- In natural semantics:

(E_{ass}) $\langle x:=a, s\rangle \to s' \Leftrightarrow s' = s[x \mapsto \mathcal{A}[\![a]\!](s)]$

(E_{skip}) $\langle \text{skip}, s\rangle \to s' \Leftrightarrow s' = s$

(E_{comp}) $\langle S_1;S_2, s\rangle \to s' \Leftrightarrow \exists s'' \in \text{State}.(\langle S_1, s\rangle \to s'' \wedge \langle S_2, s''\rangle \to s')$

(E_{if}) $\langle \underline{\text{if}}\ b\ \underline{\text{then}}\ S_1\ \underline{\text{else}}\ S_2, s\rangle \to s' \Leftrightarrow$
$$(\mathcal{B}[\![b]\!](s) = tt \wedge \langle S_1, s\rangle \to s')$$
$$\vee\ (\mathcal{B}[\![b]\!](s) = ff \wedge \langle S_2, s\rangle \to s')$$

(E_{while}) $\langle \underline{\text{while}}\ b\ \underline{\text{do}}\ S, s\rangle \to s' \Leftrightarrow$
$$(\mathcal{B}[\![b]\!](s) = tt \wedge \exists s''.(\langle S, s\rangle \to s'' \wedge \langle \underline{\text{while}}\ b\ \text{do}\ S, s''\rangle \to s'))$$
$$\vee\ (\mathcal{B}[\![b]\!](s) = ff \wedge s' = s)$$

- Showing semantic equivalence, for example:

$\underline{\text{while}}\ b\ \underline{\text{do}}\ S \sim \underline{\text{if}}\ b\ \underline{\text{then}}\ S; (\underline{\text{while}}\ b\ \underline{\text{do}}\ S)\ \underline{\text{else}}\ \text{skip}$

$$\langle \underline{\text{if}}\ b\ \underline{\text{then}}\ S; (\underline{\text{while}}\ b\ \underline{\text{do}}\ S)\ \underline{\text{else}}\ \text{skip}, s\rangle \to s'$$
$$\overset{E_{if}}{\Leftrightarrow} (\mathcal{B}[\![b]\!](s) = tt \wedge \langle S; \underline{\text{while}}\ b\ \underline{\text{do}}\ S, s\rangle \to s')$$
$$\vee\ (\mathcal{B}[\![b]\!](s) = ff \wedge \langle \text{skip}, s\rangle \to s')$$
$$\overset{E_{comp}, E_{skip}}{\Leftrightarrow} (\mathcal{B}[\![b]\!](s) = tt \wedge \exists s''.(\langle S, s\rangle \to s'' \wedge \langle \underline{\text{while}}\ b\ \underline{\text{do}}\ S, s''\rangle \to s'))$$
$$\vee\ (\mathcal{B}[\![b]\!](s) = ff \wedge s' = s)$$
$$\overset{E_{while}}{\Leftrightarrow} \langle \underline{\text{while}}\ b\ \underline{\text{do}}\ S, s\rangle \to s'$$