KTH Electrical Engineering

Exam in EG2050 System Planning,
19 March 2015, 8:00–13:00, Q22, Q26

Allowed aids

In this exam you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten**, **single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

PART I (MANDATORY)

Write all answers on the answer sheet provided. Motivations and calculations do not have to be presented.

Part I can yield 40 points in total. The examinee is guaranteed to pass if the score is at least 33 points. If the result in part I is at least 31 points, then there will be a possibility to complement for passing the exam with the grade E.

Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

a) (2 p) A balance responsible player has the following responsibilities: I) Economical responsibility that the system during each trading period (for example one hour) is supplied as much energy as consumed by the customers of the player, II) Physical responsibility that the system during each trading period (for example one hour) is supplied as much energy as consumed by the customers of the player, III) Physical responsibility that the system continuously is supplied as much power as consumed by the customers of the player.

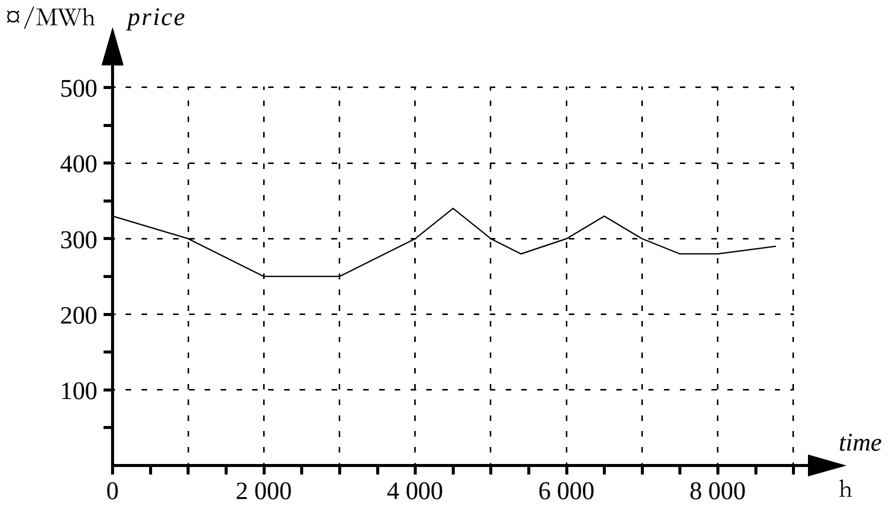
1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. I and II are true but not III.

b) (2 p) We use the notion "ahead trading" to describe all the trading which occurs before the hour of delivery (or any other trading period). In the ahead market it is possible to trade with the following contracts: I) Balance power, i.e., when a balance responsible player is selling any surplus in their balance to the system operator, or when a balance responsible player is buying from the system operator to cover for any deficit in their balance, II) Take-and-pay contracts, i.e., when the customer subscribes to a specific maximal power, and during the duration of the contract is allowed to buy any amount of energy per trading period as long as the maximal power is not exceeded, III) Regulation power, i.e., when a player at request from the system operator is supplying more power to the system (up-regulation) or when a player at request from the system operator is supplying less power to the system (down-regulation).

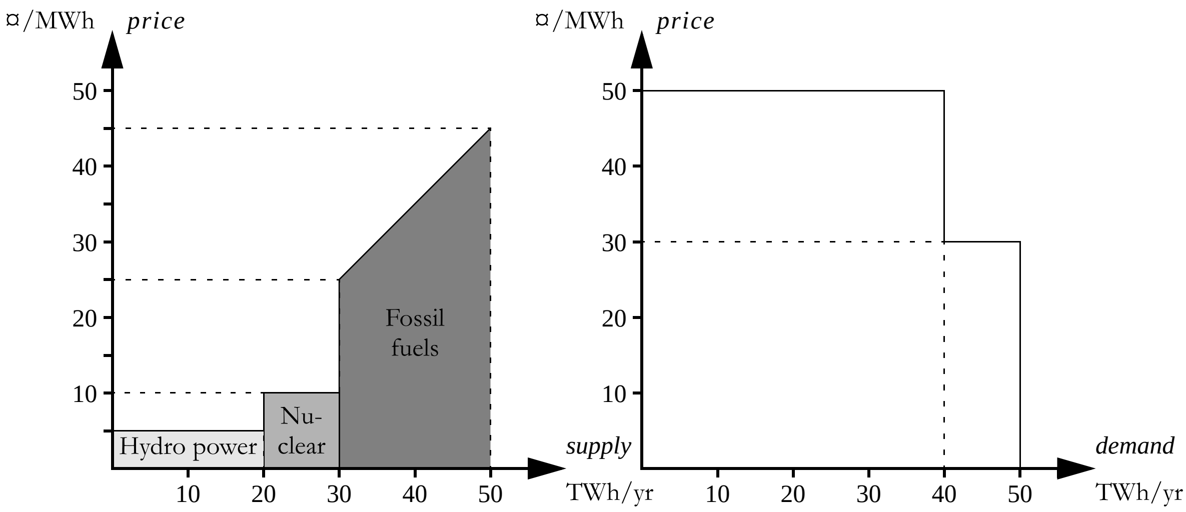
1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. I and III are true but not II.

Problem 2 (6 p)

a) (2 p) The figure below shows the electricity price in a certain electricity market during one year. Assume that there is perfect competition in this electricity market and that all players have access to perfect information. How much will be generated during this year in a power plant with the variable operation cost 300 ¤/MWh and the installed capacity 500 MW? Notice that the answer should be given in TWh!



b) (2 p) The figures below show the supply and demand curves for of a certain hour in the electricity market of Land. What will the electricity price become in this electricity market if we assume perfect competition, perfect information and that there are neither transmission, reservoir nor capacity limitations?



c) (2 p) Strålinge is a nuclear power plant with a generation capacity of 8 TWh/year. The variable costs of the power plant are 100 ¤/MWh and the fixed costs of the company are 2 400 M¤/year. Assume that the electricity price in one particular year is 350 ¤/MWh. Will Strålinge make a profit or a loss during this year?

