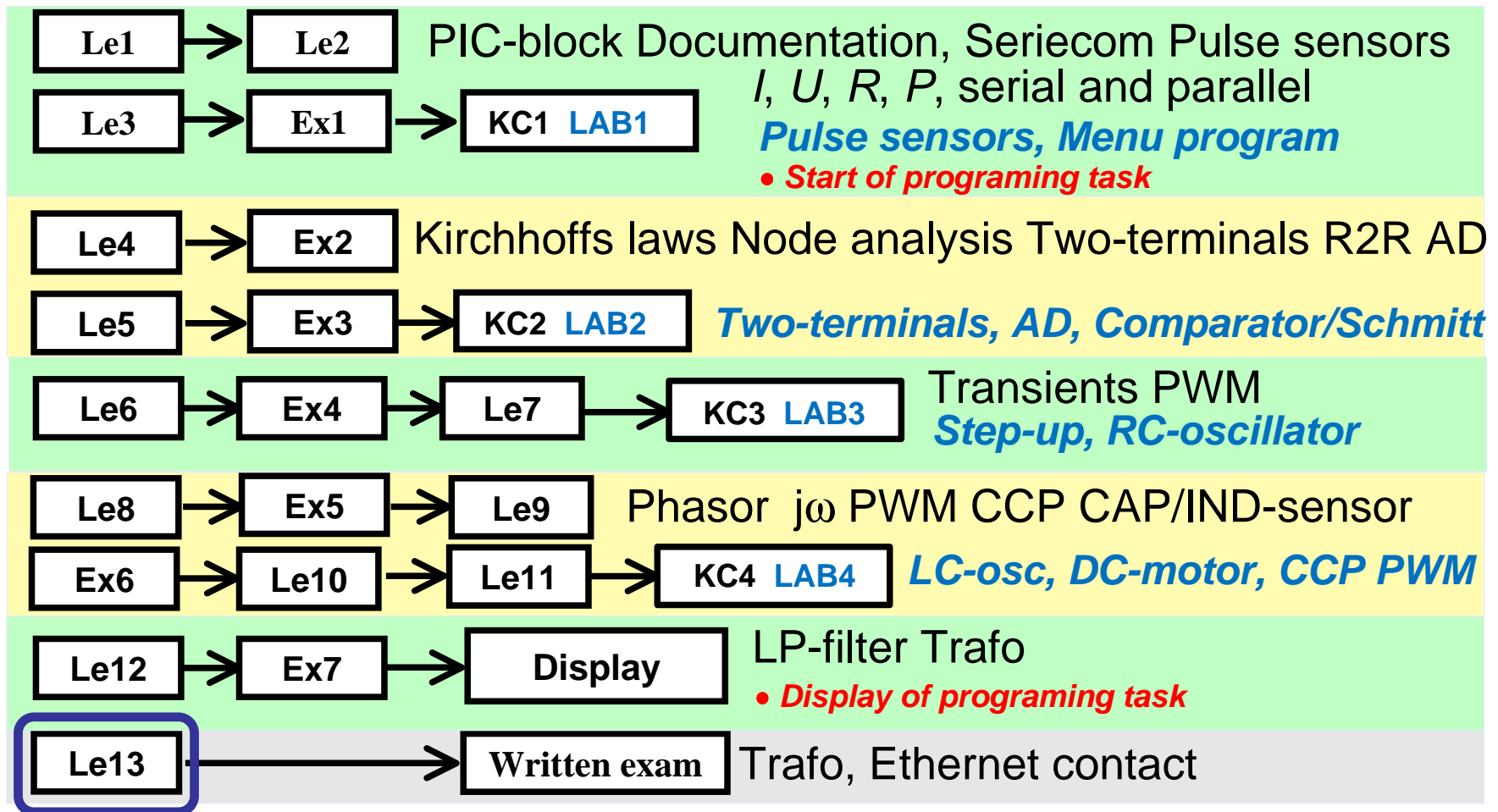
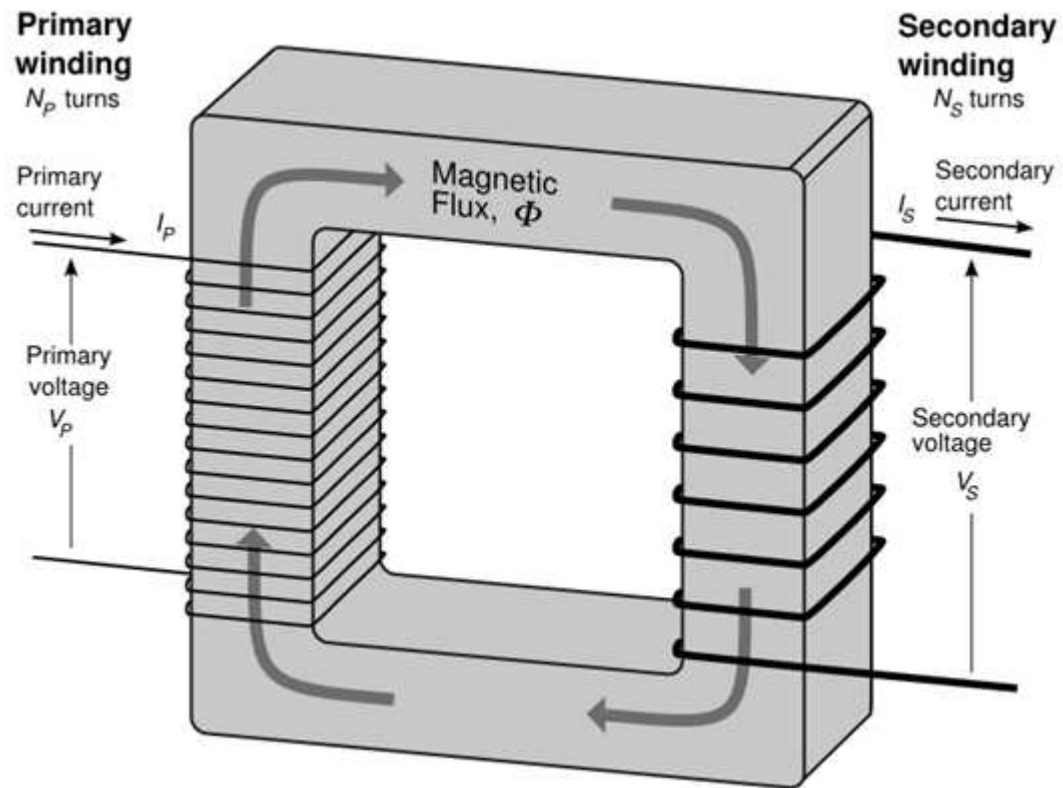


