Ytintegraler

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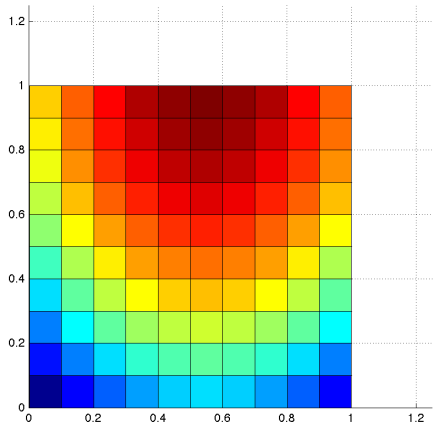
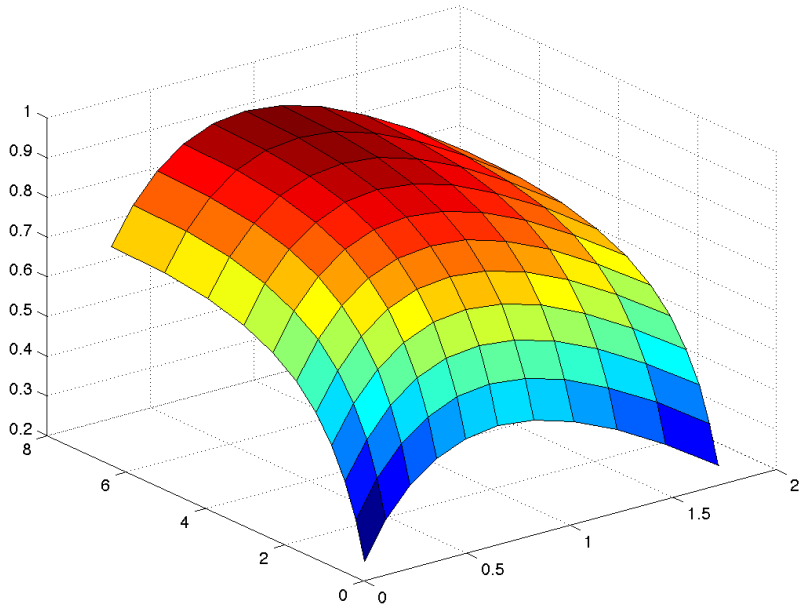
Numerisk analys, Matematik, KTH

SF1669, VT 2015

Arean av en parameteriserad yta



$\mathbf{r}(u,v)$ $\longrightarrow$

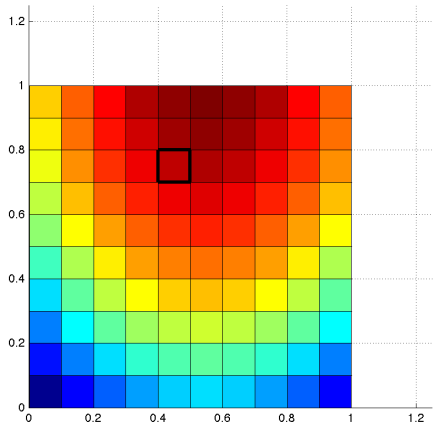


$(u, v) \in E$

$(x, y, z) \in S$

Arean av S =?

Arean av en parameteriserad yta



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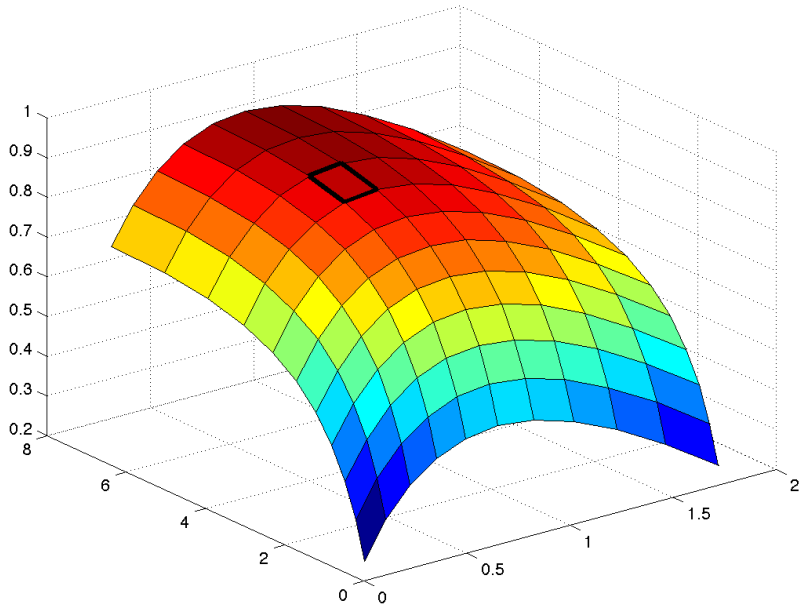

