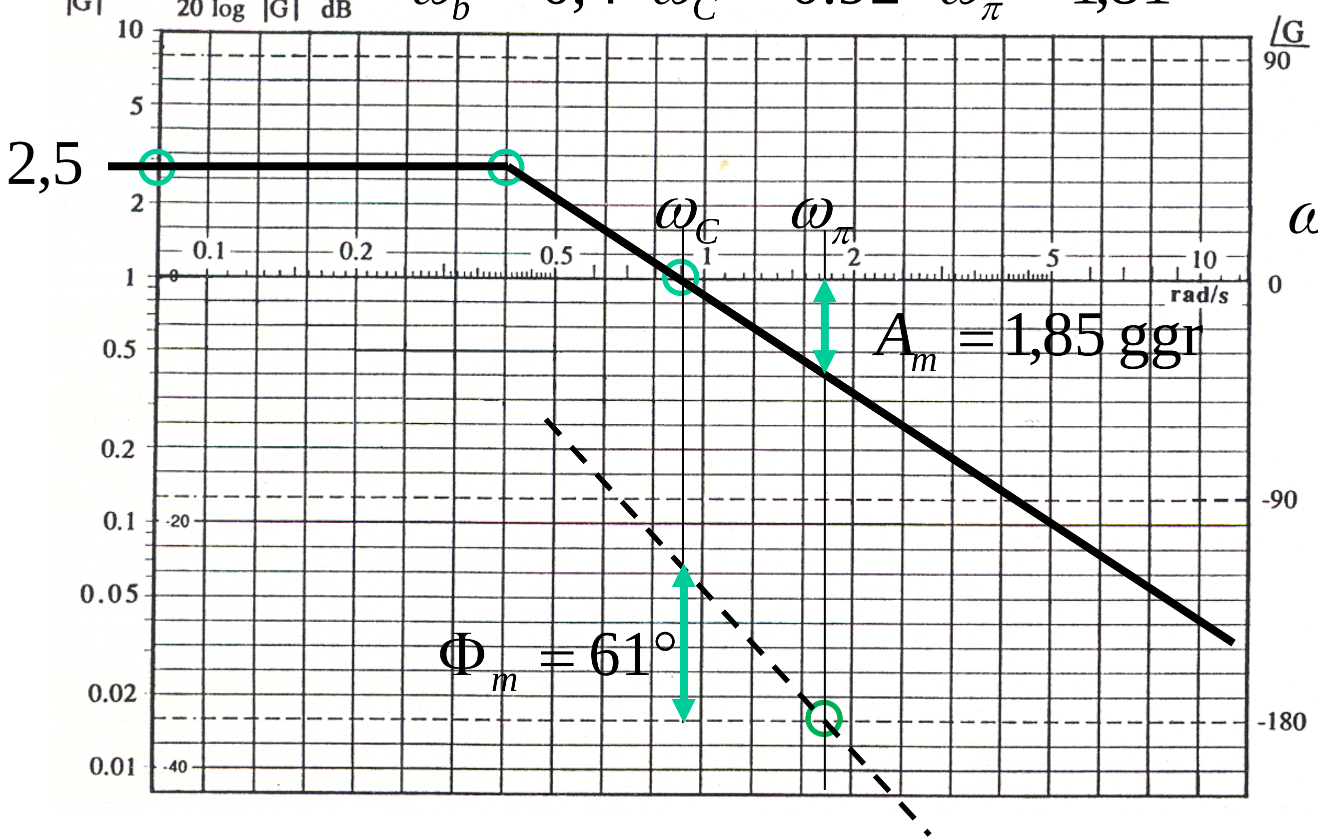
$$A(\omega = \omega_\pi = 1{,}79) = \frac{2{,}5}{\sqrt{1+(2{,}5 \cdot 1{,}79)^2}} = 0{,}54$$