# IE1206 Embedded Electronics



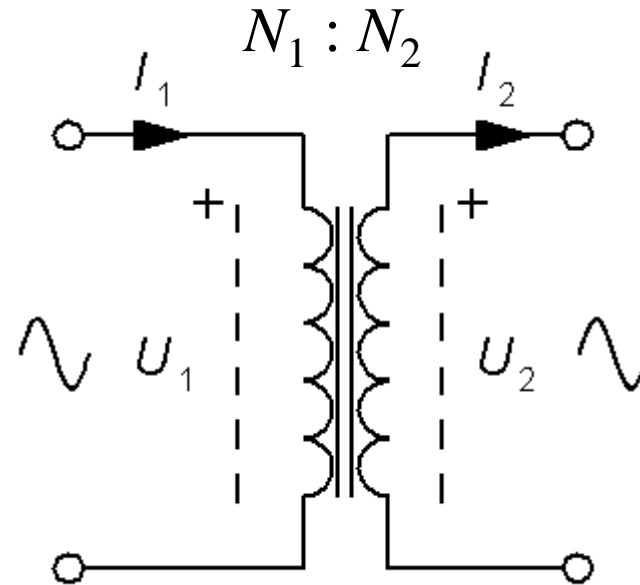
# Transformer



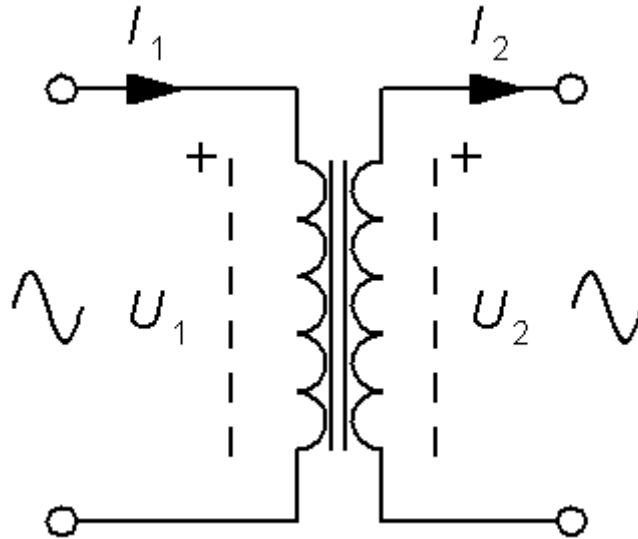
# Voltage ratio

$$U_1 = N_1 \frac{d\Phi}{dt} \quad U_2 = N_2 \frac{d\Phi}{dt}$$

$$\boxed{\frac{U_1}{U_2} = \frac{N_1}{N_2}}$$



# Ideal transformer $I_0 = 0$



$$N_1 \cdot I_0 = N_1 \cdot I_1 - N_2 \cdot I_2$$

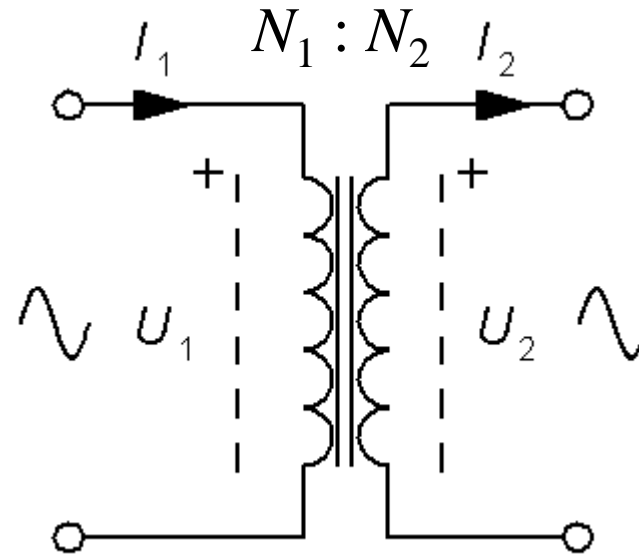
Magnetising current  $I_0 \approx 0$  is *small* compared to the work currents  $I_1$  and  $I_2$ . The transformer itself has a high inductance.

