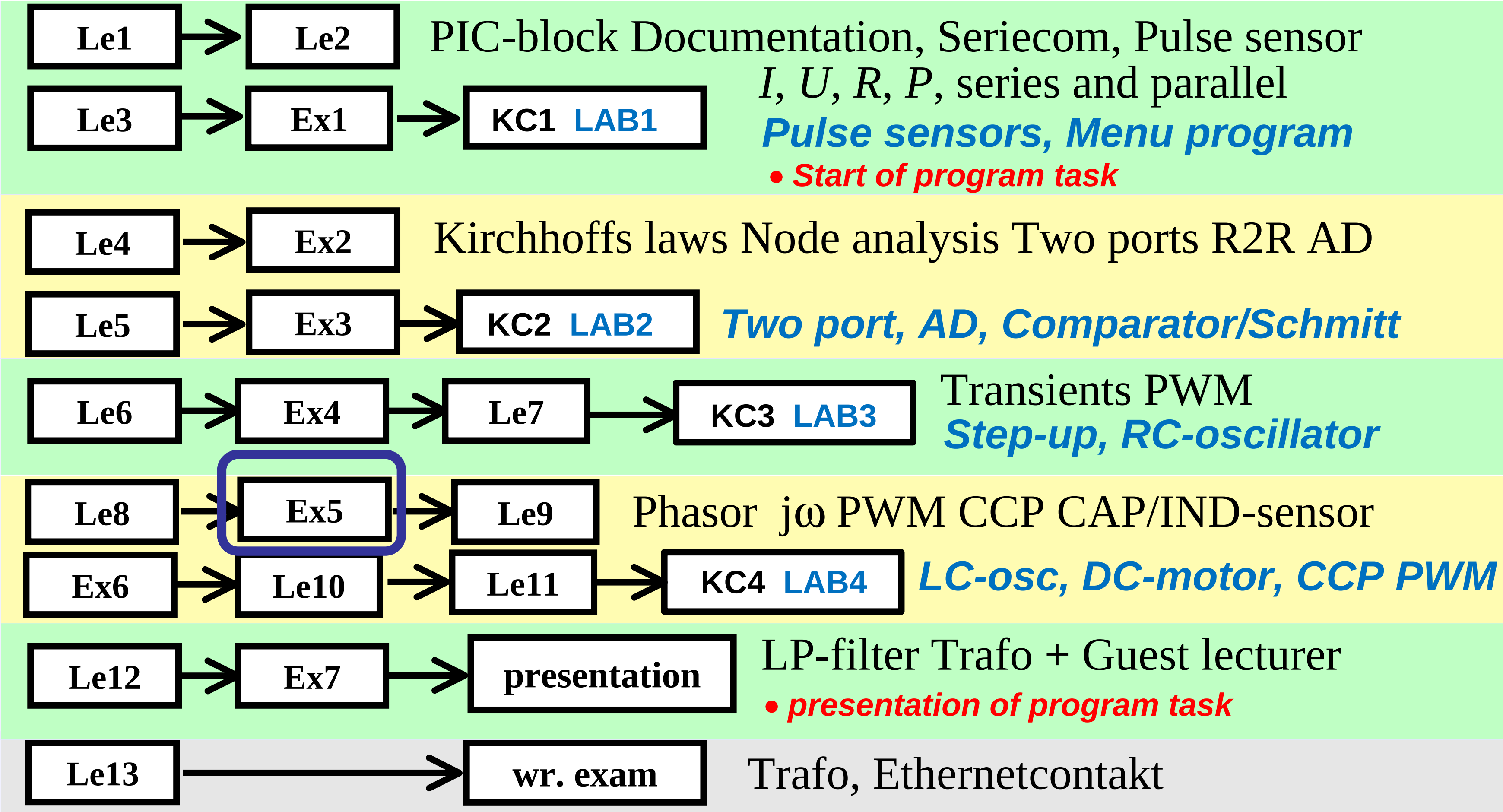
IE1206 Embedded Electronics



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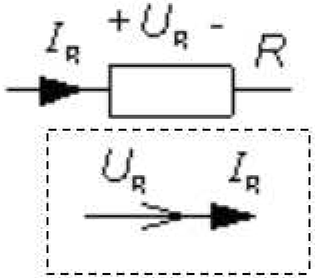
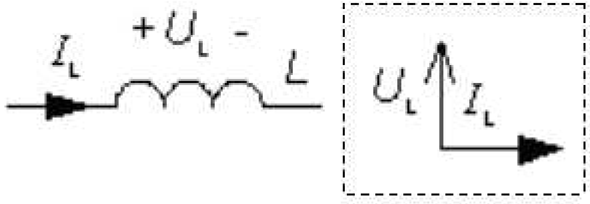
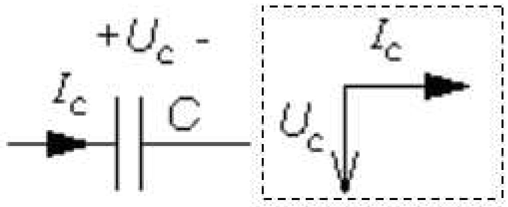
Phasor - vector







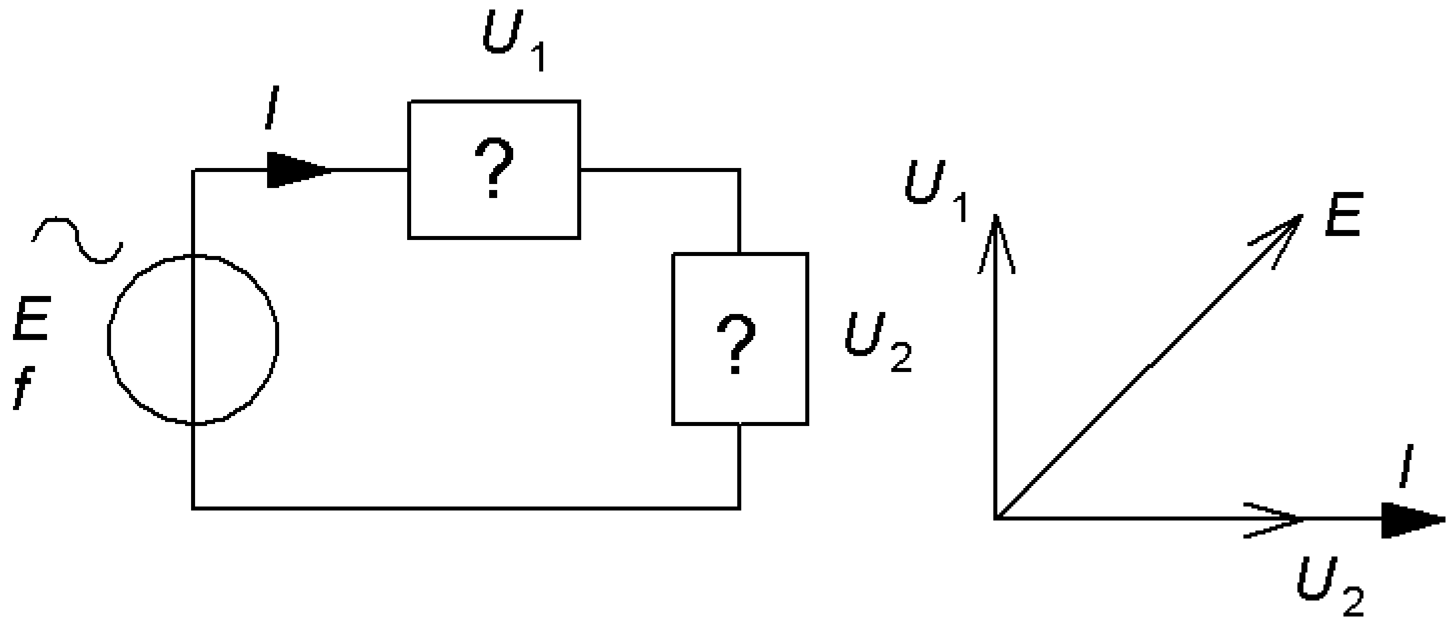
$$\omega = 2\pi\ f$$

$$\left|X_L\right| = \omega \cdot L$$

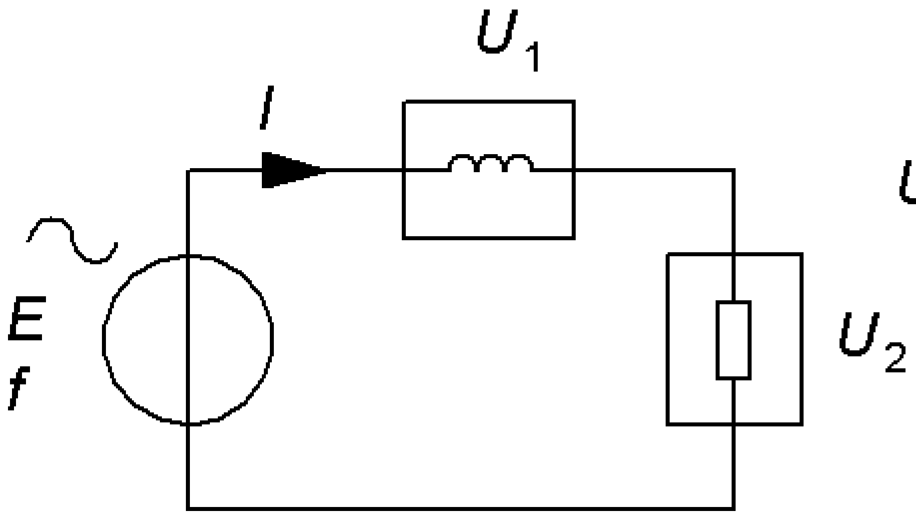
$$\left|X_C\right| = \frac{1}{\omega \cdot C}$$

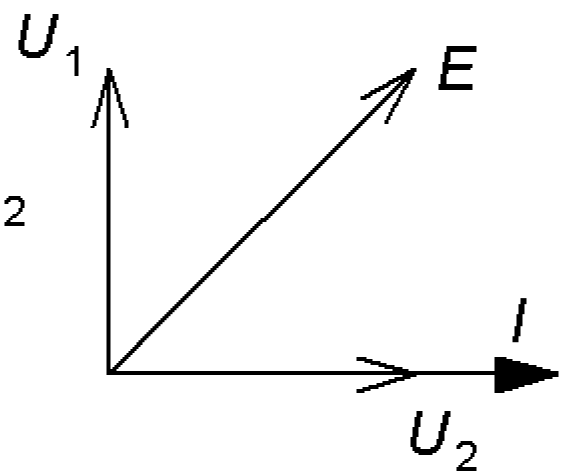
$$\boxed{Z = \frac{U}{I}}$$

What’s inside the circuit? (11.4)