$$A_m = \frac{1}{A(\omega_\pi)} = \frac{1}{0{,}54} = 1{,}85 \text{ ggr}$$

William Sandqvist william@kth.se

10.7 Lösning, Marginaler

$$\omega_b = 0{,}4 \quad \omega_C = 0.92 \quad \omega_\pi = 1{,}81$$

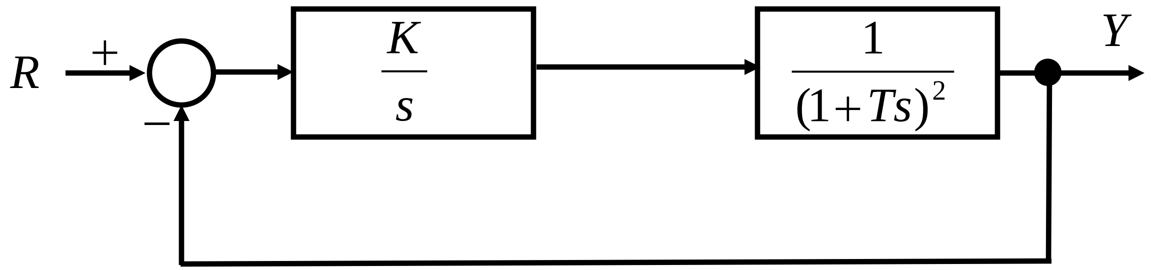


$$\omega_\pi = 1{,}79$$

$$\varphi(\omega_C) =$$
$$= -119^\circ$$

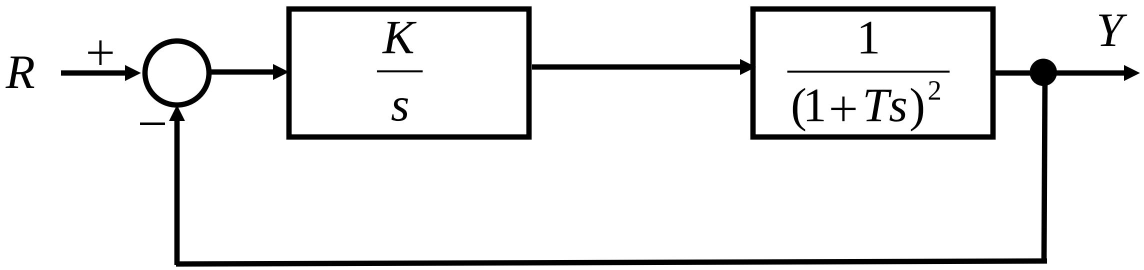
William Sandqvist william@kth.se

10.9 Marginal med Rouths metod



Välj K så att $A_m = 2$ ggr.

10.9 Lösning, Rouths metod



$$\frac{Y}{R} = \frac{\frac{K}{s}\cdot\frac{1}{(1+Ts)^2}}{1+\frac{K}{s}\cdot\frac{1}{(1+Ts)^2}} = \frac{K}{\boxed{T^2s^3+2Ts^2+s+K}}$$

10.9 Lösning, Rouths metod

$$T^2 s^3 + 2Ts^2 + s + K$$

Routh tabell

T^2	1	0
$2T$	K	0
$\frac{2T - T^2 K}{2T}$	0	0
K	0	0

$$T > 0$$

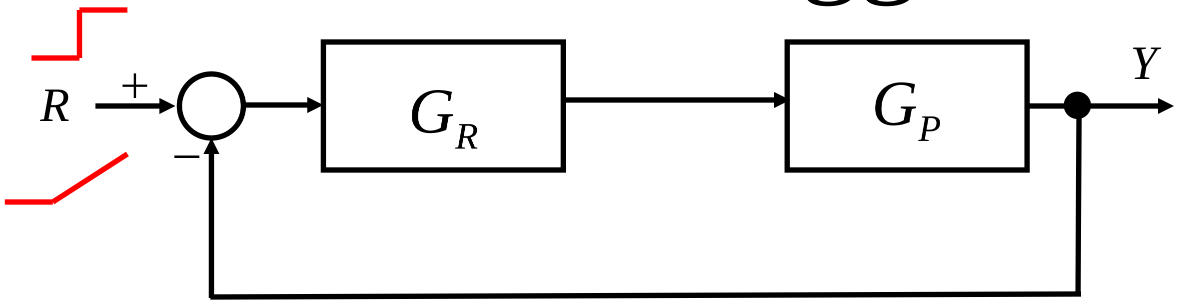
$$\frac{2 - TK}{2} > 0 \Rightarrow TK < 2$$