# Current ratio

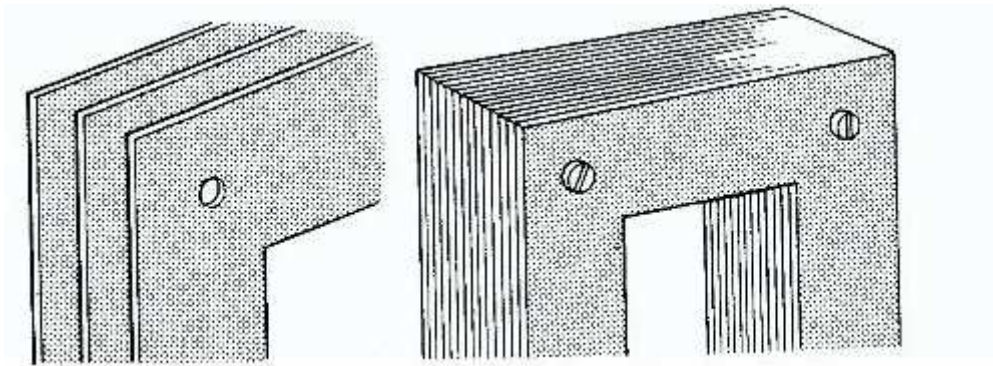
$$P_1 = P_2 \quad (P_0, I_0 = 0)$$

$$U_1 \cdot I_1 = U_2 \cdot I_2 \quad \Rightarrow$$

$$\frac{I_2}{I_1} \approx \frac{U_1}{U_2} = \frac{N_1}{N_2}$$

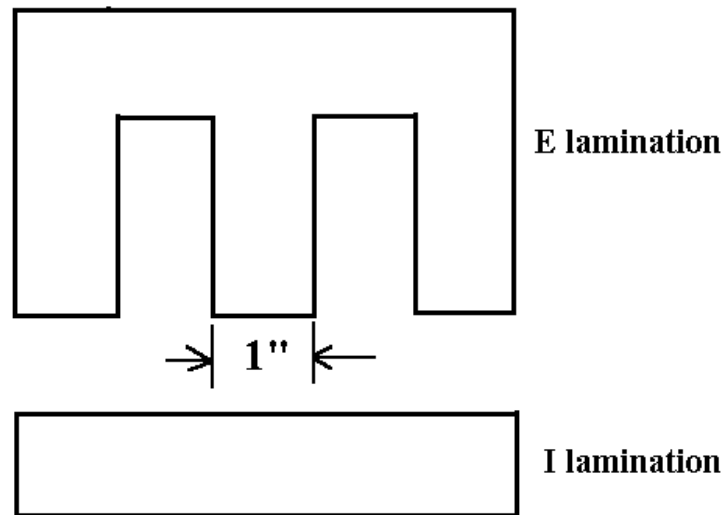


# Eddy current losses



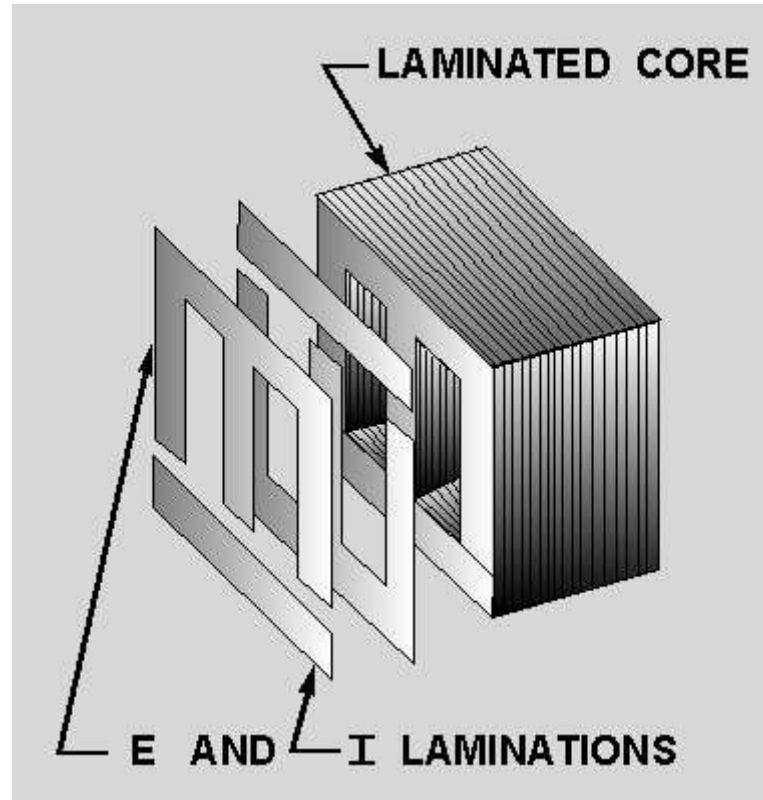
Eddy currents – currents inside the iron core is prevented with lacquered ( = isolation ) sheet metal.

# E I -core



- EI-core is very economical to manufacture !

# E I -core





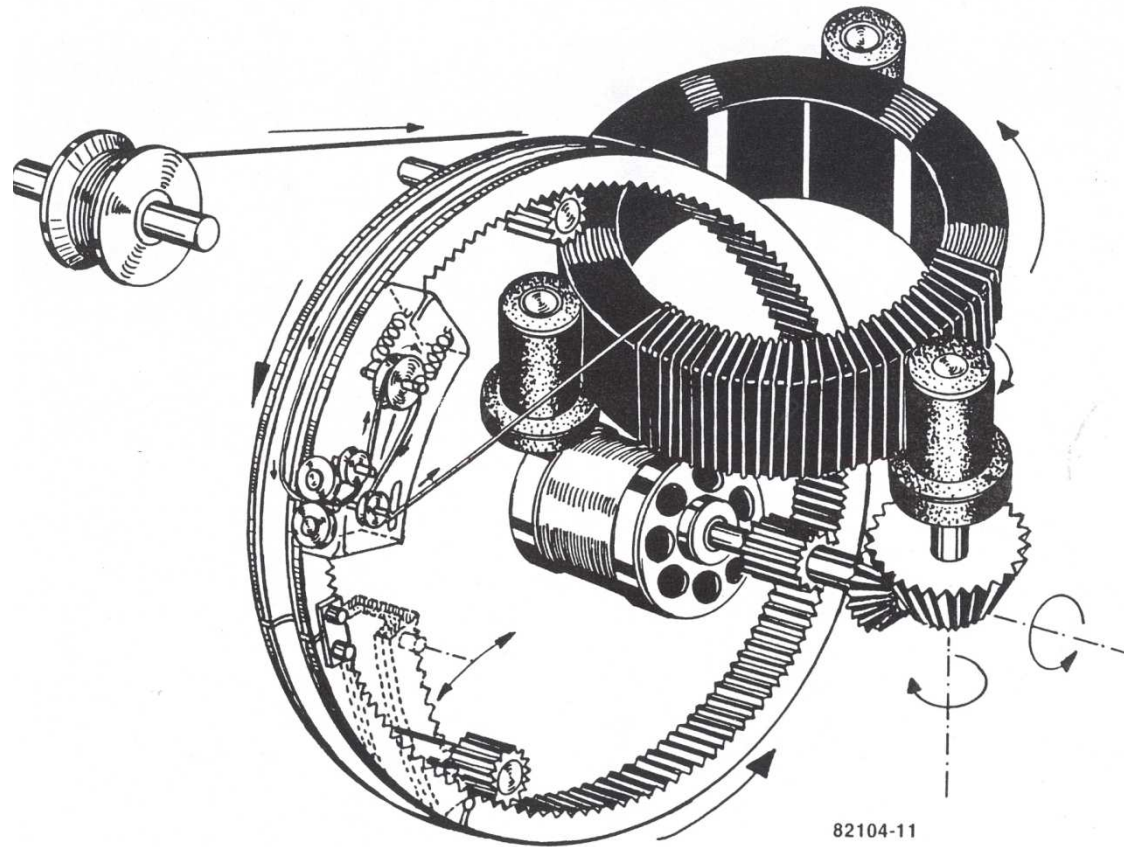
# Toroid



Toroid core has a low leakage field – so it will not disturb nearby electronics!