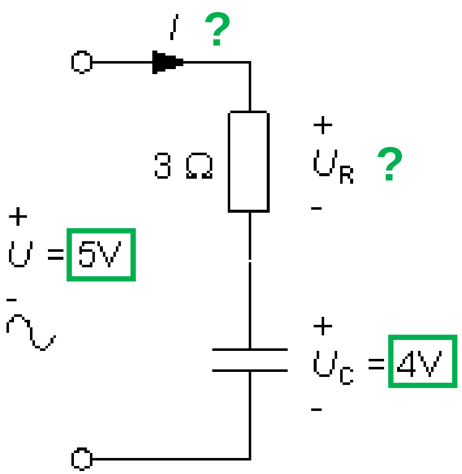


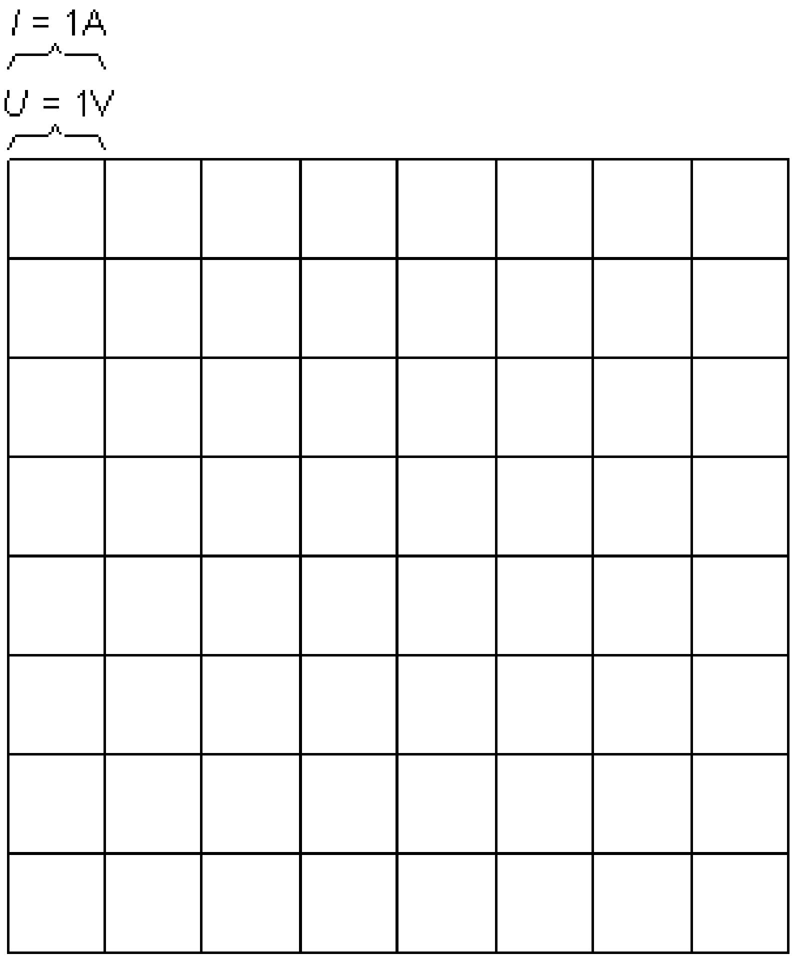
What's inside the circuit? (11.4)



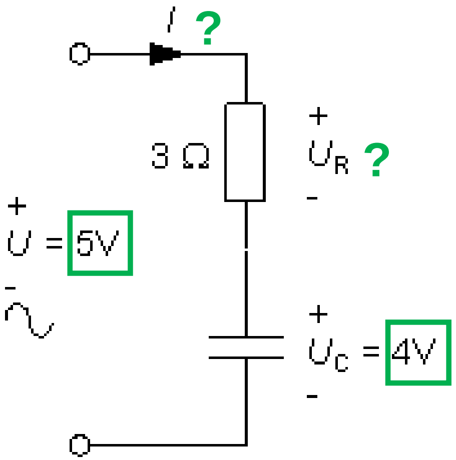


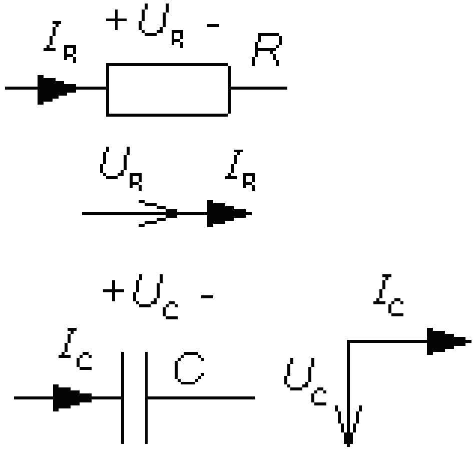
Phasor chart





Phasor chart





$$\underline{I}_R = \underline{I}_C = \underline{I}$$

$$\underline{U}_R \perp \underline{U}_C$$

$$U^2 = U_R^2 + U_C^2$$

The two voltages have 90° phase angle. Pythagorean theorem applies!

$$U_R = \sqrt{U^2 - U_C^2} = \sqrt{5^2 - 4^2} = \boxed{3} \quad I = \frac{U_R}{R} = \frac{3}{3} = \boxed{1}$$

Now all values are known!

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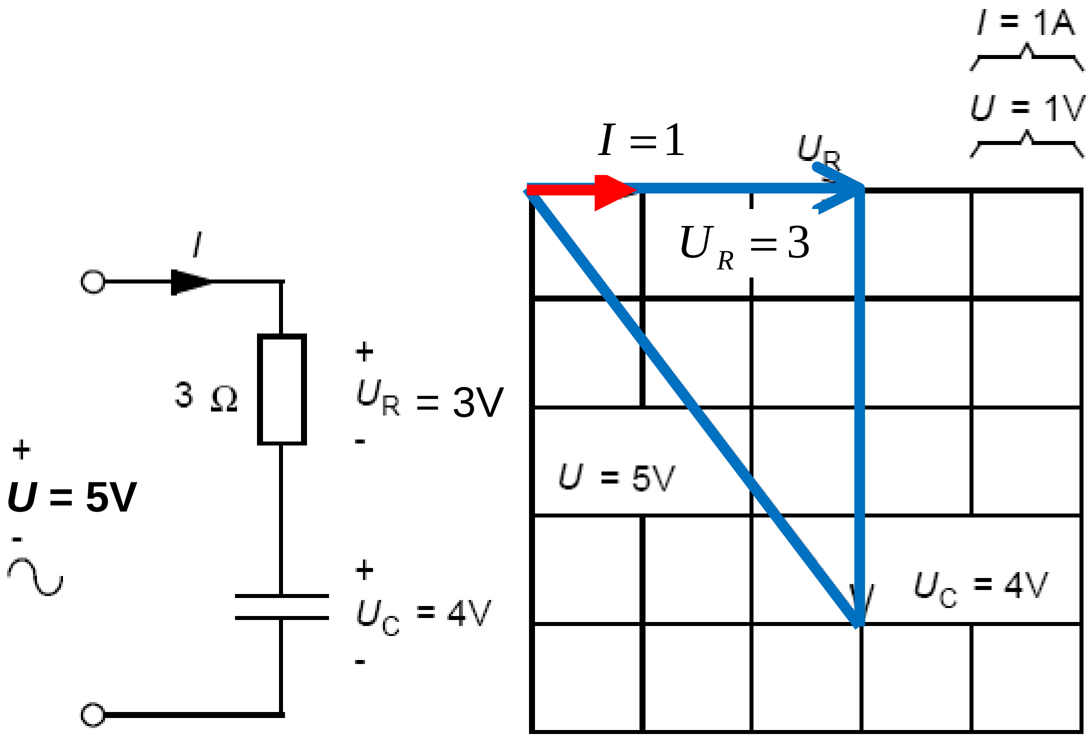
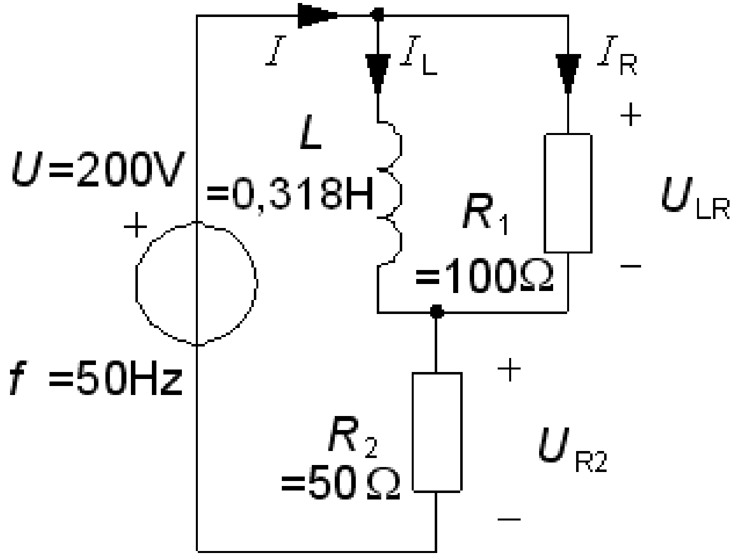
Phasor chart



Phasor chart (11.6)

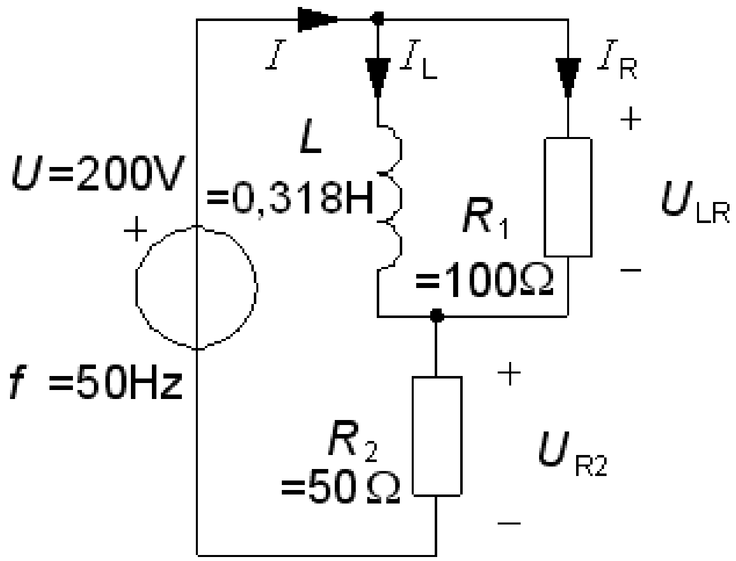
U = 200 V, f = 50 Hz,
L = 0,318 H, R_1 = 100 Ω,
R_2 = 50 Ω.