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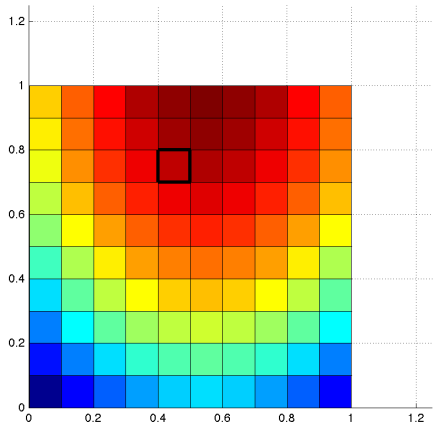

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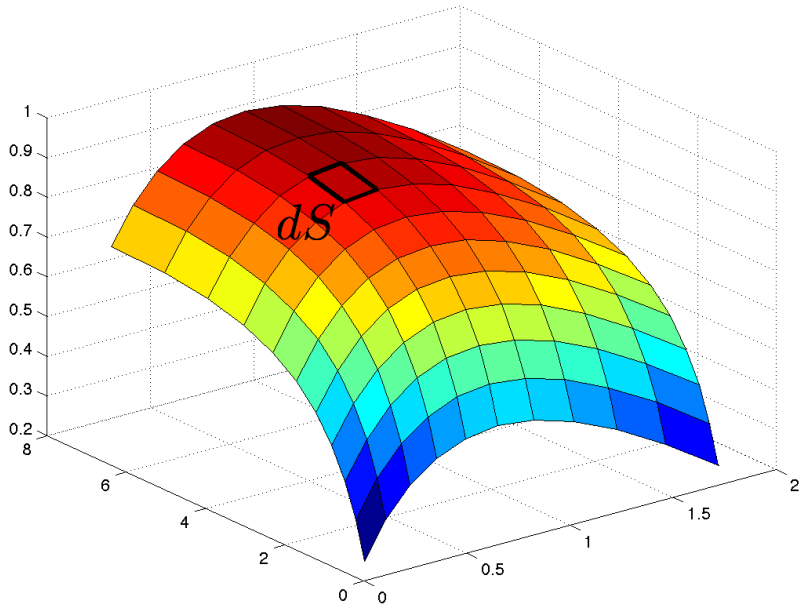

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$$\text{Arean av } S = \iint_S dS$$

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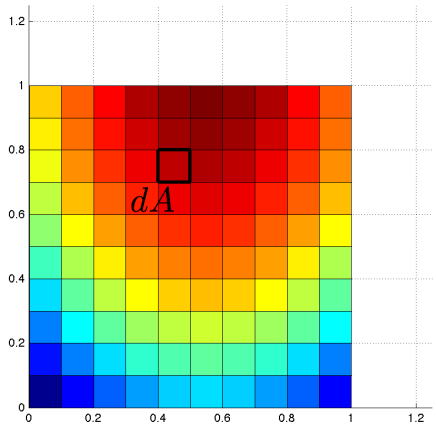

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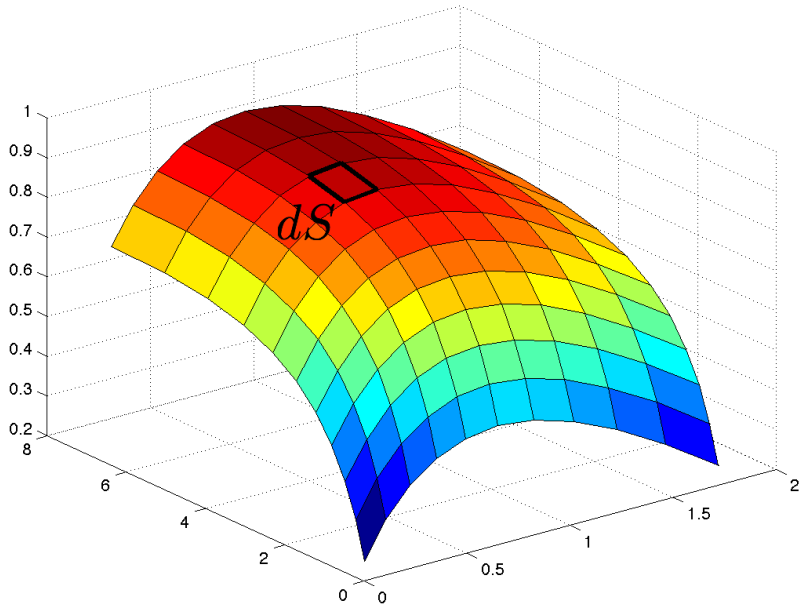