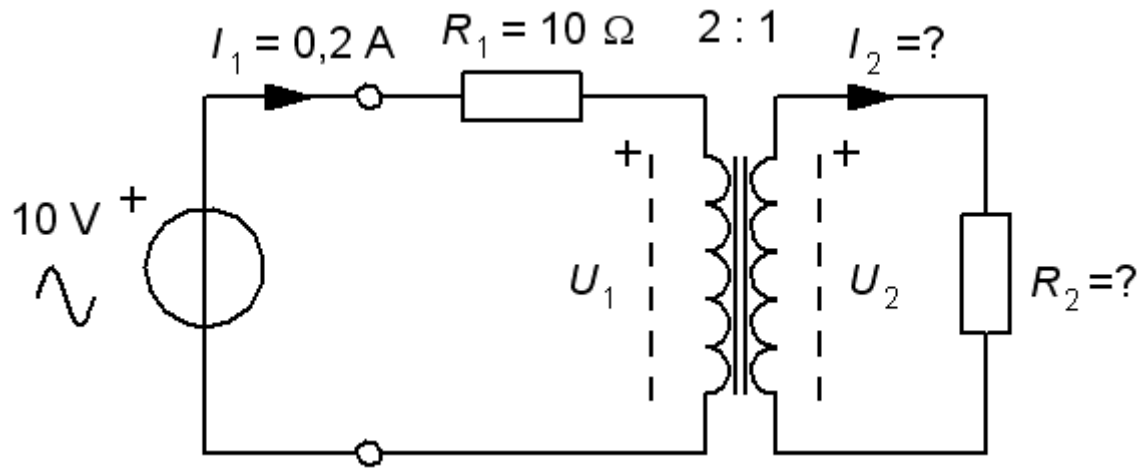
*How do one wind such a transformer?*

# Automatic Winding of toroidal core

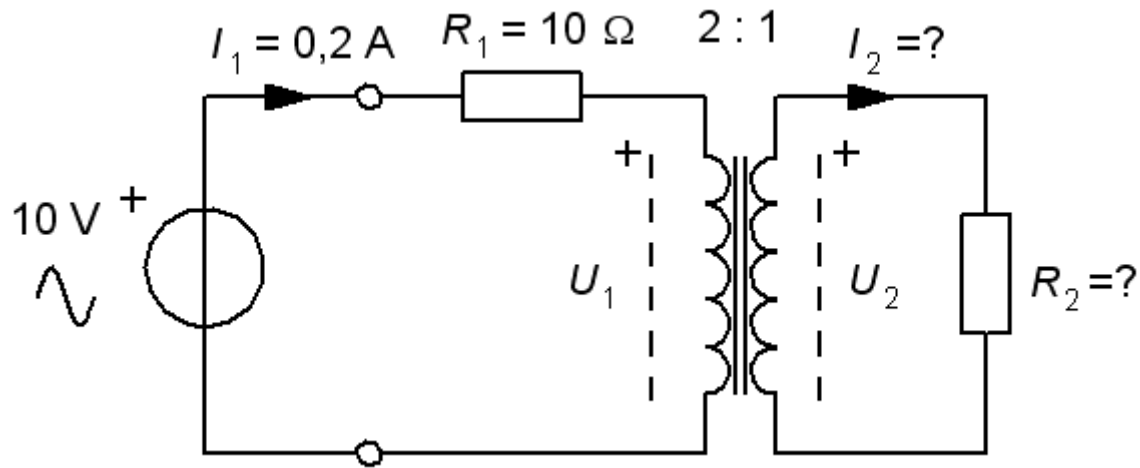


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# Transformer (15.4)

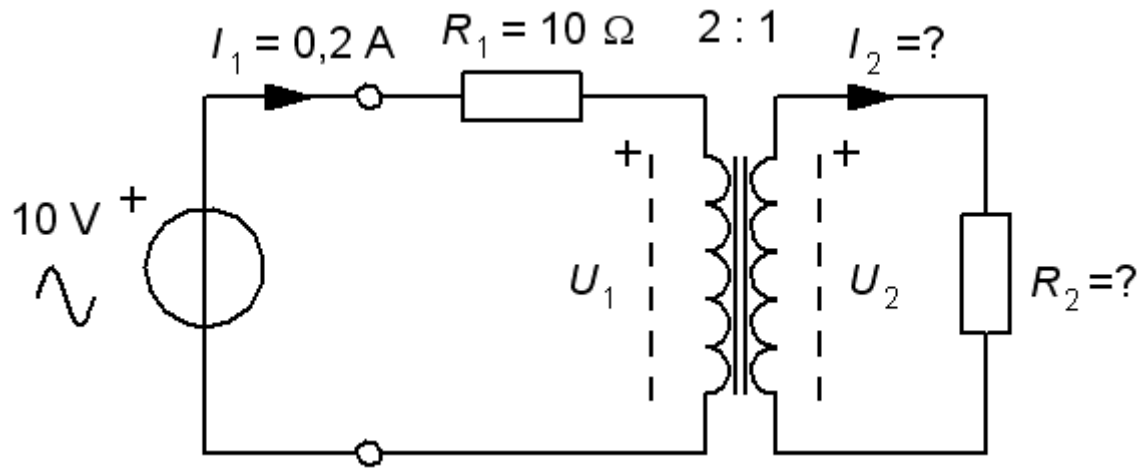


# Transformer (15.4)



$$\frac{U_1}{U_2} = \frac{2}{1} \Rightarrow$$
$$\frac{I_1}{I_2} = \frac{1}{2}$$

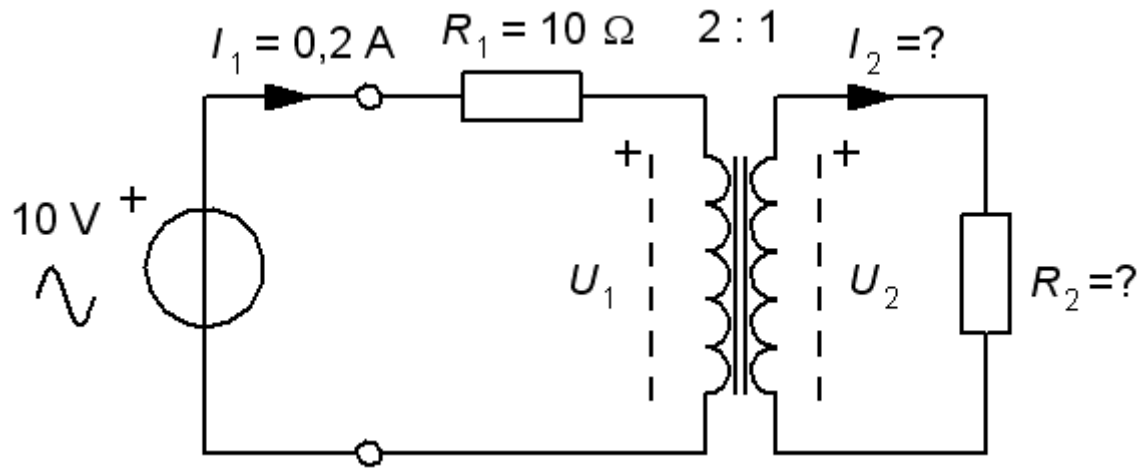
# Transformer (15.4)



