

EXAMPLE - Compute iteratively $S_{ds} \llbracket \text{while } \neg(x=0) \text{ do } x := x-2 \rrbracket$

$$F(g)(s) = \text{cond}(B \llbracket \neg(x=0) \rrbracket, g \circ S_{ds} \llbracket x := x-2 \rrbracket, id_{state})(s)$$

$$= \begin{cases} g(S_{ds} \llbracket x := x-2 \rrbracket(s)) & \text{if } B \llbracket \neg(x=0) \rrbracket(s) = \text{tt} \\ id_{state}(s) & \text{if } B \llbracket \neg(x=0) \rrbracket(s) = \text{ff} \end{cases}$$

$$F(g)(s) = \begin{cases} g(S[x \mapsto s(x)-2]) & \text{if } s(x) \neq 0 \\ s & \text{if } s(x) = 0 \end{cases}$$

$$F^0(\emptyset) = \emptyset \quad \text{or} \quad F^0(\emptyset)(s) = \text{undef}$$

$$F^1(\emptyset)(s) = F(\emptyset)(s)$$

exec. body
zero times

$$= \begin{cases} \text{undef} & \text{if } s(x) \neq 0 \\ s & \text{if } s(x) = 0 \end{cases}$$

$$F^2(\emptyset)(s) = F(F^1(\emptyset))(s)$$

$$= \begin{cases} F^1(\emptyset)(S[x \mapsto s(x)-2]) & \text{if } s(x) \neq 0 \\ s & \text{if } s(x) = 0 \end{cases}$$

exec. body
at most once

$$= \begin{cases} \text{undef} & \text{if } s(x) \neq 0 \wedge S[x \mapsto s(x)-2](x) \neq 0 \\ S[x \mapsto s(x)-2] & \text{if } s(x) \neq 0 \wedge S[x \mapsto s(x)-2](x) = 0 \\ s & \text{if } s(x) = 0 \end{cases}$$

$$= \begin{cases} \text{undef} & \text{if } s(x) \neq 0 \wedge s(x) \neq 2 \\ S[x \mapsto s(x)-2] & \text{if } s(x) = 2 \\ s & \text{if } s(x) = 0 \end{cases}$$

$$= \begin{cases} \text{undef} & \text{if } s(x) \neq 0 \wedge s(x) \neq 2 \\ S[x \mapsto 0] & \text{if } s(x) = 0 \vee s(x) = 2 \end{cases}$$

- 2nd -

We guess the i -th approximant (can be proved by math. induction):

exec. body
at most i times

$$F^i(\emptyset)(s) = \begin{cases} s[x \mapsto 0] & \text{if } \exists k: 0 \leq k < i. s(x) = 2k \\ \text{undef} & \text{otherwise} \end{cases}$$

$$\begin{aligned} (\text{FIX } F)(s) &= \left(\bigcup_{i \geq 0} F^i(\emptyset) \right)(s) \\ &= \begin{cases} s[x \mapsto 0] & \text{if } \exists k \geq 0. s(x) = 2k \\ \text{undef} & \text{otherwise} \end{cases} \end{aligned}$$

reflect whether this is what we expect for $\text{while } \neg(x=0) \text{ do } x := x-2$ (s) .