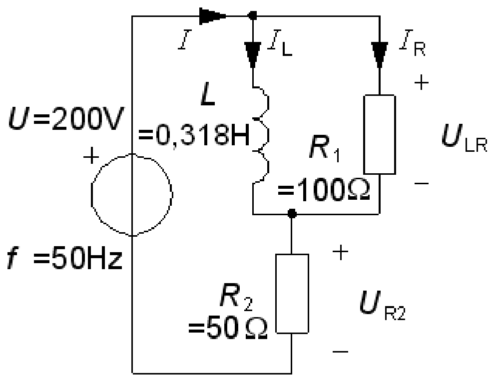
Phasor chart (11.6)

U = 200 V, f = 50 Hz,
L = 0,318 H, R_1 = 100 Ω,
R_2 = 50 Ω.

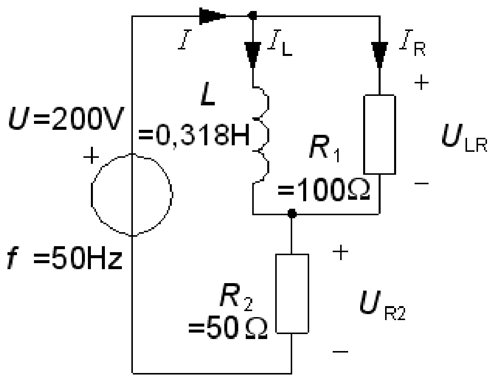
$|X_L| = \omega \cdot L = 2\pi \cdot 50 \cdot 0{,}318$
$= 100\ \Omega$



Phasor chart (11.6)

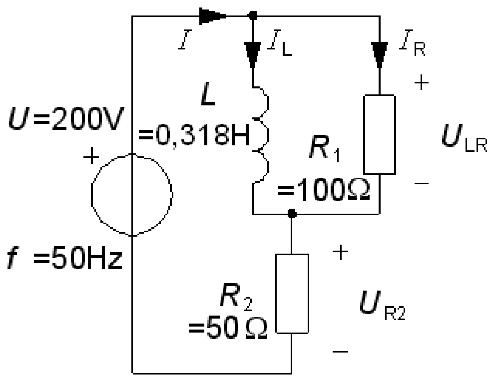


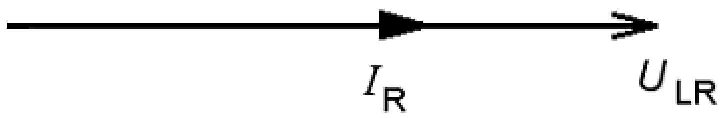
Phasor chart (11.6)



Choose U_{LR} as reference phase (= horizontal).

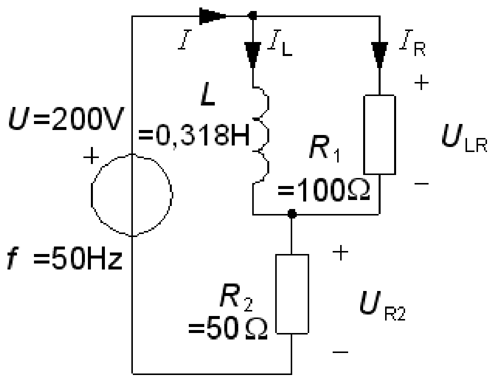
Phasor chart (11.6)

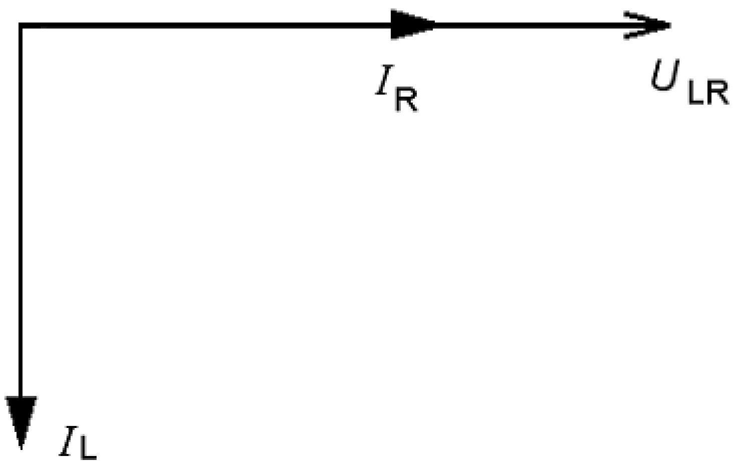




The current I_R has the same direction as U_{LR}.

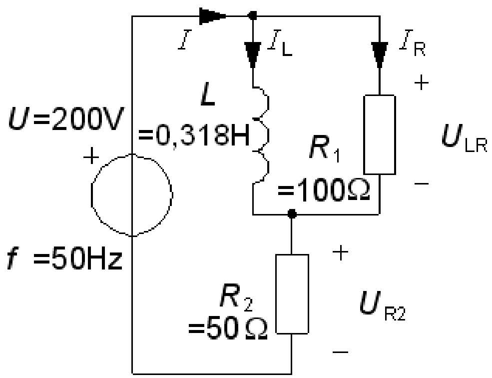
Phasor chart (11.6)

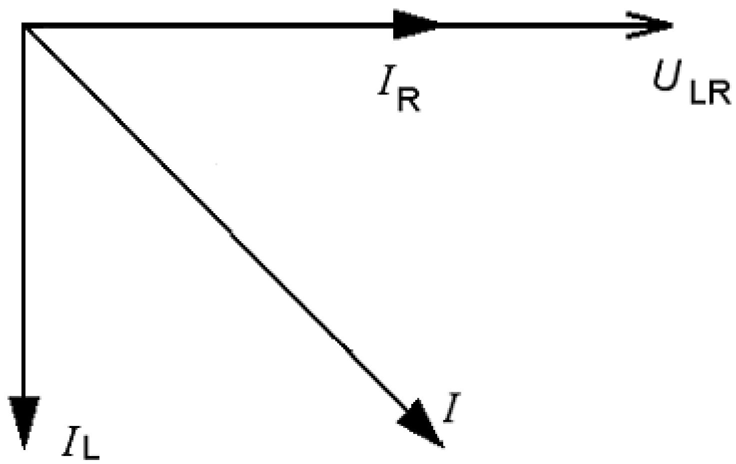




The current I_L lags 90° behind U_{LR} and has an equally long pointer as I_R because R_1 and L has the same impedance.
($|X_L| = 100\ \Omega$, $R_1 = 100\ \Omega$)

Phasor chart (11.6)





The two currents I_R and I_L can be added as vectors to the current I. I is $\sqrt{2}$ longer than I_R and I_L
(Pythagorean theorem applies!).

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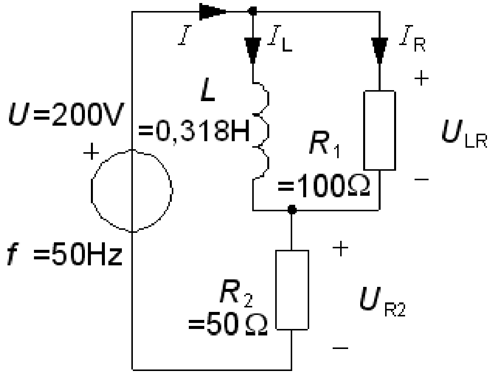
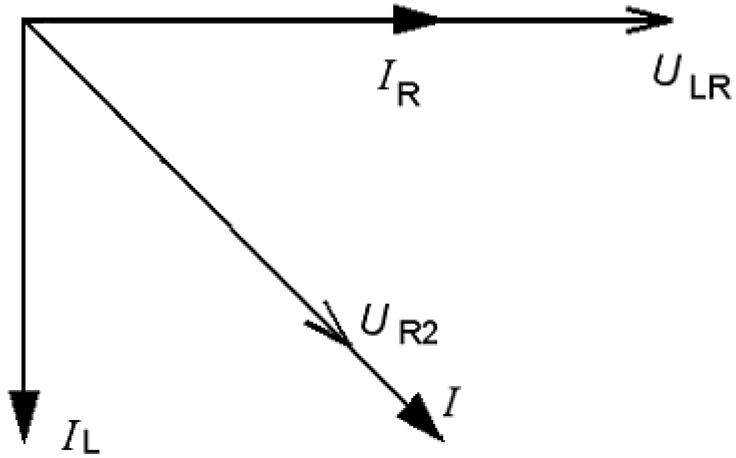
Phasor chart (11.6)



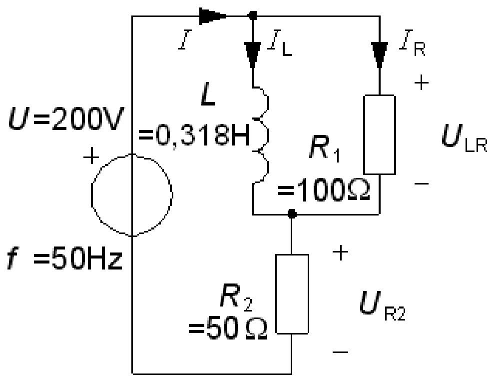


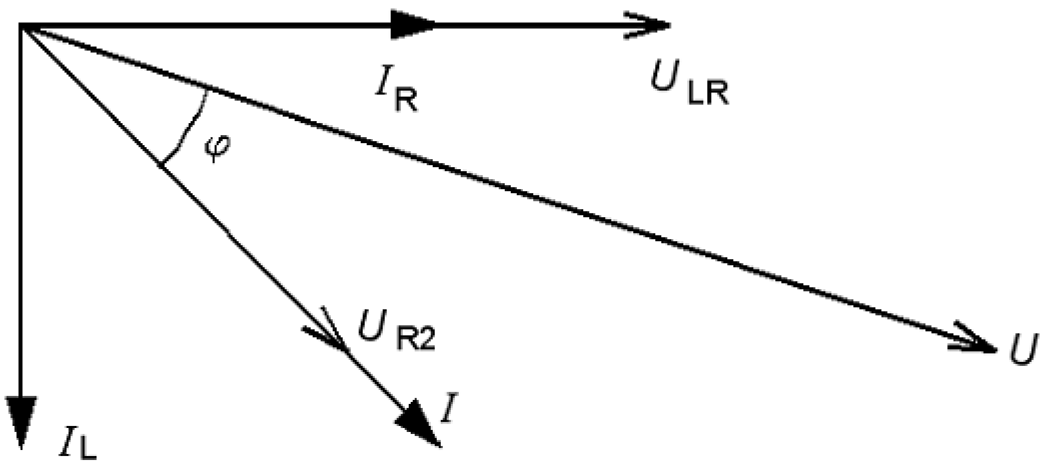
Current I passes through the lower resistor R_2. The voltage drop U_{R2} gets the same direction as I.