$$\frac{U_1}{U_2} = \frac{2}{1} \Rightarrow$$
$$\frac{I_1}{I_2} = \frac{1}{2}$$

$$10 - R_1 \cdot I_1 - U_1 = 0 \Rightarrow U_1 = 10 - 0,2 \cdot 10 = 8$$

# Transformer (15.4)

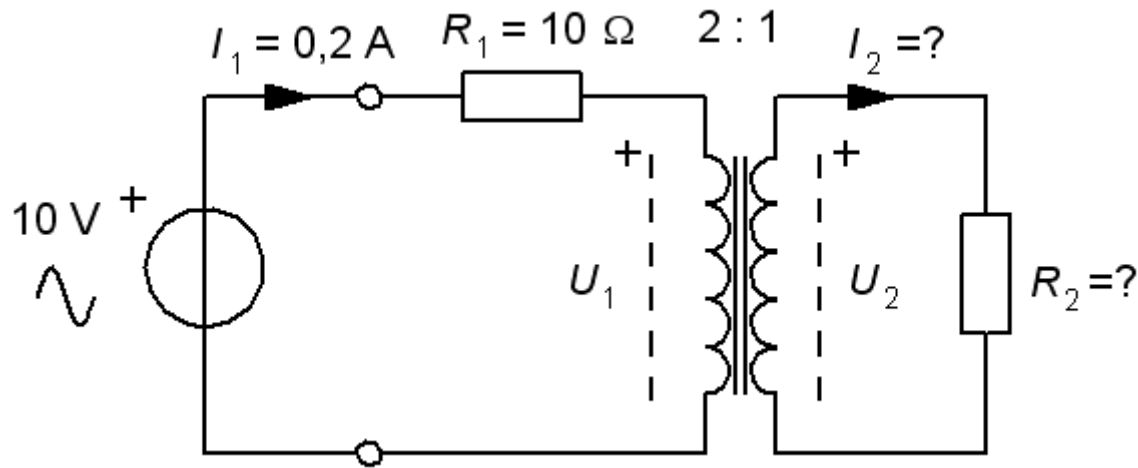


$$\frac{U_1}{U_2} = \frac{2}{1} \Rightarrow$$
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$$10 - R_1 \cdot I_1 - U_1 = 0 \Rightarrow U_1 = 10 - 0,2 \cdot 10 = 8$$

$$U_2 = U_1 \cdot \frac{1}{2} = \frac{8}{2} = 4$$

# Transformatorn (15.4)



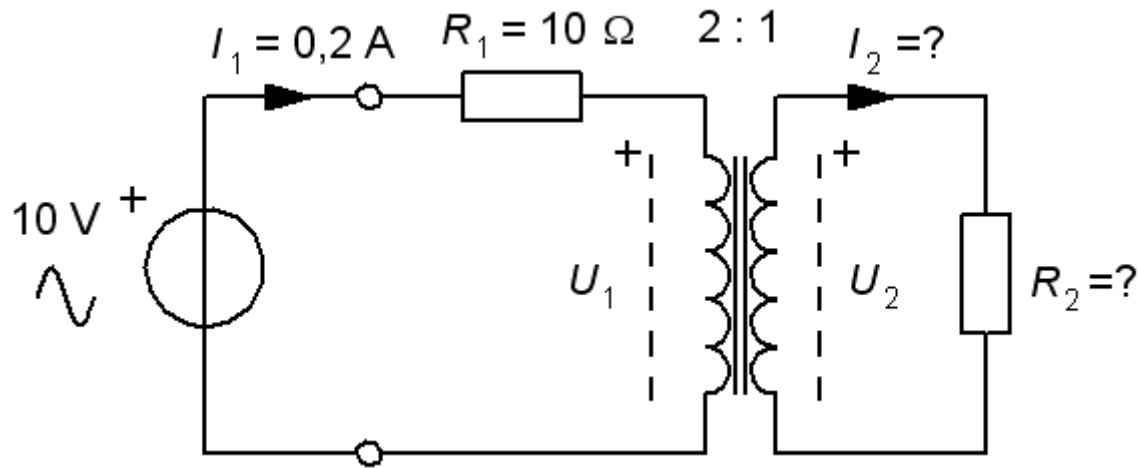
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# Transformer (15.4)



$$\frac{U_1}{U_2} = \frac{2}{1} \Rightarrow$$
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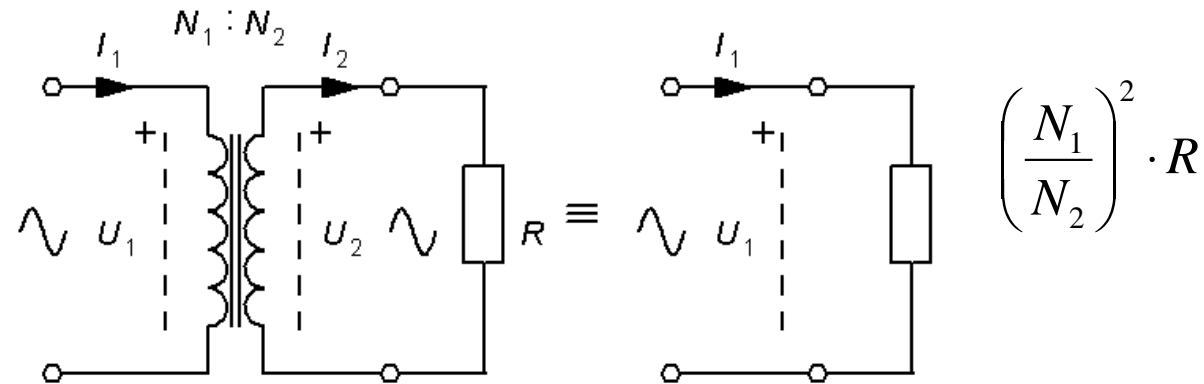
$$10 - R_1 \cdot I_1 - U_1 = 0 \Rightarrow U_1 = 10 - 0,2 \cdot 10 = 8$$

$$U_2 = U_1 \cdot \frac{1}{2} = \frac{8}{2} = 4 \quad I_2 = I_1 \cdot \frac{2}{1} = 0,4$$