$$K > 0$$

$$K_{\max} = \frac{2}{T} \quad A_m = 2 \quad \Rightarrow \frac{K_{\max}}{2} \quad \Rightarrow \boxed{K = \frac{1}{T}}$$

William Sandqvist william@kth.se

10.11 Statisk noggrannhet



$$a) \quad G_R = \frac{2s+3}{s+2} \quad G_P = \frac{2}{s(5s+1)}$$

$$b) \quad G_R = \frac{s+2}{s+1} \quad G_P = \frac{10}{s+4}$$

10.11 a lösning statiska fel

$$a)\quad G_R = \frac{2s+3}{s+2} \qquad G_P = \frac{2}{s(5s+1)}$$

$$a \Rightarrow e_0 = 0$$

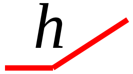
Stegformad börvärdesändring. Om G_R eller G_P är integrerande blir det *inget* kvarvarande fel, $e_0 = 0$

Rampformad börvärdesändring.

$$h \qquad e_l = \lim_{s \to 0} \frac{h}{s \cdot (1 + G_R \cdot G_P)}$$

fortsättning …

10.11 a lösning statiska fel

h 

$$G_R = \frac{2s+3}{s+2} \qquad G_P = \frac{2}{s(5s+1)}$$

$$\boxed{e_l = \lim_{s\to 0} \frac{h}{s\cdot(1+G_R\cdot G_P)}} = \lim_{s\to 0} \frac{h}{s\cdot\left(1+\frac{2s+3}{s+2}\cdot\frac{2}{s(5s+1)}\right)} =$$

$$= \lim_{s\to 0} \frac{h}{\cancel{s}\left(\frac{s(s+2)(5s+1)+2(2s+3)}{\cancel{s}(s+2)(5s+1)}\right)} = \lim_{s\to 0} \frac{h}{\frac{2\cdot 3}{2\cdot 1}} = \boxed{\frac{h}{3}}$$

10.11 b lösning statiska fel

$$b)\quad G_R = \frac{s+2}{s+1} \quad G_P = \frac{10}{s+4}$$

a Kvarvarande fel efter stegformad börvärdesändring.

$$e_0 = \lim_{s\to 0} \frac{a}{1+G_R \cdot G_P} = \lim_{s\to 0} \frac{a}{1+\frac{s+3}{s+1}\cdot\frac{10}{s+4}} = \frac{a}{1+\frac{20}{1\cdot 4}} = \frac{a}{6}$$

h Kvarvarande fel efter rampformad börvärdesändring.