U_{LR} has the length $I_R \cdot 100$, U_{R2} has length $I \cdot 50$.

Because $I = I_R \cdot \sqrt{2}$ we get $U_{R2} = U_{LR} / \sqrt{2}$.

Phasor chart (11.6)





Voltage U can finally be determined as the vector sum of U_{LR} and U_{R2}.

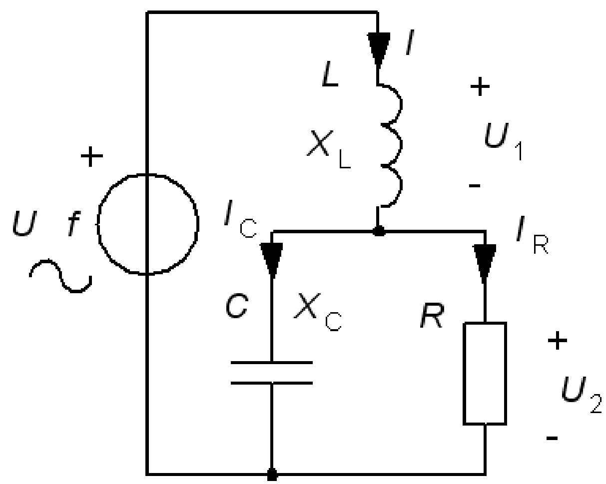
- Phase φ is the angle between U and I.
- Z is the ratio between lenghts of U and I.

The current after the voltage - inductive character

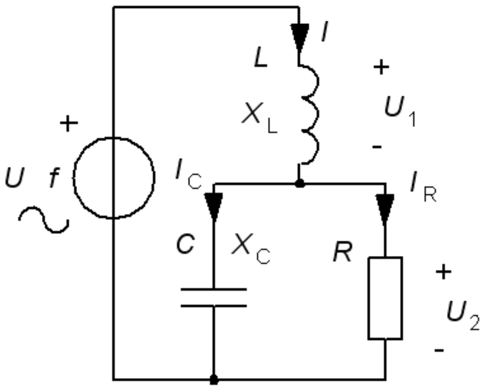
Phasor chart (11.7)

Draw the phasor chart for this circuit. At the frequency *f* applies that $|X_C| = R$ and $|X_L| = R/2$.

U_2 is a suitable reference phase.

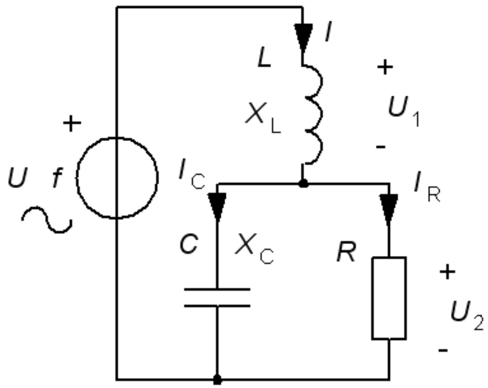


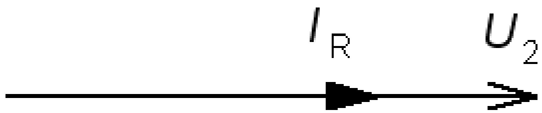
Phasor chart (11.7)



Start with U_2 as reference phase (= horizontal).

Phasor chart (11.7)





Current I_R has the same direction as U_2.

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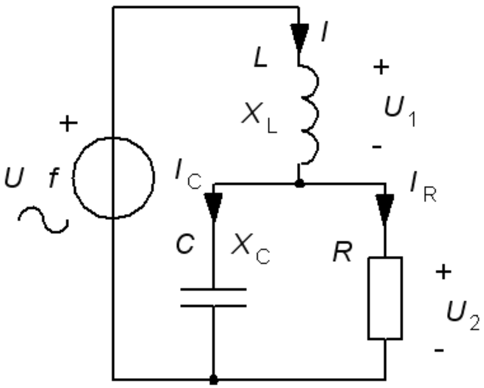
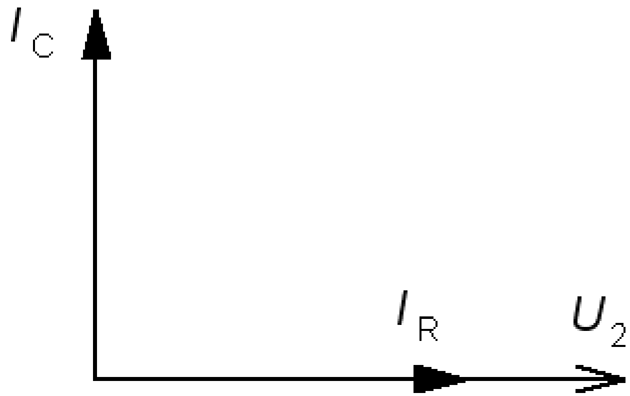
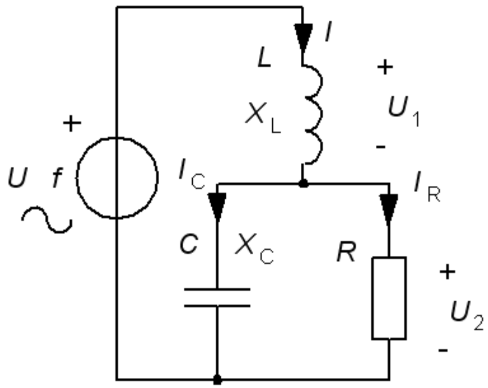
Phasor chart (11.7)

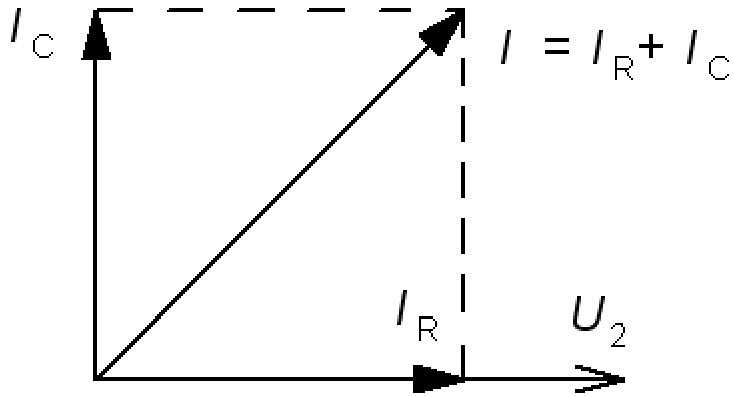




Current I_C leads 90° before U_2 and is equally long as I_R because $X_C = R$.

Phasor chart (11.7)

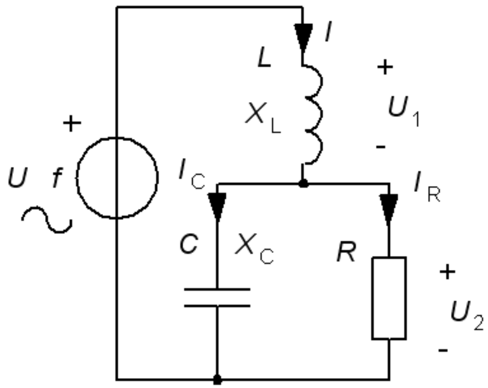


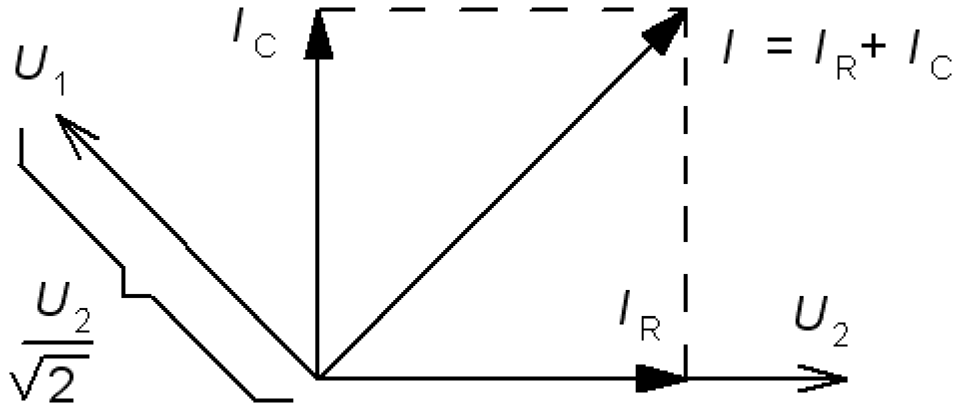


Current I_C and I_R are summed to I.

I is √2 times longer than I_C and I_R (Pythagorean theorem applies).

Phasor chart (11.7)





U_1 leads 90° before I.