Problem 3 (6 p)

Consider a power system where the primary control is divided in a normal operation reserve and a disturbance reserve. The normal operation reserve has the gain 4 375 MW/Hz and is mainly designed to manage normal variations in for example load and wind power generation. The disturbance reserve has the gain 2 000 MW/Hz and is mainly designed to manage outages in larger power plants. The normal operation reserve is available in the frequency range 49.9–50.1 Hz and the disturbance reserve is available in the frequency range 49.5–49.9 Hz.

a) (2 p) At 8:10 there is balance between production and consumption in the system and the frequency is 49.78 Hz. At this time the generation in an offshore wind farm is increased by 20 MW. The wind farm is not participating in the primary control. What will the frequency be once the primary control has stabilised the frequency in the system again?

b) (2 p) At 8:15 there is balance between production and consumption in the system and the frequency is 49.80 Hz. At this time up-regulation bids corresponding to in total 375 MW are activated. These up-regulation bids to do not change the gain of the system. What will the frequency be once the primary control has stabilised the frequency in the system again?

c) (2 p) At 8:20 there is balance between production and consumption in the system and the frequency is 49.92 Hz. At this time the hydro power plant Språnget is started, which means that the generation in the system is increased by 350 MW. The gain in Språnget is set to 625 MW/Hz and is available in the frequency range 49.9–50.1 Hz. What will the frequency be once the primary control has stabilised the frequency in the system again?

Problem 4 (12 p)

Stads energi AB owns a thermal power plant with three blocks. The company sells power to customers with firm power contracts, but the they also have the possibility to trade a certain volume per hour at the local power exchange ElKräng. Assume that the company has formulated their short-term planning problem as a MILP problem and that the following symbols have been introduced:

Indices for the power plants: Block I - 1, Block II - 2, Block III - 3.

β_{Gg} = variable operation cost in power plant g, g = 1, 2, 3,
C_g^+ = start-up cost in power plant g, g = 1, 2, 3,
D_t = contracted load during hour t, t = 1, …, 24,
$G_{g,t}$ = generation in power plant g, hour t, g = 1, 2, 3, t = 1, …, 24,
$\bar{G}_g$ = installed capacity in power plant g, g = 1, 2, 3,
$\underline{G}_g$ = minimal generation when power plant g is committed, g = 1, 2, 3,
λ_t = expected electricity price at ElKräng hour t, t = 1, …, 24,
p_t = purchase from ElKräng hour t, t = 1, …, 24,
$\bar{p}_t$ = maximal purchase from ElKräng hour t, t = 1, …, 24,
r_t = sales to ElKräng hour t, t = 1, …, 24,
$\bar{r}_t$ = maximal sales to ElKräng hour t, t = 1, …, 24,
$s_{g,t}^+$ = start-up variable for power plant g, hour t, g = 1, 2, 3, t = 1, …, 24,
$u_{g,0}$ = unit commitment of power plant g at the beginning of the planning period, g = 1, 2, 3,
$u_{g,t}$ = unit commitment of power plant g, hour t, g = 1, 2, 3, t = 1, …, 24.

a) (4 p) Formulate the objective function if the aim of the planning problem is to maximise the income of sold electricity at ElKräng minus the costs of purchasing electricity from ElKräng and minus the costs of the thermal power plant. Use the symbols defined above.

b) (4 p) Formulate the load balance constraint of Stads energi AB for hour t. Use the symbols defined above.

c) (4 p) Formulate the limits for the optimisation variables defined above for the planning problem of Stads energi AB. Please also state the possible index values for each limit!

Problem 5 (12 p)

Ebbuga is a small town in Eastern Africa. The town is not connected to a national grid, but has a local system of its own, which is supplied by four identical diesel generator sets. Each diesel generator set has an installed capacity of 200 kW and the operation cost is 10 ¤/kWh.

Table 1 shows some partial results of a probabilistic production cost simulation of the power system in Ebbuga.

Table 1 Results from a probabilistic production cost simulation of the power system in Ebbuga.

	x = 200	x = 400	x = 600	x = 800	x = 1 000	x = 1 200	$x = \infty$
$\tilde{F}_0(x)$	1.0000	0.2000	0.0000	0.0000	0.0000	0.0000	0.0000
$\int_0^x \tilde{F}_0(\xi)d\xi$	200.0	320.0	340.0	340.0	340.0	340.0	340.0
$\tilde{F}_1(x)$	1.0000	0.3200	0.0300	0.0000	0.0000	0.0000	0.0000
$\int_0^x \tilde{F}_1(\xi)d\xi$	200.0	332.0	367.0	370.0	370.0	370.0	370.0
$\tilde{F}_2(x)$	1.0000	0.4220	0.0735	0.0045	0.0000	0.0000	0.0000
$\int_0^x \tilde{F}_2(\xi)d\xi$	200.0	342.2	391.8	399.6	400.0	400.0	400.0
$\tilde{F}_3(x)$	1.0000	0.5087	0.1258	0.0148	0.0007	0.0000	0.0000
$\int_0^x \tilde{F}_3(\xi)d\xi$	200.0	350.9	414.3	428.4	429.9	430.0	430.0
$\tilde{F}_4(x)$	1.0000	0.5824	0.1832	0.0315	0.0028	0.0001	0.0000
$\int_0^x \tilde{F}_4(\xi)d\xi$	200.0	358.2	434.8	456.3	459.7	460.0	460.0

a) (3 p) What is the availability of the first power plant?
Hint: Examine how $\tilde{F}_1(400)$ is calculated!

b) (3 p) Use probabilistic production cost simulation to calculate the expected operation cost per hour in Ebbuga.

c) (2 p) Generate a value of the available generation capacity in one of the diesel generators using the inverse transform method and the random number 0.18 from a *U*(0, 1)-distribution.

d) (4 p) Assume that the electricity market in Ebbuga is simulated using stratified sampling. The results of the fifteen first scenarios of the Monte Carlo-simulation are compiled in table 2. Which estimates of *ETOC* and *LOLP* are obtained from these results?