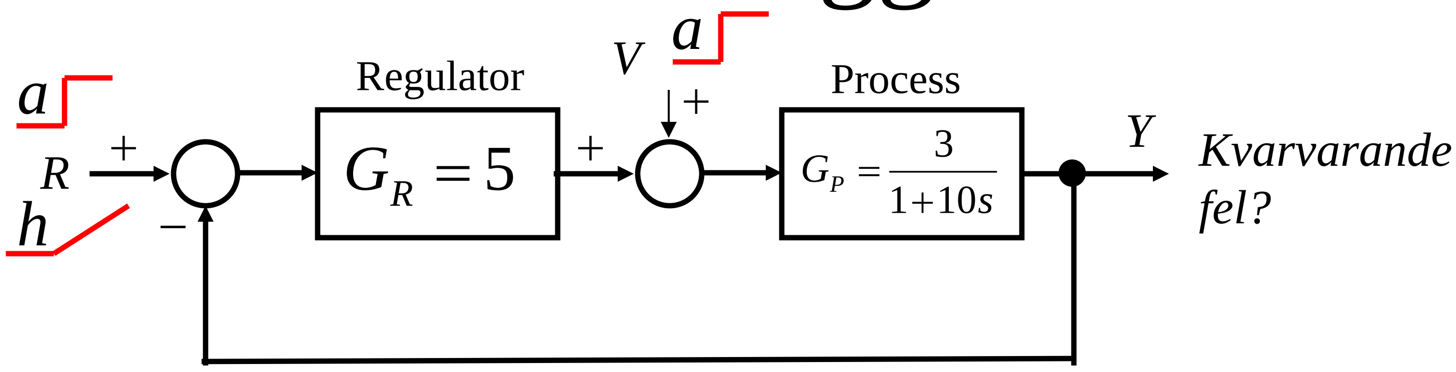
$$e_l = \lim_{s\to 0} \frac{h}{s\cdot(1+G_R \cdot G_P)}$$

$$s \cdot (1+G_R \cdot G_P) = 0$$

ingen integrering! $\Rightarrow e_1 = \infty$

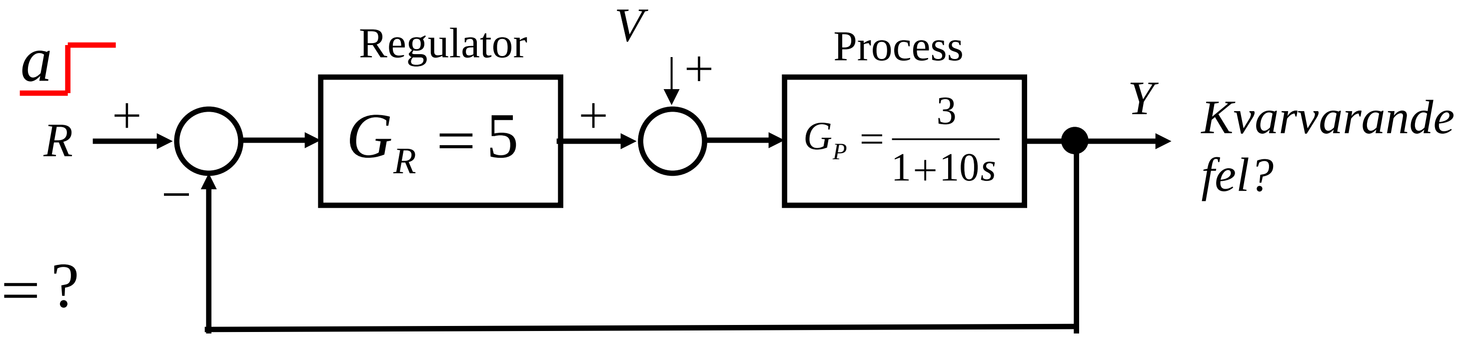
William Sandqvist william@kth.se

10.12 Statisk noggrannhet



$$a)\ \frac{Y}{R} \quad e_0 = ? \qquad b)\ \frac{Y}{R} \quad e_1 = ? \qquad c)\ \frac{Y}{V} \quad e_V = ?$$