The length is $U_1 = I \cdot X_L = \sqrt{2} \cdot I_R \cdot R/2 = I_R \cdot R/\sqrt{2}$

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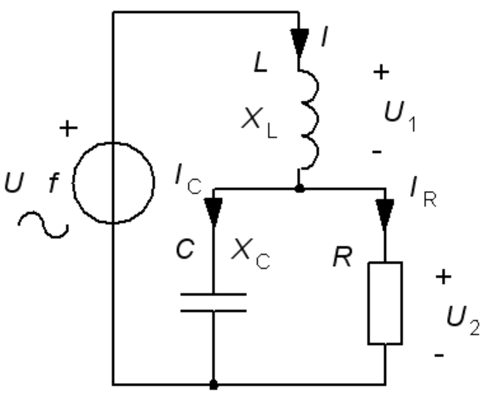
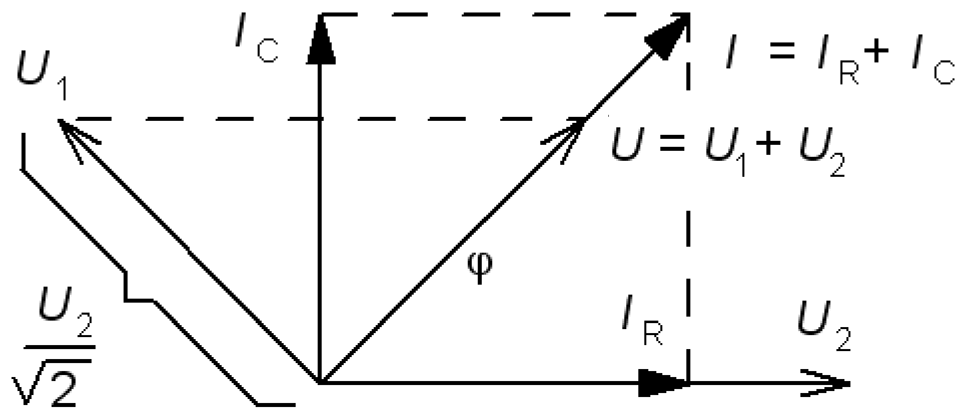
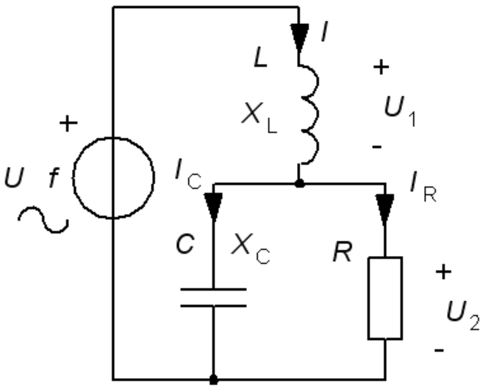
Phasor chart (11.7)

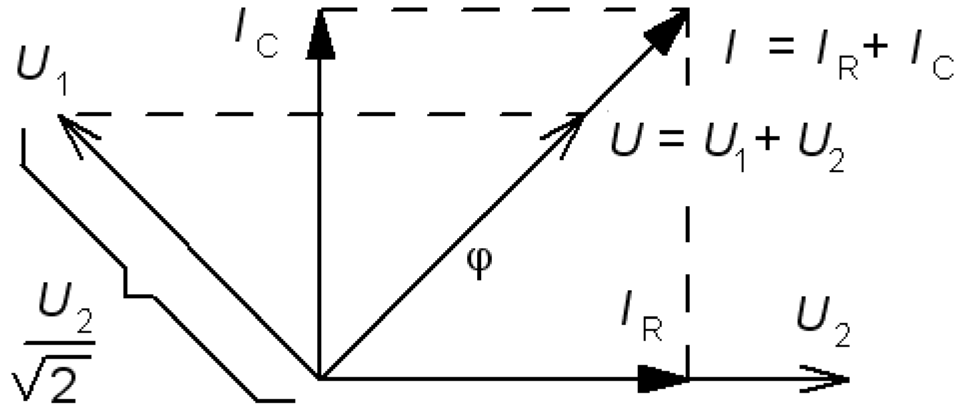




Voltage U_1 and U_2 are summed to voltage U.

Phasor chart (11.7)





One can see from the chart that U becomes equal long to U_1.
The angle φ = 0 and thereby U and I are in phase.

Inductive or capacitive character?

Complex numbers, jω-method

Complex OHM's law for *R* *L* and *C*.

$$\underline{U}_{\mathrm{R}} = \underline{I}_{\mathrm{R}} \cdot R$$

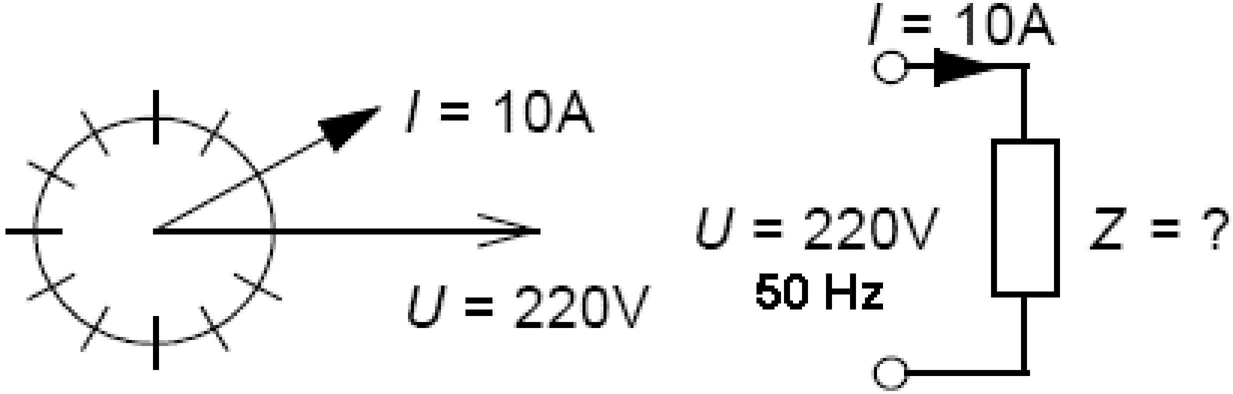
$$\underline{U}_{\mathrm{L}} = \underline{I}_{\mathrm{L}} \cdot \mathrm{j} X_{\mathrm{L}} = \underline{I}_{\mathrm{L}} \cdot \mathrm{j}\omega L \qquad X_{\mathrm{L}} = \omega L$$

$$\underline{U}_{\mathrm{C}} = \underline{I}_{\mathrm{C}} \cdot \mathrm{j} X_{\mathrm{C}} = \underline{I}_{\mathrm{C}} \cdot \frac{1}{\mathrm{j}\omega C} \qquad X_{\mathrm{C}} = -\frac{1}{\omega C} \qquad \omega = 2\pi \cdot f$$

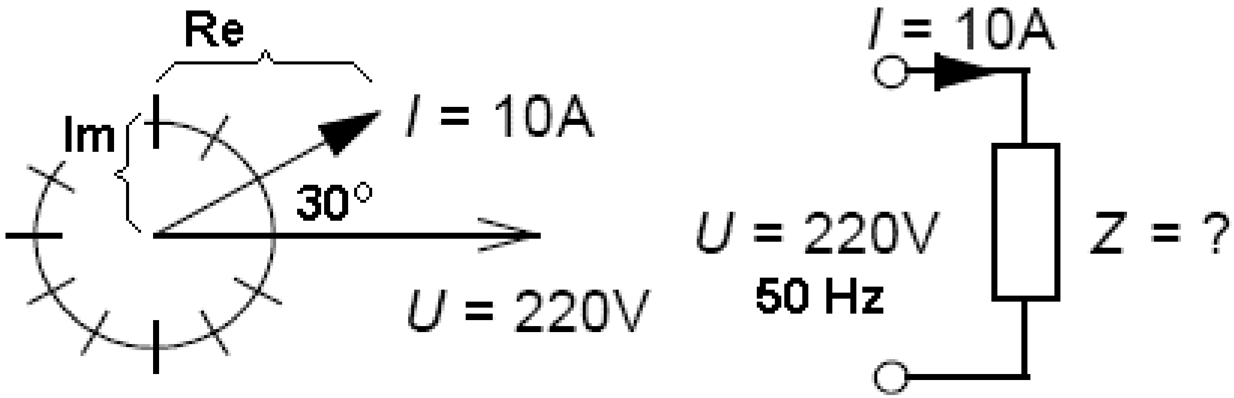
Complex OHM's law for *Z*.

$$\boxed{\underline{U} = \underline{I} \cdot \underline{Z}} \qquad Z = \frac{U}{I} \qquad \varphi = \arg(\underline{Z}) = \arctan\left(\frac{\mathrm{Im}[\underline{Z}]}{\mathrm{Re}[\underline{Z}]}\right)$$

jω Impedance (12.2)

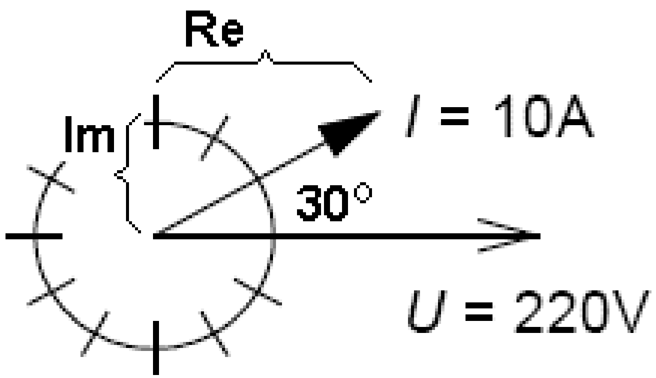


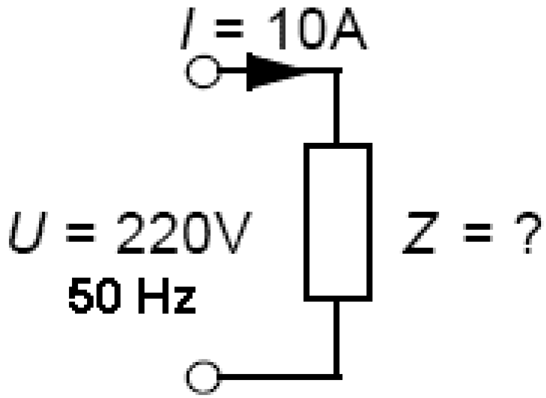
jω Impedance (12.2)



One can imagine that the phasor chart shows the complex plane, and then splitting the current phasor in real part and imaginary part:

jω Impedance (12.2)

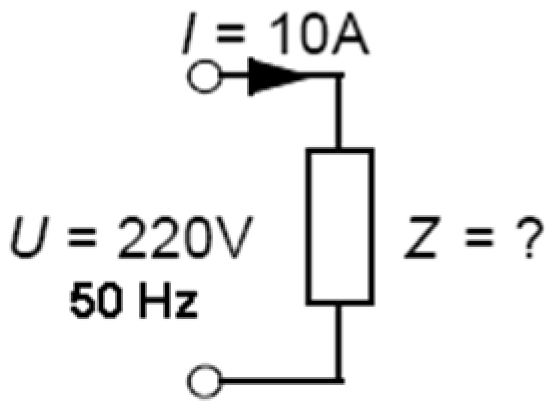




$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{220}{10 \cdot \left(\cos(30°) + j \cdot \sin(30°)\right)} =$$

$$= \frac{220}{8,6 + 5j} \cdot \frac{(8,6 - 5j)}{(8,6 - 5j)} = \frac{1892 - 1100j}{99} = 19,1 - 11,1j$$

jω Impedance (12.2)



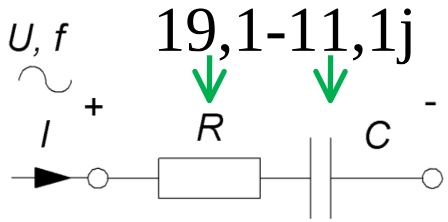
$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = 19{,}1 - 11{,}1\mathrm{j}$$

- One possible solution is then a series circuit with *R* and *C*

$$R = 19{,}1\,\Omega \qquad X_C = -\frac{1}{\omega C} = -11{,}1$$

$$C = -\frac{1}{2\pi \cdot 50 \cdot (-11{,}1)} = 287\ \mu\mathrm{F}$$

Capacitor has negative reaktance.



