Home assignment 4: Faraday rotation

To be handed during the class in 20 February (13:15-15:00).

Background:

In a crystal the birefringence can be observed when the wave vector is perpendicular to the optical axis. The polarisation vectors of the ordinary and the extraordinary waves are then perpendicular and parallel to the optical axis, respectively, thus inducing different responses. In a magnetised plasma there is a type of birefringence called Faraday rotation that appears as a result of the Lorentz force ($q\mathbf{v} \times \mathbf{B}$). This force in responsible for the gyrotropic term in the dielectric tensor, $\propto b_k \epsilon_{ijk}$, where b_i is the unit vector in the direction of the magnetic field. It also induces an intrinsic *direction of rotation* around the magnetic lines of force, which creates a difference in the response to left and right hand circularly polarised waves.

Questions:

Let us consider a uniform magnetised plasma inside a vaccum vessel. Let the vessel have a window through which we shine a laser beam with frequency ω. A detector is installed at the opposite end of the vacuum vessel to measure the polarisation of the light after passing through the plasma. Let the distance between the window and the detector be L.

The dielectric response tensor can here be described by the cold plasma theory as

$$K_{ij}(\omega) = S(\omega)(\delta_{ij} - b_i b_j) + P(\omega) b_i b_j - iD(\omega) b_k \epsilon_{ijk}, \tag{1}$$

where b_i is the unit vector in the direction of the magnetic field. Here we will assume that S and P are independent of the magnetic field strength B, while $D = D_0 B$.

a) Show that K_{ij} satiesfies the Onsager relations.

b) Assume that the magnetic field is along the ray path and the wave vector in the z-direction. What are the dispersion relations for the two transverse plasma waves?

c) What are the polarisation vectors of these two waves?

d) Let the incoming laser beam be linearly polarised with the electric field in the x-direction. Describe this wave as it enters the plasma in terms of the polarisation vectors of the two plasma waves.

e) What is the electric wave field at the detector?

f) What polarisation will the detector measure? Describe in a one, two, or more, sentences how the plasma changes the polarisation.

g) If the detector measures linear polarisation in the y-direction, what is the strength of the magnetic field (expressed in terms of S, D_0, P and L)?

h) If the laser light passes through a quarter wave plate before entering the plasma, what polarisation will the detector measure?