10.12 a lösning statiska fel

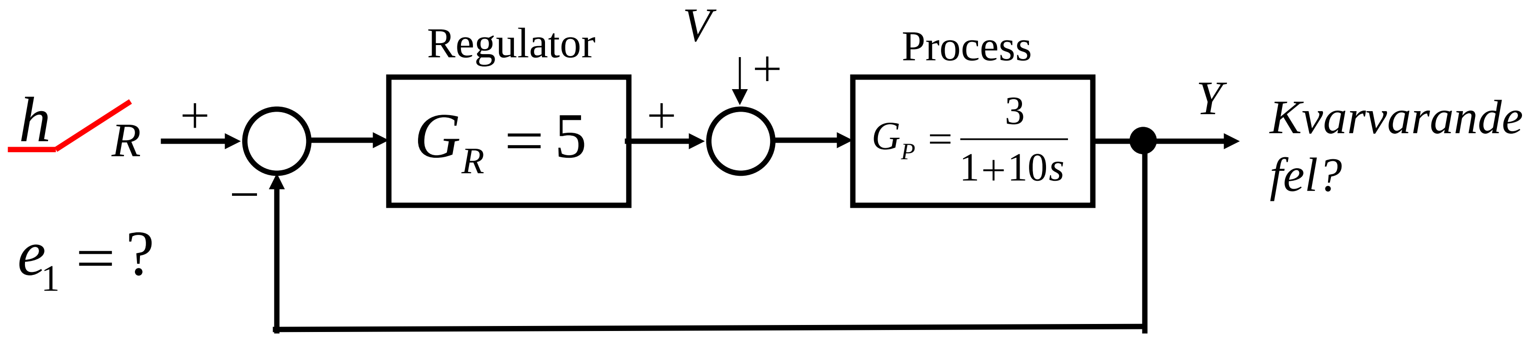


$$a)\ \frac{Y}{R} \quad e_0 = ?$$

$$e_0 = \lim_{s \to 0} \frac{a}{1 + G_R \cdot G_P} = \lim_{s \to 0} \frac{a}{1 + 5 \cdot \frac{3}{1+10s}} = \frac{a}{1 + \frac{15}{1}} = \frac{a}{16}$$