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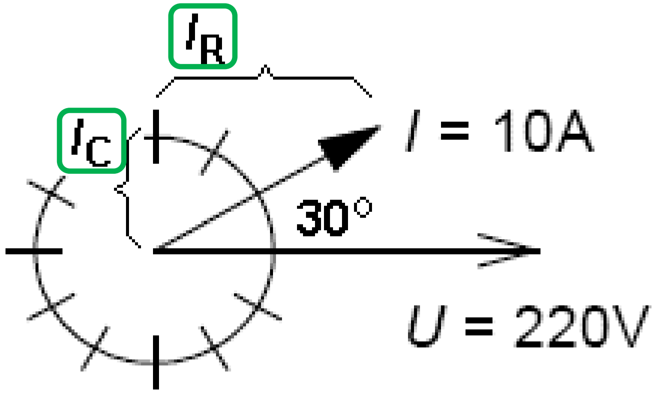
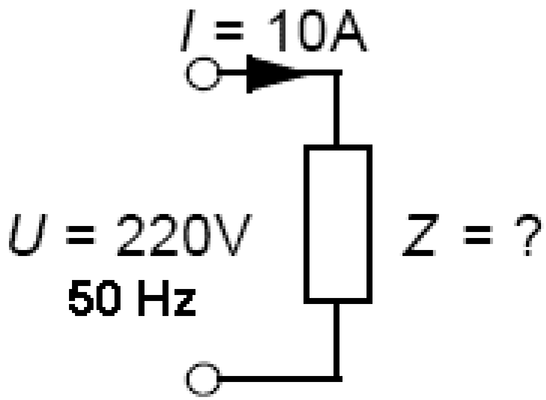
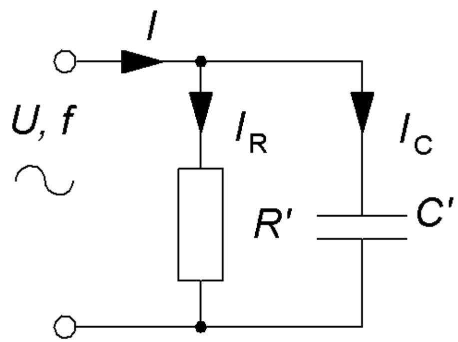
jω Impedance (12.2)





• Another possible solution is a parallel circuit with R' and C' one then thinks on I as divided in to **current composants** I_R and I_C which are perpendicular to each other.



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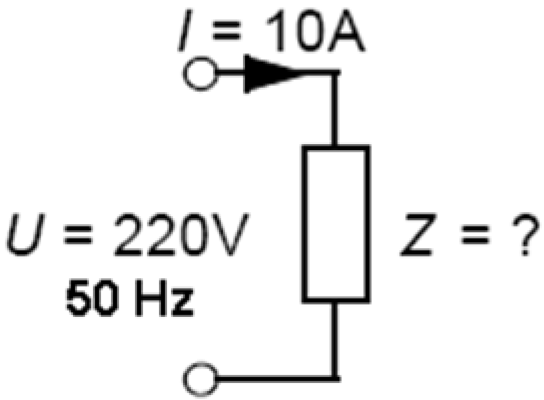
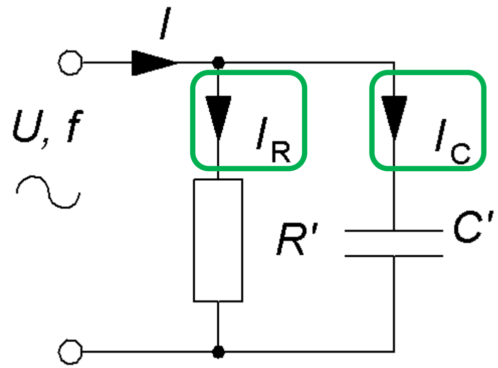
jω Impedance (12.2)



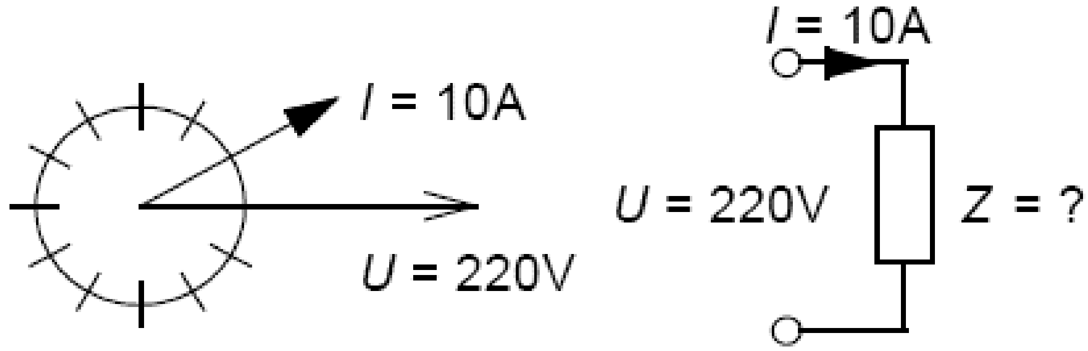


$$R' = \frac{U}{I_R} = \frac{U}{I\cos 30°} = \frac{220}{10 \cdot 0{,}87} = 25{,}3\ \Omega$$

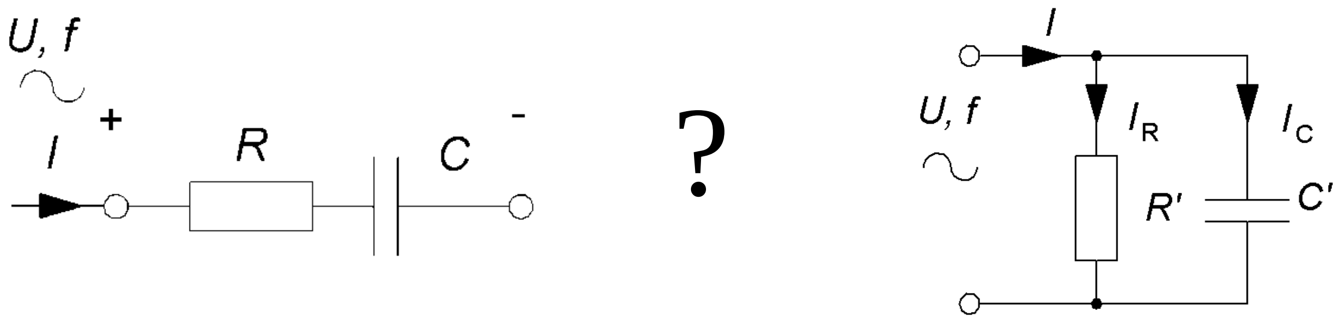
$$|X'_C| = \frac{U}{I_C} = \frac{U}{I\sin 30°} = \frac{220}{10 \cdot 0{,}5} = 44\ \Omega$$

$$|X'_C| = \frac{1}{\omega C'} \quad \Rightarrow \quad C' = \frac{1}{2\pi \cdot 50 \cdot 44} = 72{,}6\ \mu\text{F}$$

jω Impedance (12.2)



Is there any way to find out which of the two proposed circuits *Z* actually contain?



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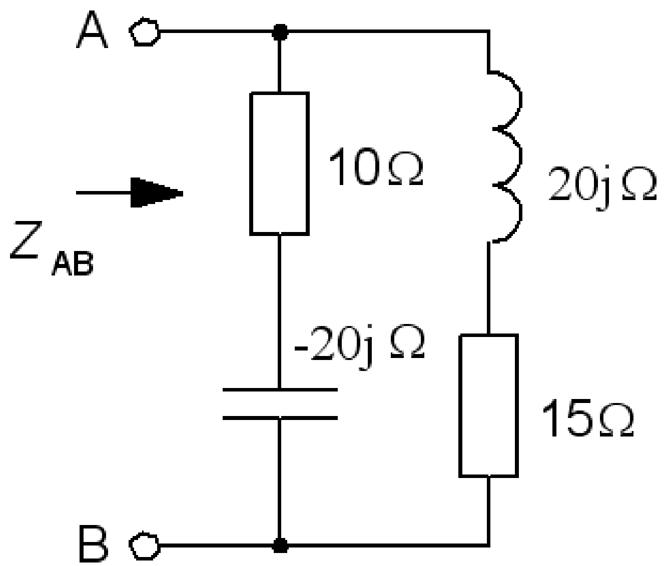
Complex impedance (12.6)

Determine the complex impedance Z_{AB} of this circuit.