Table 2 Results from a Monte Carlo simulation of Ebbuga.

Stratum	Stratum weight	Observations of *TOC* [¤/h]	Observations of *LOLO*
1	0.893	5 800, 3 400, 4 200, 2 500, 3 200	0, 0, 0, 0, 0
2	0.076	3 900, 5 300, 6 000, 3 800, 5 900	0, 0, 1, 0, 0
3	0.031	4 000, 4 000, 4 000, 4 000, 4 000	1, 1, 1, 1, 1

PART II (FOR HIGHER GRADES)

All introduced symbols must be defined. Solutions should include sufficient detail that the argument and calculations can be easily followed.

The answer to each problem must begin on a new sheet, but answers to different parts of the same problem (a, b, c, etc.) can be written on the same sheet. The fields *Namn* (Name), *Blad nr* (Sheet number) and *Uppgift nr* (Problem number) must be filled out on every sheet.

Part II gives a total of 60 points, but this part will only be marked if the candidate has obtained at least 33 points in part I. Then the results of parts I and II and the bonus points will be added together to determine the examination grade (A, B, C, D, E).

Problem 6 (10 p)

Consider a simplified model of the electricity market in Land, where it is assumed that there is perfect competition, that all players have access to perfect information and that there are neither transmission nor reservoir limitations. However, generation capacity as well as demand are varying during the year. To get a rough estimate of the electricity price in Land, the year can be divided in different periods, as listed in table 3. The demand in Land is not price sensitive, and can be assumed to be constant within each time period.[1] Data for the electricity generation are given in table 4. The variable operation costs are assumed to be linear within the intervals; the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

Table 3 Time periods in the electricity market of Land.

Period	Duration [h/year]	Demand [MWh/h]
Winter (peak)	60	12 000
Winter (high load)	3 700	10 500
Winter (low load)	4 000	9 000
Summer	1 000	5 000

Table 4 Generation capacity in the electricity market of Land.

Power source	Production capability [MWh/h]				Variable costs [¤/MWh]
	Winter (peak)	Winter (high load)	Winter (low load)	Summer	
Hydro	6 000	5 000	3 800	3 000	40
Nuclear	3 000	3 000	3 000	0	100
Combined heat and power (biomass)	2 000	2 000	2 000	1 400	100–300
Coal condensing	900	900	900	900	350–440
Oil condensing	600	600	600	600	500–620
Gas turbines	500	500	500	500	800–900

Two companies in Land are currently investigating if they should make investments in their

1. In reality the load is of course varying, but these variations are assumed to be managed by the frequency control system, which is not considered here.

power plants. The following options are considered:

- **New combined cycle gas power plant.** Industriel AB are considering to build a new combined cycle gas power plant. The installed capacity of the new power plant would be 400 MW and would be available in all time periods. The investment costs would be 100 M¤/year and the variable costs would be 300 ¤/MWh.
- **Prolongation of Strålinge.** The company Strålinge AB owns a nuclear power plants with the capacity 1 000 MW. Notice that Strålinge (as well as all other nuclear power plants in Land) isshut down for loading of fuel and general maintenance during the summer period. Strålinge was built more than 40 years ago, and major investments are necessary in order to keep operating this nuclear power plant. The costs of the necessary investments is estimated to 1 900 M¤/year. The variable costs would be the same (i.e., 100 ¤/MWh) after the investment.

a) (5 p) Assume that Industriel AB is the first company to take their investment decision. This means that the company will consider the simplified model described above (i.e., Industriel AB assumes that Strålinge will not be shut down) and then decide if their investment is profitable. The result is then made public. Strålinge AB will now investigate if it is profitable to prolong the nuclear power plant considering the decision of Industriel AB. Which investments will be carried out?

b) (5 p) Which investements will be carried out if it is Strålinge AB who makes the decision first and then Industriel AB investigate the profitability of their investment considering the decision of Strålinge AB?

Problem 7 (10 p)

The power system in Rike is divided in two price areas. There is a lot of hydro power in the northern part of the system, but most of the load is located in the southern part. The primary control of Rike is divided in a normal operation reserve and a disturbance reserve. The normal operation reserve is available in the frequency range 49.9–50.1 Hz and has a total gain of 3 000 MW/Hz, where 2 500 MW/Hz is provided by power plants in northern Rike. The disturbance reserve is available in the frequency range 49.5–49.9 Hz and has a total gain of 2 500 MW/Hz, where 2 000 MW/Hz is provided by power plants in northern Rike.

a) (6 p) There are eight parallel AC transmission lines between the two areas, where each line has a maximal transmission capacity of 500 MW. Moreover, there is an HVDC line, which also can transfer up to 500 MW. The power flow on the HVDC line is controlled manually by the system operator, Riksnät, and is not affected neither by the frequency of the system or how generation and load is divided.