William Sandqvist william@kth.se

10.12 b lösning statiska fel



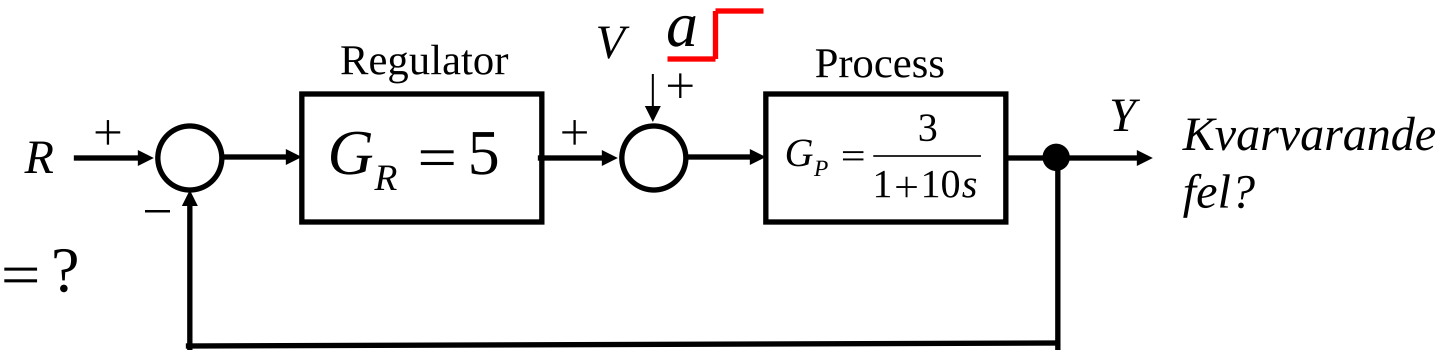
b) $\frac{Y}{R}$ $e_1 = ?$

$$e_l = \lim_{s\to 0} \frac{h}{s \cdot (1 + G_R \cdot G_P)} = 0$$

ingen integrering! $\Rightarrow e_1 = \infty$

William Sandqvist william@kth.se

10.12 c lösning statiska fel

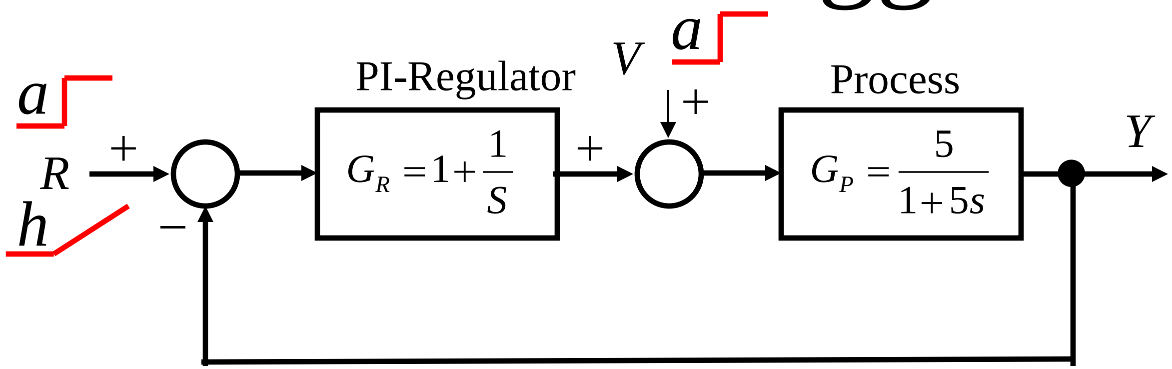


c) $\frac{Y}{V} \quad e_V = ?$

$$\boxed{e_V = \lim_{s\to 0} \frac{-G_P a}{1+G_R\cdot G_P}} = \lim_{s\to 0} \frac{-\frac{3a}{1+10s}}{1+5\cdot\frac{3}{1+10s}} = \frac{a}{1+\frac{15}{1}} = \boxed{-\frac{3a}{16}}$$

William Sandqvist william@kth.se

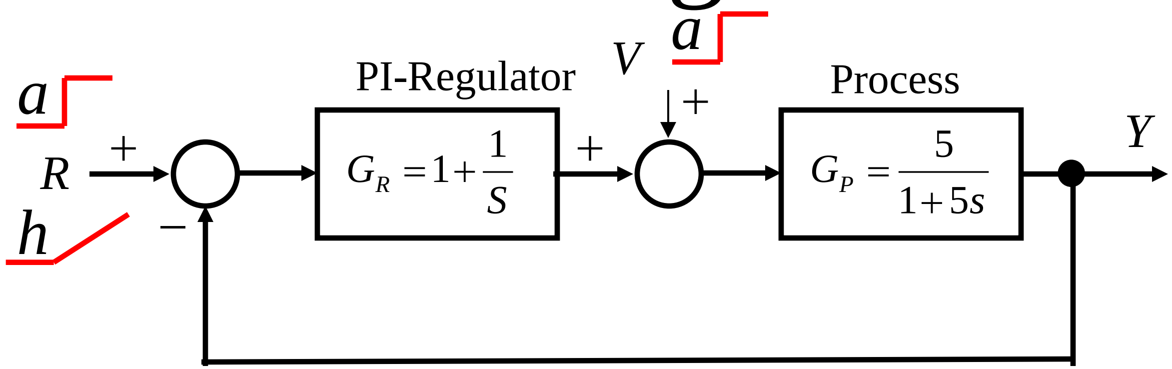
10.13 Statisk noggrannhet



Kvarvarande fel?

$$\frac{Y}{R} \quad e_0 = ? \qquad \frac{Y}{R} \quad e_1 = ? \qquad \frac{Y}{V} \quad e_V = ?$$

10.13 lösning statiska fel



Kvarvarande fel?

$$G_R \cdot G_P = \left(1 + \frac{1}{s}\right) \cdot \left(\frac{5}{1+5s}\right)$$