$$R_2 = \frac{U_2}{I_2} = \frac{4}{0,4} = 10 \Omega$$

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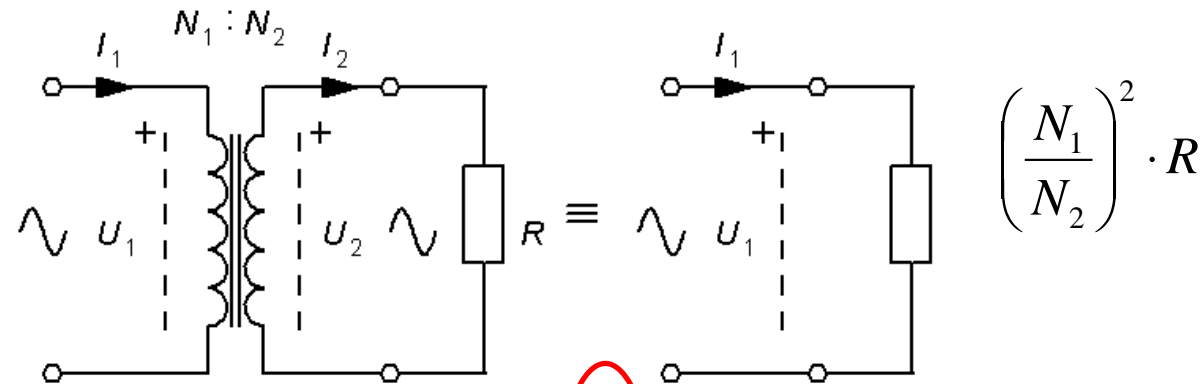
# Transforming impedances



$$R_1 = \frac{U_1}{I_1} \quad R_2 = \frac{U_2}{I_2} \quad \frac{U_1}{I_1} = \frac{\frac{N_1}{N_2} U_2}{\frac{N_2}{N_1} I_2} = \left( \frac{N_1}{N_2} \right)^2 \cdot \frac{U_2}{I_2}$$

$$R_{1 \leftarrow 2} = \left( \frac{N_1}{N_2} \right)^2 \cdot R_2$$

# Transforming impedances



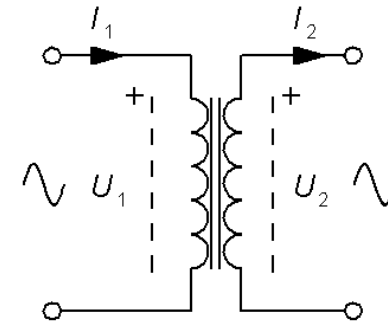
$$R_1 = \frac{U_1}{I_1} \quad R_2 = \frac{U_2}{I_2} \quad \frac{U_1}{I_1} = \frac{\frac{N_1}{N_2} U_2}{\frac{N_2}{N_1} I_2} = \left(\frac{N_1}{N_2}\right)^2 \cdot \frac{U_2}{I_2}$$

$$R_{1 \leftarrow 2} = \left(\frac{N_1}{N_2}\right)^2 \cdot R_2$$

# Ex. Transforming impedances

A transformer has the voltage ratio  
240V/120V.

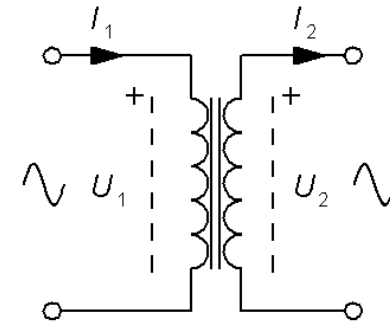
We have two capacitors  $1\mu\text{F}$  and  $16\mu\text{F}$ .  
How should one connect to get  $5\mu\text{F}$  ?



# Ex. Transforming impedances

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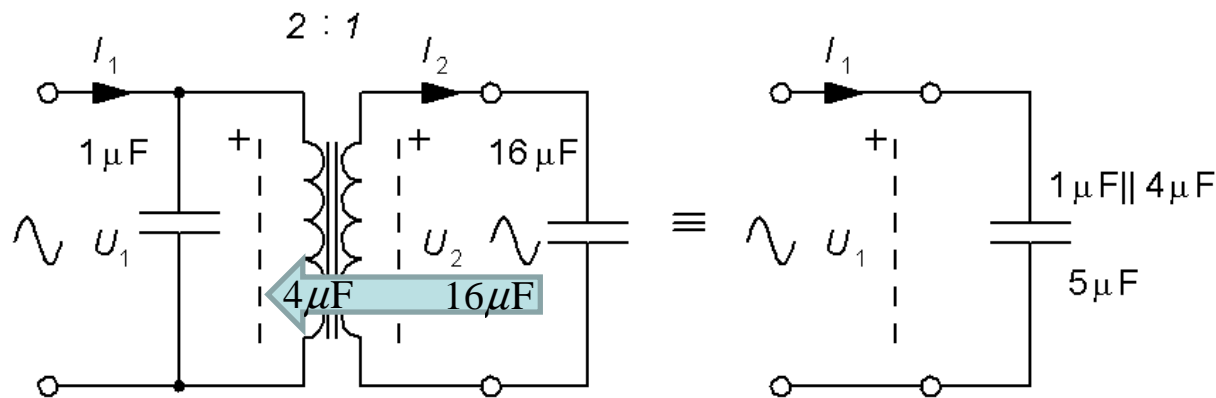
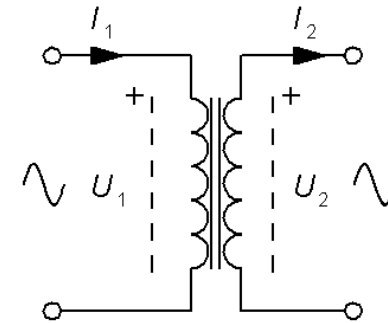


$$Z_2 = \frac{1}{\omega C} \Rightarrow$$
$$Z_{1 \leftarrow 2} = \frac{1}{\omega C} \cdot 2^2 = \frac{1}{\omega(C/4)}$$

# Ex. Transforming impedances

A transformer has the voltage ratio 240V/120V.