Hence, the total transmission capacity between the northern and southern parts is 4 500 MW, but the entire capacity may not be utilised by the players of the electricity market, because Riksnät must have sufficient margins for primary control as well as to manage outages in transmission lines. The requirement is that the system should be able to survive that one transmission line is disconnected, while the normal operation reserve and disturbance reserve are fully utilised to cover load increase and generation outages in southern Rike. (The load and generation that is not part of the primary control can thus be assumed to be constant in northern Rike.) How large transmission capacity may the players of the electricity market use (i.e., what is the largest acceptable transmission when the frequency is exactly 50 Hz) in order to fulfil this requirement?

b) (4 p) Assume that the HVDC line is equipped with a frequency sensitive control system, making the transmission on this line a linear function of the frequency in the system: $P = a + bf$, where a and b are two constants that can be chosen freely. Can such a control system be used to increase the part of the transmission capacity from north to south that is available to the market? How should then the constants a and b be set?

Problem 8 (20 p)

In the power exchange ElKräng, players in the electricity market can sell and purchase electricity for each hour during the next day. The two most important bids are sell bids and purchase bids. A sell bid is valid for a specific hour t, $t = 1, \ldots, 24$, and comprises a certain maximal volume, $\bar{r}_{i,t}$, $i = 1, \ldots, N_t$,[2] $t = 1, \ldots, 24$, as well as a requested price, $\beta_{Ri,t}$, $i = 1, \ldots, N_t$, $t = 1, \ldots, 24$. A purchase bid is valid for a specific hour t, $t = 1, \ldots, 24$, and comprises a certain maximal volume, $\bar{p}_{j,t}$, $j = 1, \ldots, M_t$,[3] $t = 1, \ldots, 24$, as well as a value of purchased electricity, $\beta_{Rj,t}$, $j = 1, \ldots, M_t$, $t = 1, \ldots, 24$. Notice that these bids do not have to be accepted as a whole; if somebody submits a 100 MW sell bid, it is possible that the player will only get to sell 50 MW.

Bids to ElKräng must be submitted at last 12:00 noon. Then ElKräng compiles the bids and decides which bids that will be accepted as well as the electricity price for each hour (all accepted bids for one hour receive the same electricity price). This is done by solving an optimisation problem, where the objective function is to maximise the value of the accepted purchase bids minus the requested price for the accepted sell bids.

a) (8 p) Formulate the planning problem of ElKräng as an LP problem. Use the notation introduced above for the parameters (it is however permitted to add further symbols if you consider it necessary).

NOTICE! The following is required to get full score for this problem:

- The symbols for the optimisation variables must be clearly defined.
- The optimisation problem should be formulated so that it is easy to determine what the objective function is, which constraints there are and which limits there are.
- The possible values for all indices should be clearly stated for each equation.

b) (2 p) How can ElKräng compute the hourly electricity prices when the planning problem from part a has been solved?

c) (10 p) ElKräng is also planning to introduce block bids. These bids will be a class of sell bids, which are valid for several consecutive hours and which can only be accepted as a whole. Sellers can choose between five types of block bids, compromising different time periods (see table 5). How must the planning problem from part a be reformulated in order to consider these block bids? Do not forget to define all new variables and parameters that you introduce!

Hint: Introduce a binary variable, $u_{k,s}$, $k = 1, \ldots, L_s$,[4] $s = 1, \ldots, 5$, which is equal to one if a certain block bid is accepted and equal to zeros if the bid is rejected.

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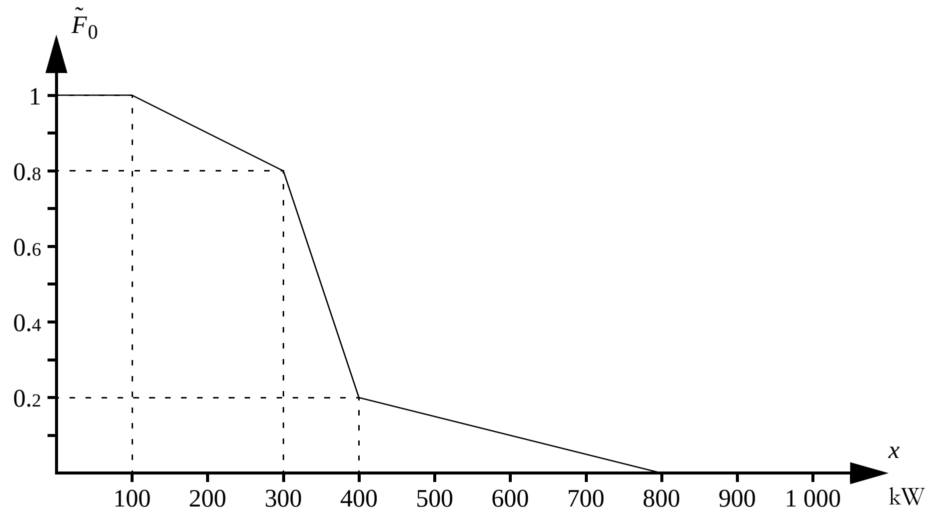
2. Where N_t is the number of sell bids that have been submitted for hour t.
3. Where M_t is the number of purchase bids that have been submitted for hour t.
4. Where L_s is the number of block bids of type s.

Table 5 Different types of block bids at ElKräng.