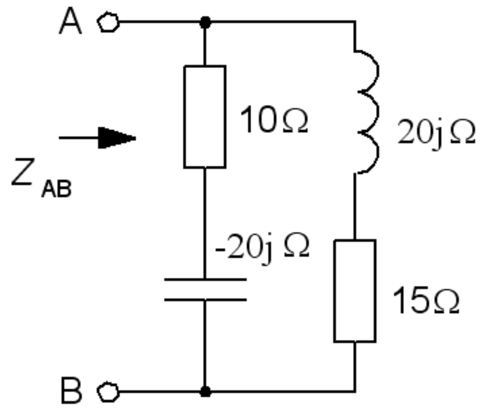


Complex numbers calculation

$$\underline{Z}_{AB} = \frac{(15+\text{j}20)\cdot(10-\text{j}20)}{15+\text{j}20+10-\text{j}20} =$$

$$= \frac{550-\text{j}100}{25} =$$

$$= 22-\text{j}4 \ \ [\Omega]$$



Here the denominator directly was a real number so there no need to multiply with the complex conjugate of the denominator, otherwise the calculations had been moore extensive…

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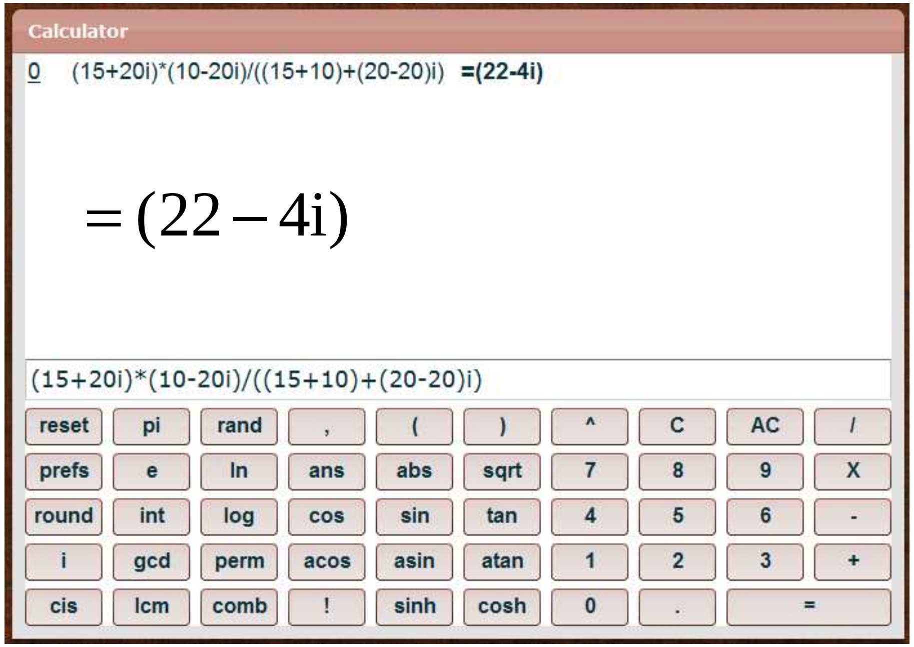
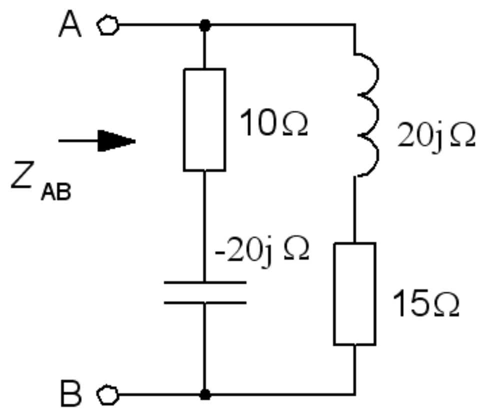
Complex numbers calculation

$$\underline{Z}_{AB} = \frac{(15+j20)\cdot(10-j20)}{15+j20+10-j20} =$$



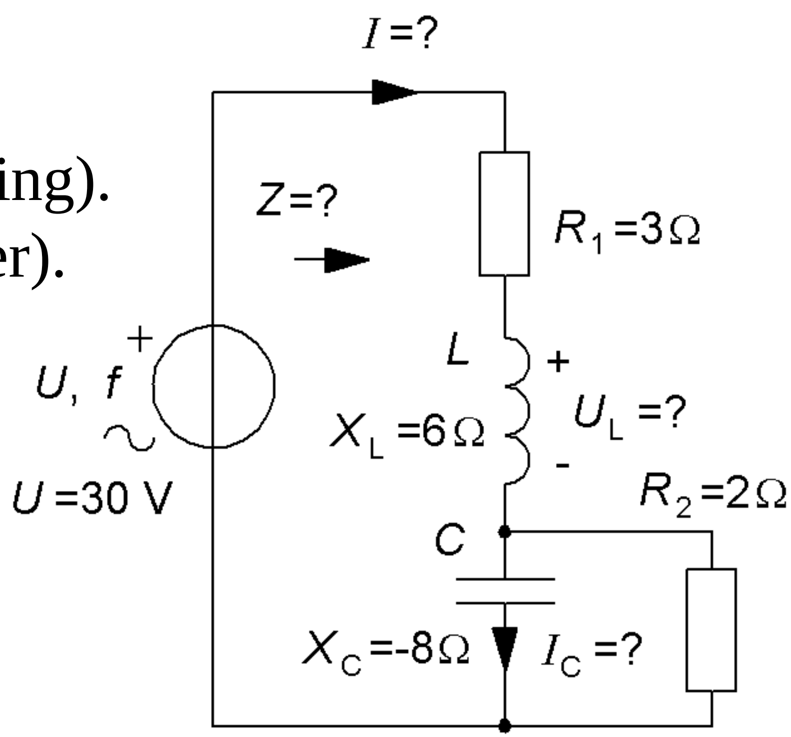
$$= (22-4i)$$



[Online Scientific Calculator]

"heavy" calculations! (12.9)

- Calculate impedance Z.
- Calculate current I.
- Calculate I_C (current branching).
- Calculate U_L (voltage divider).



Calculate impedance Z

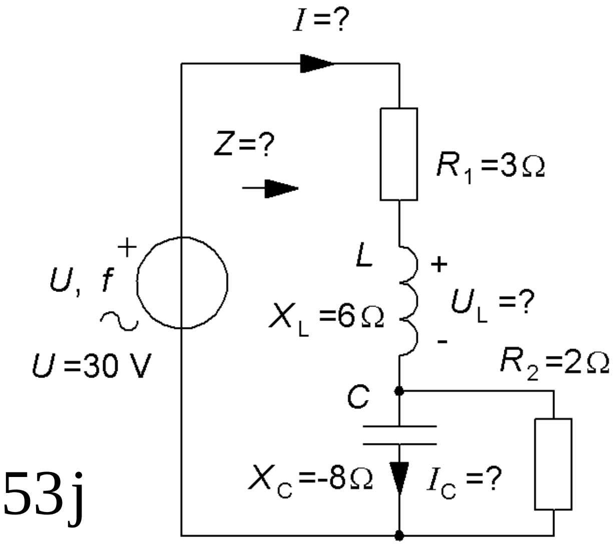
$$\underline{Z}_{R\|C} = \frac{2\cdot(-8\text{j})}{2-8\text{j}}\cdot\frac{(2+8\text{j})}{2+8\text{j}} =$$

$$= 1{,}88 - 0{,}47\,\text{j}$$

$$\underline{Z} = R_1 + \text{j}X_L + \underline{Z}_{R\|C} =$$

$$= 3 + 6\text{j} + (1{,}88 - 0{,}47\,\text{j}) = 4{,}88 + 5{,}53\text{j}$$

$$Z = \sqrt{4{,}88^2 + 5{,}53^2} = 7{,}38\ \Omega$$



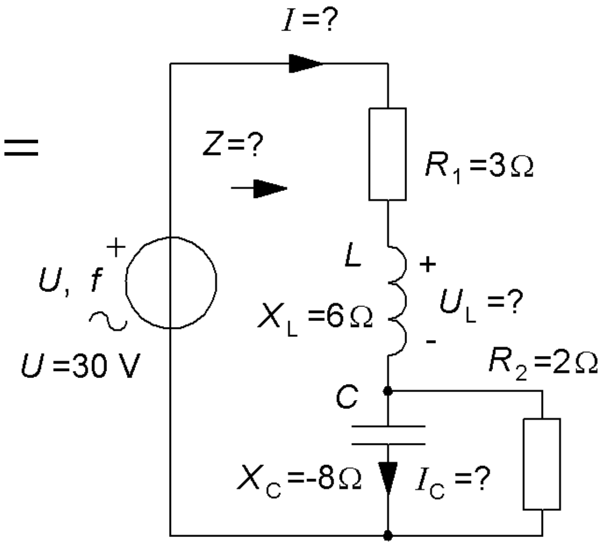
Calculate current *I*

U is reference phase, real.

$$\underline{I} = \frac{U}{\underline{Z}} = \frac{30}{4{,}88 + 5{,}53\text{j}} \cdot \frac{(4{,}88 - 5{,}53\text{j})}{(4{,}88 - 5{,}53\text{j})} =$$

$$= \frac{146{,}5 - 165{,}9\,\text{j}}{4{,}88^2 + 5{,}53^2} = 2{,}7 - 3\text{j}$$

$$I = \sqrt{2{,}7^2 + 3^2} = 4\ \text{A}$$

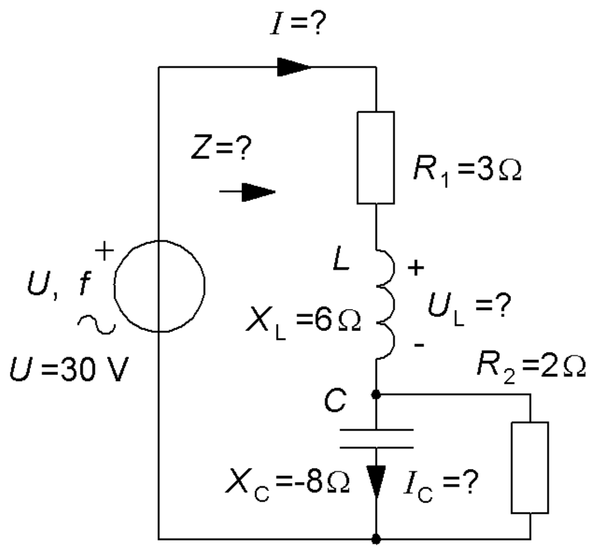


Calculate current I_C

$$\underline{I}_C = \underline{I}\frac{R_2}{R_2 + \mathrm{j}X_C} = (2,7 - 3\mathrm{j})\cdot\frac{2}{2-8\mathrm{j}} =$$

$$= \frac{(2,7-3\mathrm{j})\cdot 2}{2-8\mathrm{j}}\cdot\frac{(2+8\mathrm{j})}{(2+8\mathrm{j})} = 0,86 + 0,46\mathrm{j}$$

$$I_C = \sqrt{0,86^2 + 0,46^2} = 0,98\ \mathrm{A}$$

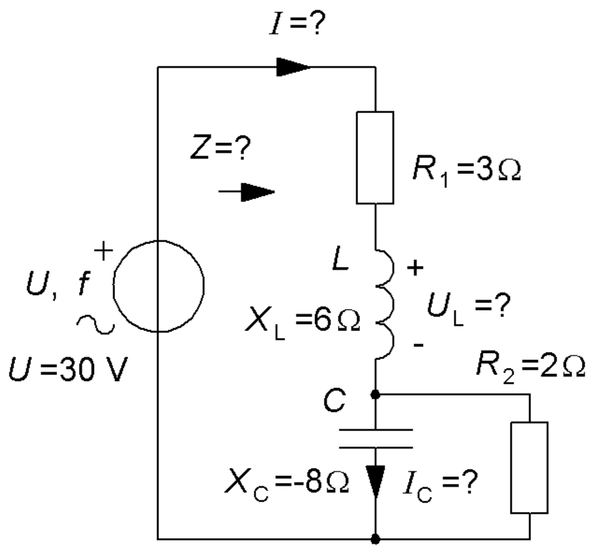


U_L complex conjugate method?