We have two capacitors  $1\mu\text{F}$  and  $16\mu\text{F}$ .  
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$$Z_2 = \frac{1}{\omega C} \Rightarrow$$

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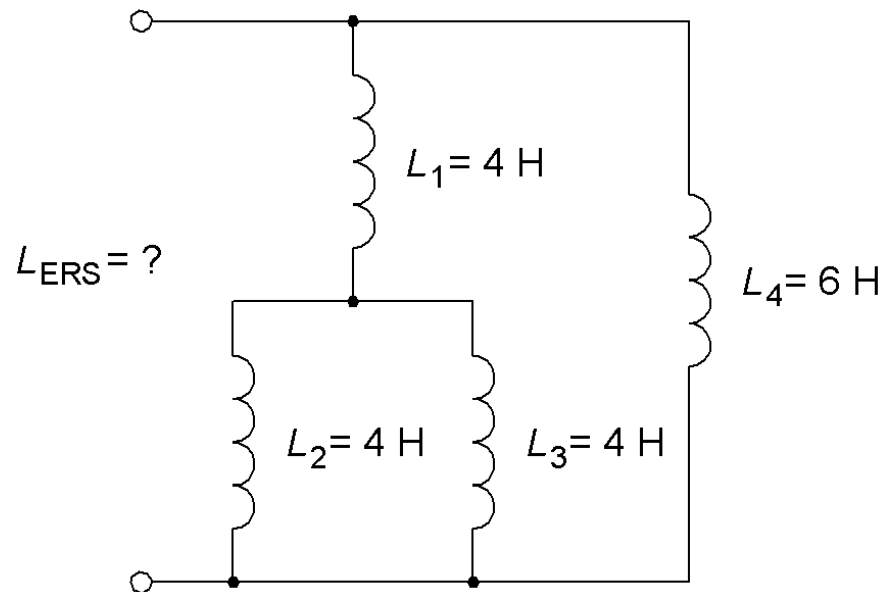
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# Series and parallel connection of inductors

(Ex. 15.6) Assuming that none of the coils parts magnetic lines of force with each other but are completely independent components, they can be treated series and parallel inductors just as if they were resistors.

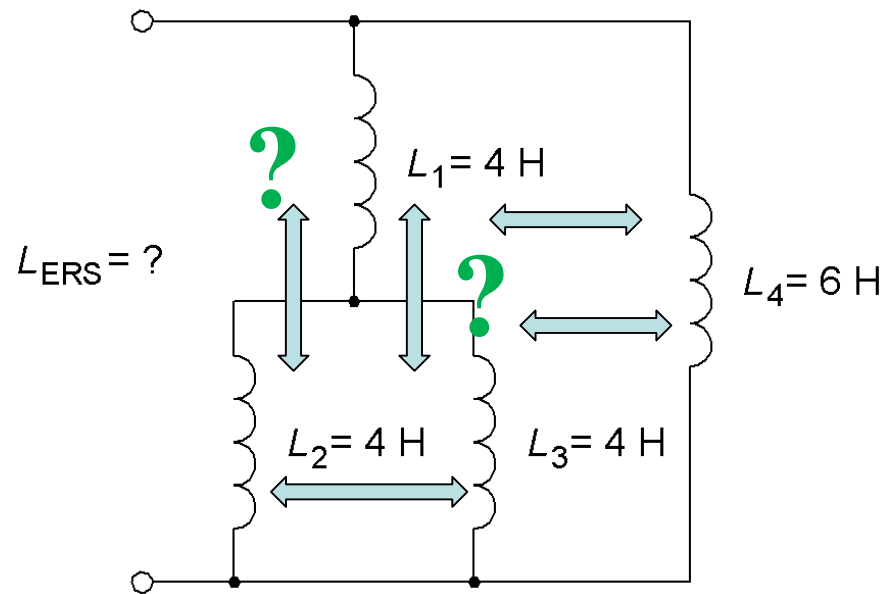
$$L_{\text{ERS}} = \frac{\left(4 + \frac{4 \cdot 4}{4 + 4}\right) \cdot 6}{4 + \frac{4 \cdot 4}{4 + 4} + 6} = 3 \text{ H}$$



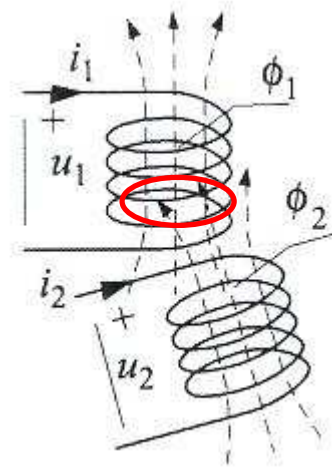
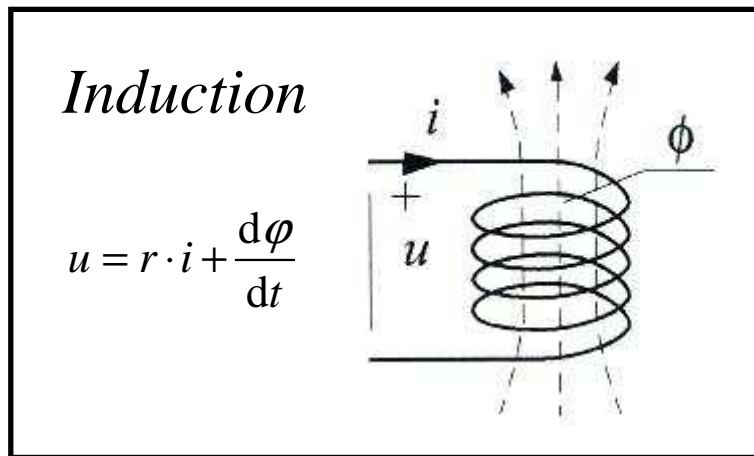
# Series and parallel connection of inductors?

*We have previously studied serial and parallel coils as if they were completely independent components that do not share magnetic lines with each other.*

We are now treating coils with interconnected flow



# Inductive coupling



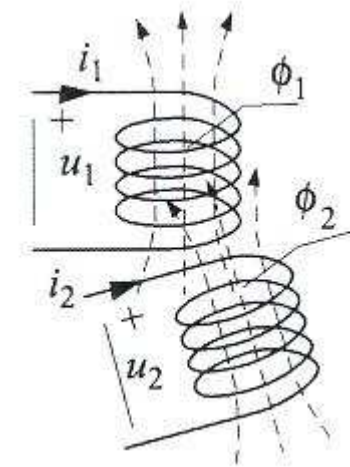
A portion of the flow in the coil 1 is interconnected with flow from the coil 2.

$$u_1 = r_1 \cdot i_1 + \frac{d\phi_1}{dt} \quad \phi_1 = i_1 \cdot L_1 + i_2 \cdot M$$

In same  
way:

$$u_2 = r_2 \cdot i_2 + \frac{d\phi_2}{dt} \quad \phi_2 = i_2 \cdot L_2 + i_1 \cdot M$$

# Inductive coupling



$\pm M$  is called mutual inductance

$$u_1 = r_1 \cdot i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$u_2 = r_2 \cdot i_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$j\omega$ -method:

$$U_1 = r_1 \cdot I_1 + j\omega L_1 I_1 + j\omega M I_2$$

$$U_2 = r_2 \cdot I_2 + j\omega L_2 I_2 + j\omega M I_1$$

An ideal transformer has  
coupling factor  $k = 1$  (100%)