Type, s	Hours, t
1	1–24
2	1–6
3	7–12
4	13–18
5	19–24

Problem 9 (20 p)

Large parts of the rural areas of Eggwanga have no access to electricity. The Eggwangan authorities have started a rural electrification programme, and as part of this programme, the Eggwanga National Electricity Supply Company Ltd. (ENESCO) has been given the task of electrifying the village Ekyaro and its surroundings. ENESCO is now investigating how much it would cost to connect Ekyaro to the national grid.



a) (5 p) The figure above show the load duration curve of Ekyaro. The line between Ekyaro and the national grid will have a capacity of 500 kW. As the electricity demand in the national grid is large compared to the generation capacity, rotating load curtailment is common practice. ENESCO estimates that Ekyaro will be disconnected in average 438 hours per year. Assume that the average generation cost of electricity fed into the national grid is 5 ¤/kWh and that the losses on the line between Ekyaro and the national grid can be neglected. Use probabilistic production cost simulation to calculate the expected annual operation cost of the power system in Ekyaro.

b) (5 p) To increase the reliability of supply in Ekyaro, ENESCO is considering installing a number of local diesel generator sets, which can be used when the line from the national grid is disconnected, or when the load in Ekyaro is higher than the capacity of the line. Each diesel generator set will have a capacity of 150 kW and an availability 90%. How many diesel generator sets are needed to keep the risk of load shedding in Ekyaro below 10%?

c) (6 p) To obtain a more accurate calculation of the expected operation cost of Ekyaro, ENESCO would like to simulate the power system in Ekyaro using a more detailed model. The objective of this model is to include that the marginal generation cost in the national grid is different at different times, and that the losses on the line between the national grid and Ekyaro amounts to 4%. ENESCO is using the following frequency function to model the marginal generation cost:

$$f(x) = \begin{cases} 0.45 & x = 2, \\ 0.35 & x = 6, \\ 0.2 & x = 10, \\ 0 & \text{all other } x. \end{cases}$$

Consider the system where Ekyaro is only supplied by the line from the national grid (i.e., there are no diesel generator sets). Define an electricity market model which can be used to analyse the scenarios in a Monte Carlo simulation of the power system in Ekyaro, where the objective of the simulation is to estimate the expected operation cost and the risk of load shedding in the system. State the scenario parameters, model constants and result variables of your model, and describe how the result variables are calculated.

d) (3 p) Use the random number 0.3 from a $U(0, 1)$-distribution to generate a random value of the marginal generation cost of the national grid. Moreover, state the complementary random number of this value.

e) (1 p) When using complementary random numbers in a Monte Carlo simulation, a negative correlation is created between the input values of the electricity market model (the scenario parameters) in different scenarios. How must this negative correlation affect the correlation between the outputs (the result variables) in different scenarios if the method should reduce the variance of the estimate, $Var[m_X]$?

1. The result variables must be negatively correlated.
2. The result variables must be independent.
3. The result variables must be positively correlated.

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Answer sheet for part I

Name: ....

Personal number: ....

Problem 1

a) Alternative .... is correct.

b) Alternative .... is correct.

Problem 2

a) .... TWh **b)** .... ¤/MWh

c) ....

Problem 3

a) .... Hz **b)** .... Hz

c) .... Hz

Problem 4

a) ....

....

b) ....

....

c) ....

....

....

....

Problem 5

a) .... % **b)** .... ¤/h

c) .... kW

d) *ETOC* .... ¤/h *LOLP* .... %

Problem 1

a) 2, **b)** 3.

Problem 2

a) The power plant will generate its installed capacity during those hours when the electricity price is higher than 300 ¤/MWh. From the figure, we can determine that this is the case during 3 000 hours, i.e., the total generation in this year will be 500 · 3 000 = 1 500 000 MWh = 1.5 TWh.

b) The electricity price is determined by the intersection of the supply and demand curves. The intersection can be found graphically by drawing both curves in the same figure. In order to have a demand of 50 TWh/year the electricity price may not exceed 30 ¤/MWh, but the supply is only 35 TWh/year at that electricity price. Hence, the demand is 40 TWh/year and then it the electricity price will have to be 35 ¤/MWh (half the interval for fossil fuel generation).

c) The income of Strålinge is 8 TWh/year · 350 ¤/MWh = 2 800 M¤/year. This is not sufficient to cover the total variable costs (8 TWh/year · 100 ¤/MWh = 800 M¤/year) and the fixed costs; hence, the power plant is making a loss this year.

Problem 3

a) At this frequency, the gain in the system is 2 000 MW/Hz. The increase in electricity generation then results in a frequency increase $\Delta f = \Delta G/R$ = 20/2 000 = 0.01 Hz, i.e., the new frequency is 49.78 + 0.01 = 49.79 Hz.

b) The disturbance reserve can decrease the generation by 0.1 · 2 000 = 200 MW if the frequency is 49.80 Hz. The frequency has then increased to 49.9 Hz. The remaining 175 MW generation decrease must be managed by the normal operation reserve, which results in a frequency increase $\Delta f = \Delta G/R$ = 175/4 375 = 0.04 Hz, i.e., the new frequency is 49.9 + 0.04 = 49.94 Hz.

c) The gain of the system is increased to 5 000 MW/Hz when Språnget is started. The increase in electricity generation then results in a frequency increase $\Delta f = \Delta G/R$ = 350/5 000 = 0.07 Hz, i.e., the new frequency is 49.92 + 0.07 = 49.99 Hz.

Problem 4

a) maximise $\sum_{t=1}^{24}\left(\lambda_t(r_t - p_t) - \sum_{g=1}^{3}(C_g^+ s_{g,t}^+ + \beta_{Gg}G_{g,t})\right)$.

b) $\sum_{g=1}^{3} G_{g,t} + p_t = D_t + r_t$.

c) The minimal and maximal generation for each hour is controlled by special constraints. The limits that have to be stated are then for the trading and the binary variables:

$0 \le p_t \le \bar{p}_i, \qquad t = 1, \ldots, 24,$

$0 \le r_t \le \bar{r}_i, \qquad t = 1, \ldots, 24,$

$s_{g,t}^+ \in \{0, 1\}, \qquad g = 1, 2, 3, t = 1, \ldots, 24,$

$u_{g,t} \in \{0, 1\}, \qquad g = 1, 2, 3, t = 1, \ldots, 24.$

Problem 5

a) $\tilde{F}_1(400) = p_1\tilde{F}_0(400) + (1 - p_1)\tilde{F}_0(200) \Rightarrow$