Integrering! $e_0 = e_V = 0$

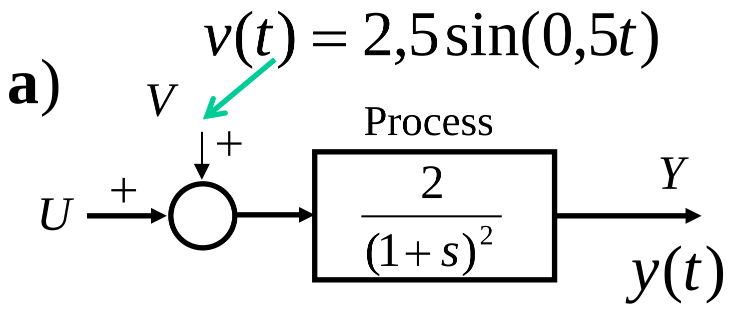
h

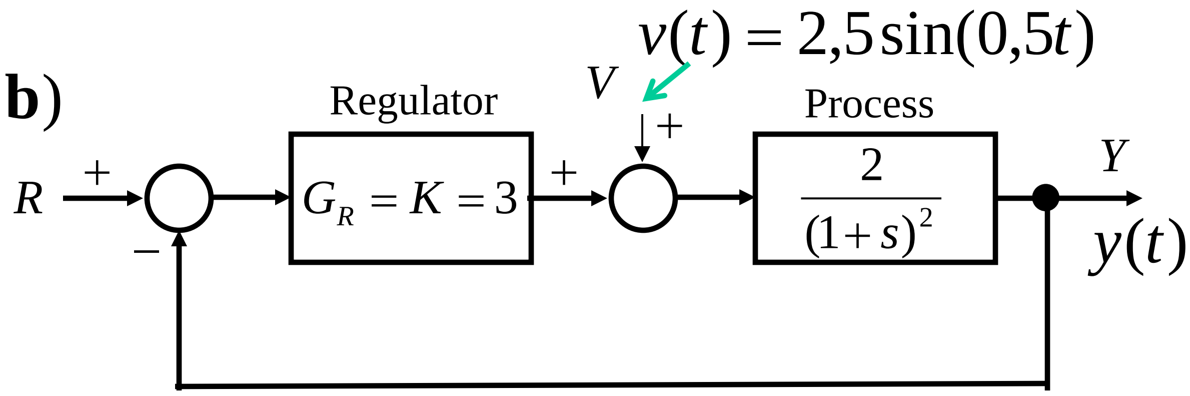
$$e_l = \lim_{s\to 0} \frac{h}{s\cdot(1+G_R\cdot G_P)} = \lim_{s\to 0} \frac{h}{s\cdot\left(1+\left(1+\frac{1}{s}\right)\cdot\frac{5}{1+5s}\right)} = \lim_{s\to 0} \frac{h}{s+\frac{5(s+1)}{1+5s}} = \frac{h}{5}$$

William Sandqvist william@kth.se

10.20 Störningsdämpning

a) Hur stor amplitud får $y(t)$ från $v(t)$?
b) Hur stor amplitud får $y(t)$ från $v(t)$ om en P-regulator med $K = 3$ används?