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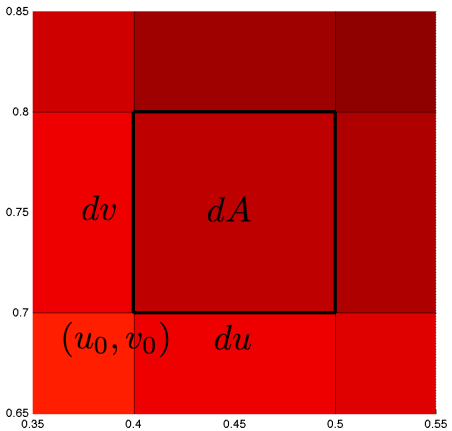

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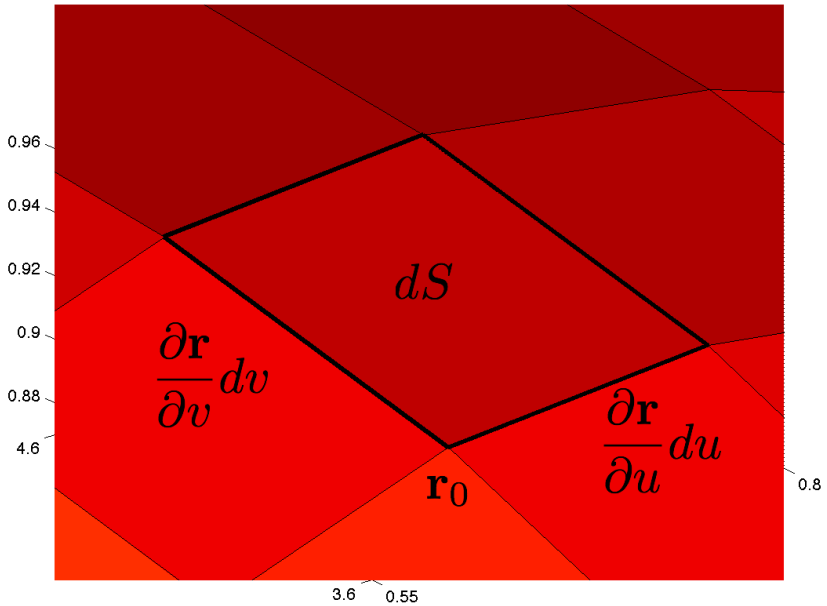

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$$dS = \left| \frac{\partial \boldsymbol{r}}{\partial u} \times \frac{\partial \boldsymbol{r}}{\partial v} \right| dudv$$

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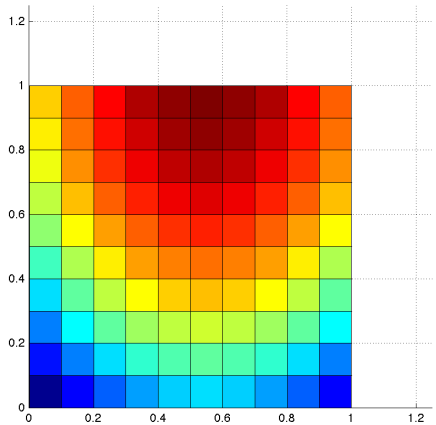