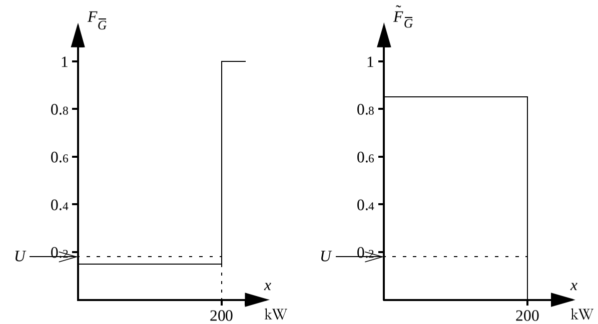
$$p_1 = \frac{\tilde{F}_1(400) - \tilde{F}_0(200)}{\tilde{F}_0(400) - \tilde{F}_0(200)} = \frac{0.32 - 1}{0.2 - 1} = 85\%.$$

b) The total electricity generation of the three power plants is calculated by

$$EG_{1234} = EENS_0 - EENS_4 = \int_0^\infty \tilde{F}_0(\xi)d\xi - \int_{800}^\infty \tilde{F}_4(\xi)d\xi = \int_0^\infty \tilde{F}_0(\xi)d\xi - \left(\int_0^\infty \tilde{F}_4(\xi)d\xi - \int_0^{800} \tilde{F}_4(\xi)d\xi\right) =$$

= 340.0 – (460.0 – 456.3) = 336.3 kWh/h; hence $ETOC$ = 10 · 336.3 = 3 363 ¤/h.

c) If we draw the distribution function or the duration curve of $\bar{G}$ it is easy to see that U = 0.18 is transformed into $\bar{G}$ = 200.



d) The following estimates are obtained of the expectation value in each stratum:

m_{TOC1} = 19 100/5 = 3 820 $\qquad m_{LOLO1}$ = 0

m_{TOC2} = 24 900/5 = 4 980 $\qquad m_{LOLO2}$ = 1/5 = 0.2

m_{TOC3} = 20 000/5 = 4 000 $\qquad m_{LOLO3}$ = 5/5 = 1

Thus, we get

$$m_{TOC} = \sum_{h=1}^{3}\omega_h m_{TOCh} = 0.893 \cdot 3\,820 + 0.076 \cdot 4\,980 + 0.031 \cdot 4\,000 \approx 3\,914 ¤/h,$$

$$m_{LOLO} = \sum_{h=1}^{3}\omega_h m_{LOLOh} = 0 + 0.076 \cdot 0.2 + 0.031 \cdot 1 = 4.62\%.$$

Problem 6

a) First we need to compute the electricity prices during the four periods if the combined cycle gas power plant is built:

- Winter peak: Demand 12 000 MWh/h ⇒ All hydro, nuclear, combined heat and power and the new combined cycle gas will be needed (in total 11 400 MWh/h) ⇒ 600 MWh/h coal condensing will be used ⇒ Electricity price λ = 410 ¤/MWh.
- Winter high load: Demand 10 500 MWh/h ⇒ All hydro, nuclear, combined heat and power and the new combined cycle gas will be needed (in total 10 400 MWh/h) ⇒ 100 MWh/h coal condensing will be used ⇒ Electricity price λ = 360 ¤/MWh.
- Winter low load: Demand 9 000 MWh/h ⇒ All hydro, nuclear and combined heat and power will be needed (in total 8 800 MWh/h) ⇒ 200 MWh/h combined cycle gas will be used ⇒ Electricity price λ = 300 ¤/MWh.
- Summer: Demand 5 000 MWh/h ⇒ All hydro, combined heat and power and the new combined cycle gas will be needed (in total 4 800 MWh/h) ⇒ 200 MWh/h coal condensing will be used ⇒ Electricity price λ = 370 ¤/MWh.

The profit of the new combined cycle gas power plant is then given by the income of sold electricity minus the variable costs summarised for all periods minus the annual fixed costs:

profit = (410 – 300) · 400 · 60 {winter peak} + (360 – 300) · 400 · 3 700 {winter high load}
+ (300 – 300) · 200 · 4 000 {winter low load} + (370 – 300) · 400 · 1 000 {summer}
– 100 000 000 {fixed costs} = 19.44 M¤/year.

The result is positive and thus the investment is profitable.

Now we can compute the profit of Strålinge considering that the gas power plant will be built. The prices will then not change and we can compute the profit of Strålinge using the same principle as above:

profit = (410 – 100) · 400 · 60 {winter peak} + (360 – 100) · 400 · 3 700 {winter high load}
+ (300 – 100) · 200 · 4 000 {winter low load} – 1 900 000 000 {fixed costs} =
= –119.4 M¤/year.

The result is negative and thus the investment is not profitable.

b) First we need to compute the electricity prices during the four periods without the combined cycle gas power plant:

- Winter peak: Demand 12 000 MWh/h ⇒ All hydro, nuclear, combined heat and power and coal condensing will be needed (in total 11 900 MWh/h) ⇒ 100 MWh/h oil condensing will be used ⇒ Electricity price λ = 520 ¤/MWh.
- Winter high load: Demand 10 500 MWh/h ⇒ All hydro, nuclear and combined heat and power will be needed (in total 10 000 MWh/h) ⇒ 500 MWh/h coal condensing will be used ⇒ Electricity price λ = 400 ¤/MWh.
- Winter low load: Demand 9 000 MWh/h ⇒ All hydro, nuclear and combined heat and power will be needed (in total 8 800 MWh/h) ⇒ 200 MWh/h coal condensing will be used ⇒ Electricity price λ = 370 ¤/MWh.
- Summer: Demand 5 000 MWh/h ⇒ All hydro and combined heat and power will be needed (in total 4 400 MWh/h) ⇒ 600 MWh/h coal condensing will be used ⇒ Electricity price λ = 410 ¤/MWh.