$$\underline{U}_L = U \frac{jX_L}{jX_L + \underline{Z}_{R||C} + R_1} =$$

$$= 30 \frac{6j}{6j + (1{,}88 - 0{,}47j) + 3} =$$

$$= 30 \frac{6j}{4{,}88 + 5{,}53j} \cdot \frac{(4{,}88 - 5{,}53j)}{(4{,}88 - 5{,}53j)} = 18{,}3 + 16{,}2j$$



$$U_L = \sqrt{18{,}3^2 + 16{,}2^2} = 24{,}4 \text{ V}$$

Amount and phase

$$\underline{Z} = \underline{Z}_1 \cdot \underline{Z}_2 \qquad \left|\underline{Z}\right| = \left|\underline{Z}_1 \cdot \underline{Z}_2\right| = \left|\underline{Z}_1\right| \cdot \left|\underline{Z}_2\right|$$

$$\arg(\underline{Z}) = \arg(\underline{Z}_1) + \arg(\underline{Z}_2)$$

$$\underline{Z} = \frac{\underline{Z}_1}{\underline{Z}_2} \qquad \left|\underline{Z}\right| = \left|\frac{\underline{Z}_1}{\underline{Z}_2}\right| = \frac{\left|\underline{Z}_1\right|}{\left|\underline{Z}_2\right|}$$

$$\arg(\underline{Z}) = \arg(\underline{Z}_1) - \arg(\underline{Z}_2)$$

U_L amount ∠ phase method?

Amount ∠ phase method, the polar form, often gives simpler calculations, but nowdays most math programs and pocket calculators handles complex numbers directly …

$$\underline{U}_L = U\frac{\mathrm{j}X_L}{\mathrm{j}X_L + \underline{Z}_{R||C} + R_1} = 30\frac{6\mathrm{j}}{6\mathrm{j} + (1{,}88 - 0{,}47\mathrm{j}) + 3} =$$

$$= 30\frac{6\mathrm{j}}{4{,}88 + 5{,}53\mathrm{j}} = 30\frac{6}{\sqrt{4{,}88^2 + 5{,}53^2}}\frac{\angle 90^\circ}{\angle \arctan\left(\frac{5{,}53}{4{,}88}\right)} =$$

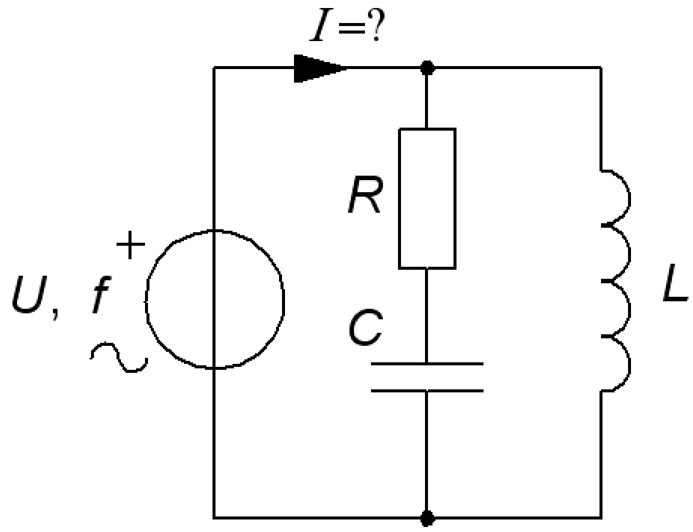
$$= 30\frac{6}{7{,}38}\angle(90^\circ - 48{,}6^\circ) = 24{,}4\angle 41{,}4^\circ$$

$$U_L = 24{,}4\ \mathrm{V}$$

Derive the complex current I (12.7)

Set up the complex current I (with U as reference phase).

Note! One does not always have give the answer of the form a+jb. The same information, but with less effort, one gets if the answer is expressed as a ratio of complex numbers. Amount and phase can if necessary be taken from the numerator and the denominator directly.



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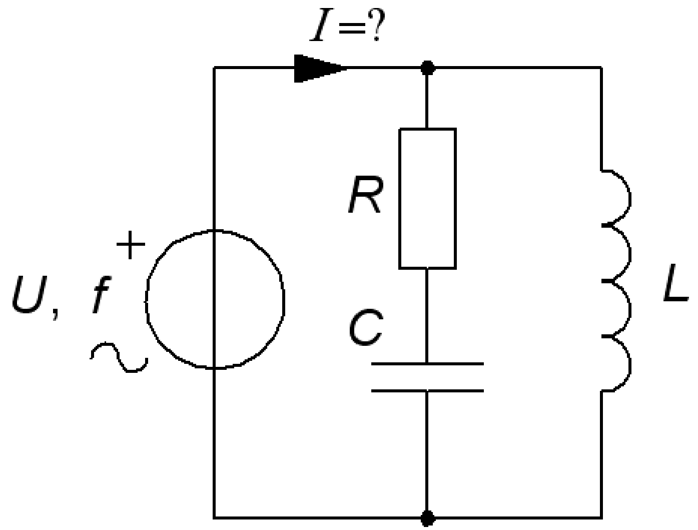
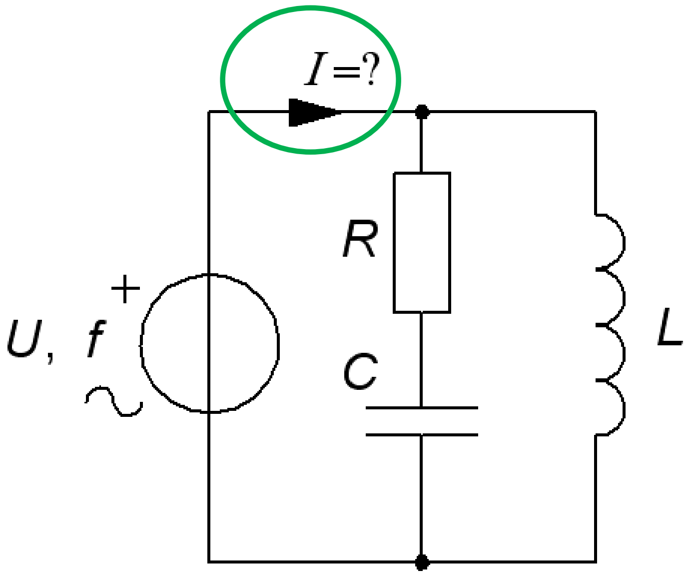


$$\underline{I} = \frac{a + \mathrm{j}b}{c + \mathrm{j}d} \quad I = \frac{|a + \mathrm{j}b|}{|c + \mathrm{j}d|} \quad \arg(\underline{I}) = \arg(a + \mathrm{j}b) - \arg(c + \mathrm{j}d)$$

Note!

Derive the complex current I (12.7)

Set up the complex current I (With U as the reference phase).



$$\underline{Z} = \frac{(R+\frac{1}{j\omega C})\cdot j\omega L}{R+j\omega L+\frac{1}{j\omega C}} = \frac{\frac{L}{C}+j\omega LR}{R+j(\omega L-\frac{1}{\omega C})}$$

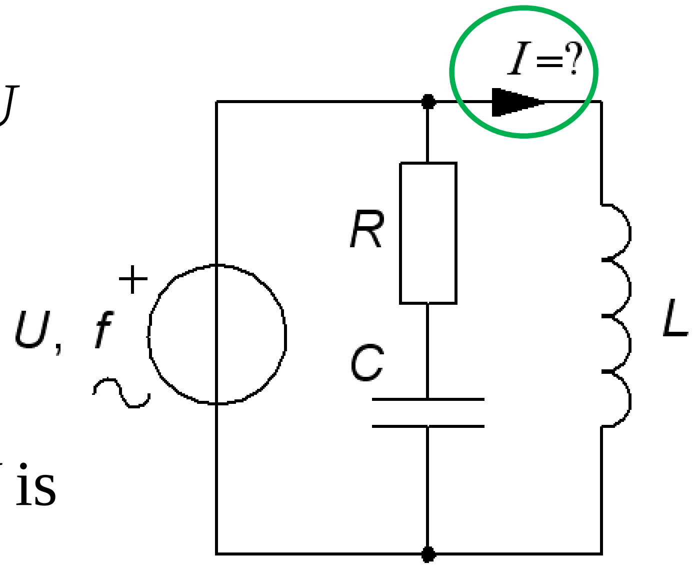
$$\underline{I} = \frac{U}{\underline{Z}} = U\frac{R+j(\omega L-\frac{1}{\omega C})}{\frac{L}{C}+j\omega LR}$$

Sufficiently simplified!

That it is U which is the reference phase can be seen that we let the voltage be a real number!

Derive the complex current *I* (12.8)

Set up the complex current *I* (With *U* as the reference phase).



Now it will be easier! The voltage *U* is located directly across the parallel branch with the inductance *L*. (We need not concern ourselves with *R* and C)

$$\underline{I} = \frac{U}{j\omega L} = -j\frac{U}{\omega L}$$

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