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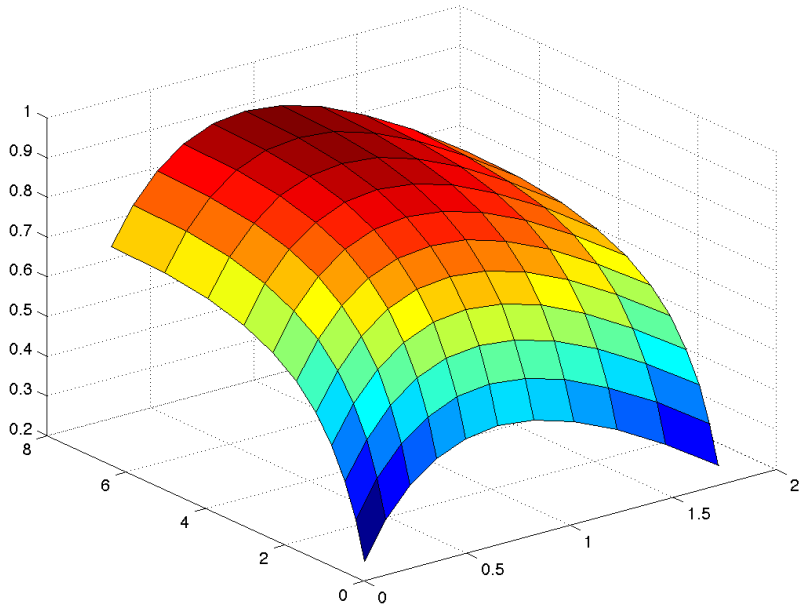

$(u, v) \in E$

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$$\text{Arean av } S = \iint_S dS = \iint_E \left| \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right| dudv$$

Fråga

Vad blir ytelementet för planet $ax + by + cz = d$?

1. $$dS = \sqrt{a^2 + b^2 + c^2 - d^2}\,dxdy$$
2. $$dS = \sqrt{1 + a^2 + b^2}\,dxdy$$
3. $$dS = \frac{d}{\sqrt{1 + a^2 + b^2}}\,dxdy$$
4. $$dS = \sqrt{1 + \frac{a^2}{c^2} + \frac{b^2}{c^2}}\,dxdy$$
5. $$dS = \sqrt{1 + \frac{a^2}{d^2} + \frac{b^2}{d^2}}\,dxdy$$

Fråga

Vad blir ytelementet för skålen $z = x^2 + y^2$?

1. $$dS = \sqrt{1 + 2x + 2y}\,dxdy$$
2. $$dS = \sqrt{1 + 4x^2 + 4y^2}\,dxdy$$
3. $$dS = \sqrt{1 + 2x^2 + 2y^2}\,dxdy$$
4. $$dS = 2\sqrt{x^2 + y^2}\,dxdy$$
5. $$dS = \sqrt{1 + 4x^2 + 2y^2}\,dxdy$$

Fråga

Vad blir ytelementet för skålen $z = x^2 + y^2$ parameteriserad med polära koordinater,

$$x = r\cos\theta, \qquad y = r\sin\theta, \qquad z = r^2.$$

1. $dS = rdrd\theta$
2. $dS = r\sin^2\theta drd\theta$
3. $dS = \sqrt{4r^2 + 1}rdrd\theta$
4. $dS = \sqrt{r}\sin\theta drd\theta$
5. $dS = \sqrt{1 + r^2}drd\theta$

Fråga

Vad blir det vektoriella ytelementet för konen $z = \sqrt{x^2 + y^2}$ parameteriserad med (x, y), med utåtriktad normal?

1. $$d\boldsymbol{S} = \begin{pmatrix} \frac{x}{\sqrt{x^2+y^2}} \\ \frac{y}{\sqrt{x^2+y^2}} \\ -1 \end{pmatrix} dxdy$$

2. $$d\boldsymbol{S} = \begin{pmatrix} \frac{-x}{\sqrt{x^2+y^2}} \\ \frac{-y}{\sqrt{x^2+y^2}} \\ 1 \end{pmatrix} dxdy$$

3. $$d\boldsymbol{S} = \begin{pmatrix} x \\ y \\ -1 \end{pmatrix} dxdy$$

4. $$d\boldsymbol{S} = \begin{pmatrix} -x \\ -y \\ 1 \end{pmatrix} dxdy$$

5. $$d\boldsymbol{S} = \begin{pmatrix} \frac{x^2}{\sqrt{x^2+y^2}} \\ \frac{y^2}{\sqrt{x^2+y^2}} \\ -1 \end{pmatrix} dxdy$$

6. $$d\boldsymbol{S} = \begin{pmatrix} \frac{1}{\sqrt{x^2+y^2}} \\ \frac{1}{\sqrt{x^2+y^2}} \\ -z \end{pmatrix} dxdy$$

Fråga

Vad blir flödesintegralen över ytan S_2?

1. π
2. 3π
3. $\pi/2$
4. -2π
5. 0