The profit of Strålinge is then calculated in the same way as above:

profit = (520 – 100) · 400 · 60 {winter peak} + (400 – 100) · 400 · 3 700 {winter high load}
+ (370 – 100) · 200 · 4 000 {winter low load} – 1 900 000 000 {fixed costs} =
= 315.2 M¤/year.

The result is positive and thus the investment is profitable.

Industriel AB will now make their decision considering that Strålinge will remain in operation. This is the same situation as in question a; hence, we can conclude that the investment in the gas power plant is profitable.

Problem 7

a) If an outage occurs on the HVDC line then the flow will be redirected to the AC lines. If an outage occurs on one of the AC lines then the flow will be divided among the other seven AC lines (the flow on the HVDC line is not changed in this case). Either way, there must be enough reserves on the AC lines to accommodate another 500 MW. To manage an outage in one line, the transmission may not exceed 7 · 500 (AC lines) + 500 (HVDC line) = 4 000 MW.

A part of these 4 000 MW must be reserved for primary control. If the entire primary control reserve in Rike is to be exported southwards then the reserve must be in total 2 500 MW/Hz · 0.1 Hz (normal operation reserve) + 2 000 MW/Hz · 0.4 Hz (disturbance reserve) = 1 050 MW. The players in the electricity market may therefore at most have access to 2 950 MW transmission capacity.

b) The suggested control system can govern how the power flow is distributed between the AC lines and the HVDC line for different frequencies. However, this distribution is not important in this case; it must still be possible to transfer 1 050 MW at the frequency 49.5 Hz without getting a total flow that exceeds 4 000 MW.

Problem 8

a) The problem we want to solve is

maximise	*value of accepted purchase bids – requested price for sell bids,*
subject to	*load balance,*
	maximal volume of each bid.

Parameters

The parameters are defined in the problem text.

Optimisation variables

$p_{j,t}$ = accepted volume from purchase bid *j*, hour *t*, $j = 1, \ldots, M_t$, $t = 1, \ldots, 24$,
$r_{i,t}$ = accepted volume from sell *i*, hour *t*, $i = 1, \ldots, N_t$, $t = 1, \ldots, 24$.

Objective function

$$\text{maximise} \quad \sum_{t=1}^{24}\left(\sum_{j=1}^{M_t}\beta_{j,t}p_{j,t} - \sum_{i=1}^{N_t}\beta_{i,t}r_{i,t}\right).$$

Constraints

$$\sum_{j=1}^{M_t} p_{j,t} = \sum_{i=1}^{N_t} r_{i,t}, \qquad t = 1, \ldots, 24.$$

Variable limits

$$0 \le p_{j,t} \le \bar{p}_{j,t}, \qquad j = 1, \ldots, M_t,\ t = 1, \ldots, 24,$$

$$0 \le r_{i,t} \le \bar{r}_{i,t}, \qquad i = 1, \dots, N_t,\ t = 1, \dots, 24.$$

b) The electricity price can be obtained directly from the solution of the optimisation problem from part a, namely from the dual variable of the load balance constraint of the corresponding hour.

c) The following new parameters are needed:

L_s = number of block bids of type s, $s = 1, \dots, 5$,
$\bar{b}_{k,s}$ = volume in block bid k, type s, $k = 1, \dots, L_s$, $s = 1, \dots, 5$,
$\beta_{k,s}$ = requested price for block bid k, type s, $k = 1, \dots, L_s$, $s = 1, \dots, 5$.

We also introduce new optimisation variables as suggested in the hint:

$u_{k,s}$ = acceptance of block bid k, type s, $k = 1, \dots, L_s$, $s = 1, \dots, 5$.

The cost of accepted block bids must be included in the objective function. In order to do that, we need to consider which hours a block bid are valid for, which gives us the following updated objective function:

$$\text{maximise} \quad \sum_{t=1}^{24}\left(\sum_{j=1}^{M_t}\beta_{j,t}p_{j,t} - \sum_{i=1}^{N_t}\beta_{i,t}r_{i,t} - \sum_{k=1}^{L_1}\beta_{k,1}\bar{b}_{k,1}u_{k,1}\right) - \sum_{t=1}^{6}\sum_{k=1}^{L_2}\beta_{k,2}\bar{b}_{k,2}u_{k,2}$$
$$- \sum_{t=7}^{12}\sum_{k=1}^{L_3}\beta_{k,3}\bar{b}_{k,3}u_{k,3} - \sum_{t=13}^{18}\sum_{k=1}^{L_4}\beta_{k,4}\bar{b}_{k,4}u_{k,4} - \sum_{t=19}^{24}\sum_{k=1}^{L_5}\beta_{k,5}\bar{b}_{k,5}u_{k,5}.$$