10.20 lösn Störningsdämpning

$$a) \quad G(s) = \frac{2}{(1+s)^2} \qquad G(\omega) = \frac{2}{1+\omega^2}$$

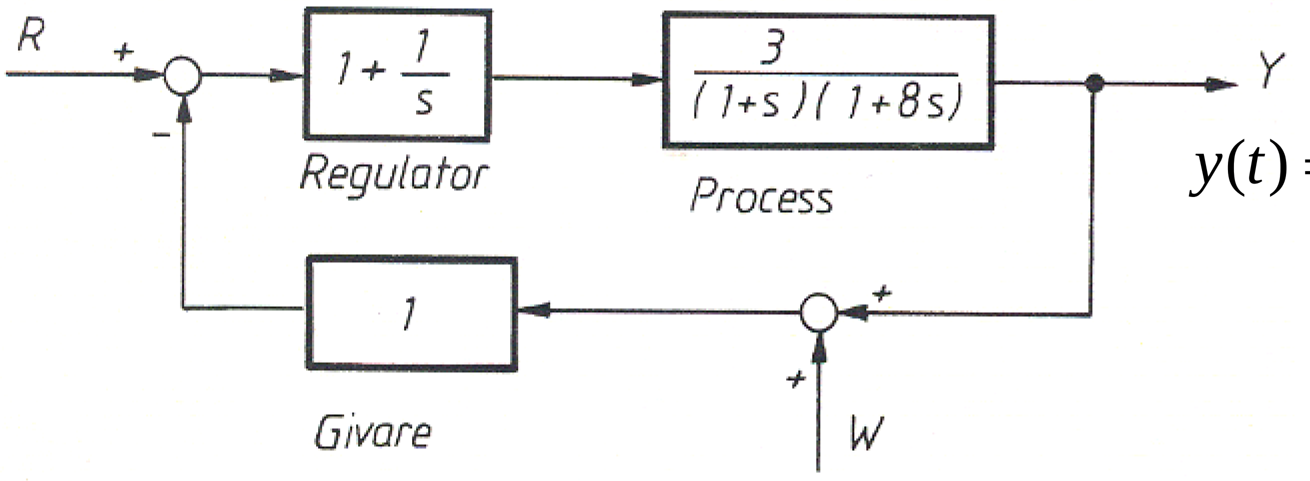
$$G(\omega = 0{,}5) = \frac{2}{1+0{,}5^2} = \boxed{1{,}6}$$

$$b) \quad G(s) = \frac{Y}{V} = \frac{G_P}{1+G_R G_P} = \frac{\frac{2}{(1+s)^2}}{1+3\cdot\frac{2}{(1+s)^2}} = \frac{2}{(1+s)^2+6} = \frac{2}{s^2+2s+7}$$

$$G(\omega) = \frac{2}{\sqrt{(7-\omega)^2+4\omega^2}} \qquad G(\omega = 0{,}5) = \frac{5}{\sqrt{6{,}75^2+1}} = \boxed{0{,}73}$$

William Sandqvist william@kth.se

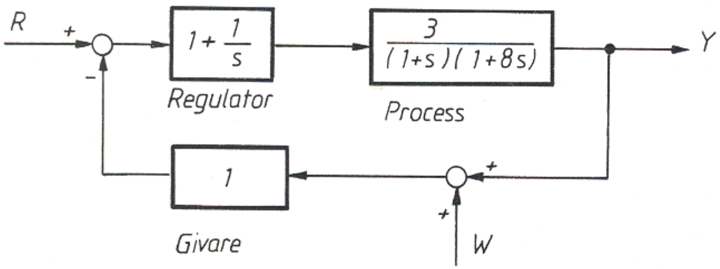
10.21 Störningsdämpning



$y(t) = ?$

$w(t) = 3{,}2\sin(2t)$

10.21 lösn Störningsdämpning



$$-W \approx R \quad G(s) = \frac{Y}{W} = \frac{-G_R G_P}{1 + G_R G_P}$$

$$G(s) = \frac{-\left(1+\frac{1}{S}\right)\cdot \dfrac{3}{(1+s)(1+8s)}}{1+\left(1+\frac{1}{s}\right)\cdot \dfrac{3}{(1+s)(1+8s)}} = \frac{3\left(1+\frac{1}{s}\right)}{(1+s)(1+8s)+3\left(1+\frac{1}{s}\right)} = \frac{3+3s}{8s^3+9s^2+4s+3}$$

$$\underline{G}(j\omega) = \frac{3+3j\omega}{-8j\omega^3-9\omega^2+4j\omega+3} \quad G(\omega=2) = \frac{3{,}2\sqrt{9+36}}{\sqrt{(3-36)^2+(8-69)^2}} = \boxed{0{,}33}$$

William Sandqvist william@kth.se