The accepted bids must also be included in the load balance constraints. Again, we need to consider which hours a block bid are valid for, which gives us the following updated constraints:

$$\sum_{j=1}^{M_t} p_{j,t} = \sum_{i=1}^{N_t} r_{i,t} + \sum_{k=1}^{L_1}\bar{b}_{k,1}u_{k,1} + \sum_{k=1}^{L_2}\bar{b}_{k,3}u_{k,3}, \qquad t = 1, \dots, 6,$$
$$\sum_{j=1}^{M_t} p_{j,t} = \sum_{i=1}^{N_t} r_{i,t} + \sum_{k=1}^{L_1}\bar{b}_{k,1}u_{k,1} + \sum_{k=1}^{L_3}\bar{b}_{k,4}u_{k,4}, \qquad t = 7, \dots, 12,$$
$$\sum_{j=1}^{M_t} p_{j,t} = \sum_{i=1}^{N_t} r_{i,t} + \sum_{k=1}^{L_1}\bar{b}_{k,1}u_{k,1} + \sum_{k=1}^{L_4}\bar{b}_{k,2}u_{k,2}, \qquad t = 13, \dots, 18,$$
$$\sum_{j=1}^{M_t} p_{j,t} = \sum_{i=1}^{N_t} r_{i,t} + \sum_{k=1}^{L_1}\bar{b}_{k,1}u_{k,1} + \sum_{k=1}^{L_5}\bar{b}_{k,2}u_{k,2}, \qquad t = 19, \dots, 24.$$

Finally, we need to state the limits of the new optimisation variables:

$$u_{k,s} \in \{0, 1\}, \qquad k = 1, \dots, L_s,\ s = 1, \dots, 5.$$

Problem 9

a) The line can be expected to be disconnected during 438 of the 8 760 hours in a year, which corresponds to5% of the time. The line can therefore be modelled as a power plant with a capacity of 500 kW, generation cost 5 ¤/kWh and availability 95%. The expected annual generation in such a power plant is calculated according to

$$EG_1 = 8\,760 \cdot 0{,}95 \int_0^{500} \tilde{F}_0(x)dx = 8\,760 \cdot 0.95(1 \cdot 100 + (1 + 0.8)/2 \cdot 200 + (0.8 + 0.2)/2 \cdot 100$$
$$+ (0.2 + 0.15)/2 \cdot 100) \approx 2.89 \text{ GWh/year}.$$

The expected operation cost in a year is then $ETOC = 5EG_1 \approx 14.46$ M¤/year.

b) The risk of load shedding in Ekyaro when there is only the line from the national grid is given by

$$LOLP_1 = \tilde{F}_1(500) = 0.95\tilde{F}_0(500) + 0.05\tilde{F}_0(500 - 500) = 0.95 \cdot 0.15 + 0.05 \cdot 1 = 19.25\%.$$

With one diesel generator set, we get

$$LOLP_2 = \tilde{F}_2(650) = 0.9\tilde{F}_1(650) + 0.1\tilde{F}_1(500) =$$
$$= 0.9(0.95\tilde{F}_0(650) + 0.05\tilde{F}_0(650 - 500)) + 0.1 \cdot 0.1925 =$$
$$= 0.9(0.95 \cdot 0.075 + 0.05 \cdot 0.95) + 0.1 \cdot 0.1925 \approx 12{,}61\%.$$

With two diesel generator sets, we get

$$LOLP_3 = \tilde{F}_3(800) = 0.9\tilde{F}_2(800) + 0.1\tilde{F}_2(650) =$$
$$= 0.9(0.9\tilde{F}_1(800) + 0.1\tilde{F}_1(800 - 150)) + 0{,}1 \cdot 0{,}126125 =$$
$$= 0.9(0.9(0.95\tilde{F}_0(800) + 0.05\tilde{F}_0(800 - 500)) + 0.1\tilde{F}_1(650)) + 0.1 \cdot 0.126125 =$$
$$= 0.9(0.9 \cdot (0.95 \cdot 0 + 0.05 \cdot 0.8) + 0.1 \cdot (0.95 \cdot 0.075 + 0.05 \cdot 0.95)) + 0.1 \cdot 0.126125 = 5{,}57\%.$$

In conclusion, two diesel generator sets are needed to keep the *LOLP* ni Ekyaro below 10%.

c) Introduce the following scenario parameters:

D = load in Ekyaro,
β = marginal production cost in the national grid,
$\bar{P}$ = available transmission capacity from the national grid.

Introduce the following model constant:

β_L = loss coefficient for transmission from the national grid.

The import from the national grid, i.e., the power that is injected on the line, is denoted P and is a result variable. It is calculated according to

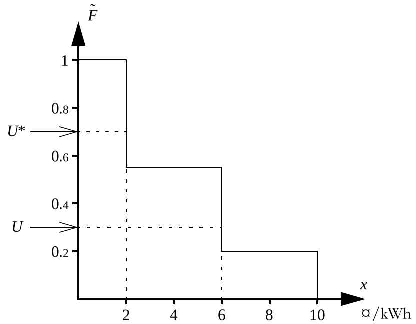
$$P = \begin{cases} 0 & \text{if } \bar{P} = 0, \\ \dfrac{D}{1 - \beta_L} & \text{if } \bar{P} = 500 \text{ and } D \le (1 - \beta_L)\bar{P} = 480, \\ 500 & \text{if } \bar{P} = 500 \text{ and } D > (1 - \beta_L)\bar{P} = 480. \end{cases}$$

Finally, we have the two result variables which are needed to estimate *ETOC* and *LOLP*: the operation cost is given by $TOC = \beta \cdot P$, and *LOLO* which is equal to 1 if $\bar{P} = 0$ (i.e., if the line is disconnected) or if $D > 480$ (i.e., if power that reaches Ekyaro if 500 kW is injected to the line), and which otherwise is equal to 0.

d) According to the definition we have

$$\tilde{F}(x) = \sum_{t > x} f(t) = \begin{cases} 1 & x < 2, \\ 0.55 & 2 \le x < 6, \\ 0.2 & 6 \le x < 10, \\ 0 & 10 \le x. \end{cases}$$

By drawing the duration curve, it is easy to see that U = 0.3 is transformed to 6 ¤/kWh and U^* = 1 – U = 0.7 is transformed to 2 ¤/kWh.



e) 1.