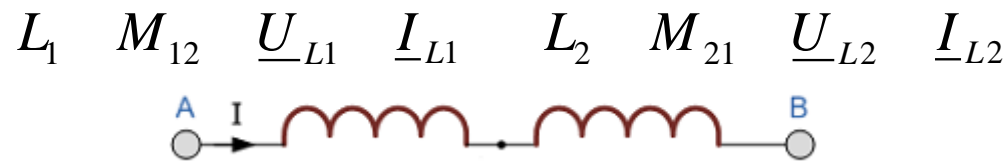
Coupling factor:

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

*The coupling factor indicates  
how much of the flow a coil has  
in common with another coil*

# Series with mutual inductance

*Derive:*



$$\underline{U}_{L1} = j\omega L_1 \underline{I}_{L1} \pm j\omega M_{12} \underline{I}_{L2} \quad \underline{U}_{L2} = j\omega L_2 \underline{I}_{L2} \pm j\omega M_{21} \underline{I}_{L1}$$

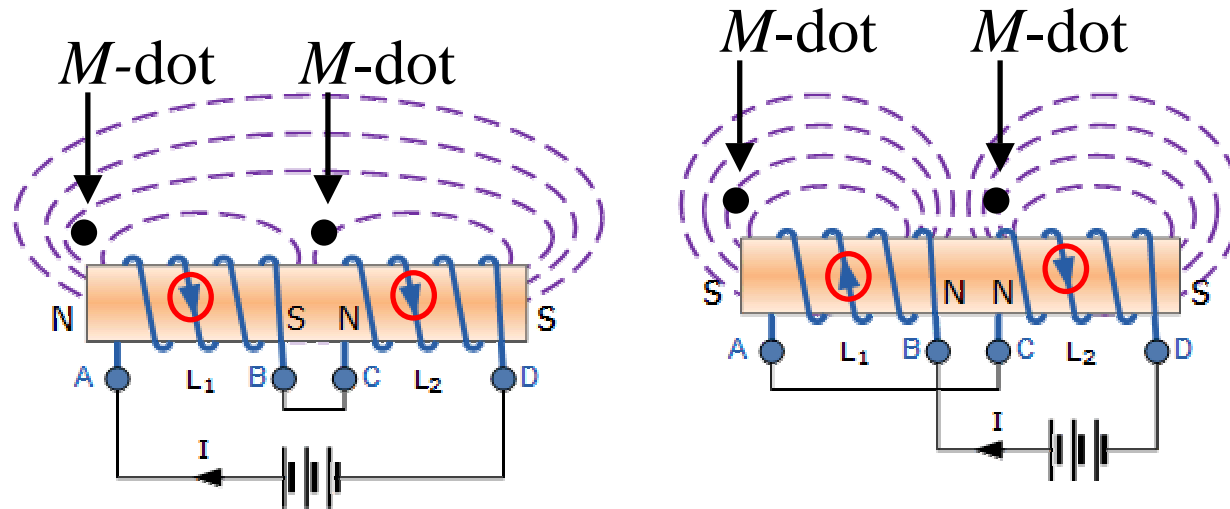
Series connection has the same current

$$\underline{I}_{L1} = \underline{I}_{L2} = \underline{I} \quad \underline{U} = \underline{U}_{L1} + \underline{U}_{L2} \quad M_{12} = M_{21} = M \quad \Rightarrow$$

$$\underline{U} = \underline{I} \cdot j\omega(L_1 \pm M + L_2 \pm M)$$

$$\frac{\underline{U}}{\underline{I}} = j\omega(L_1 + L_2 \pm 2M)$$

# Series with mutual inductance



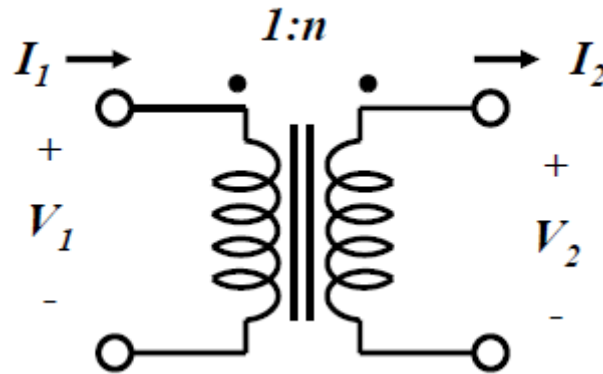
Series connection has the same current  $I_1 = I_2 = I$

$$L_{TOT} = L_1 + L_2 + 2M$$

$$L_{TOT} = L_1 + L_2 - 2M$$

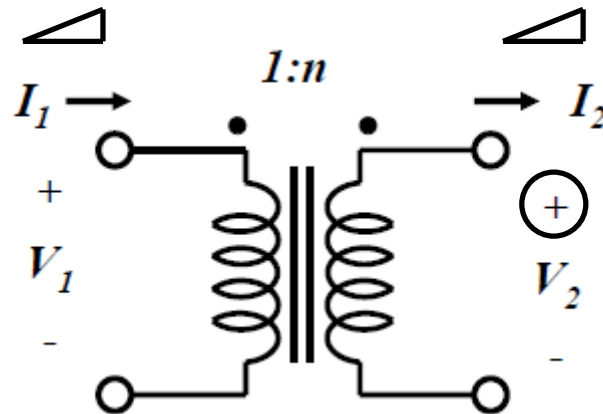
$M$  can contribute or counter act to the flow, this gives  $\pm$  sign. Therefore, coil winding polarity is usually indicated by a dot convention in schematics.

# ”Dot” convention



An increasing current *in* to a dot results in induced voltages with directions that would give increasing currents **out** of other dots.

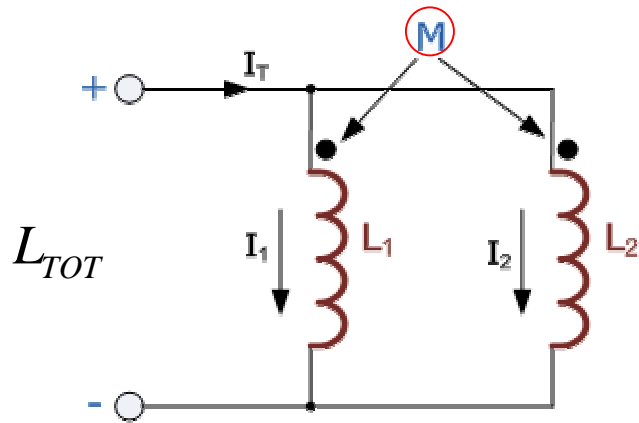
# ”Dot” convention



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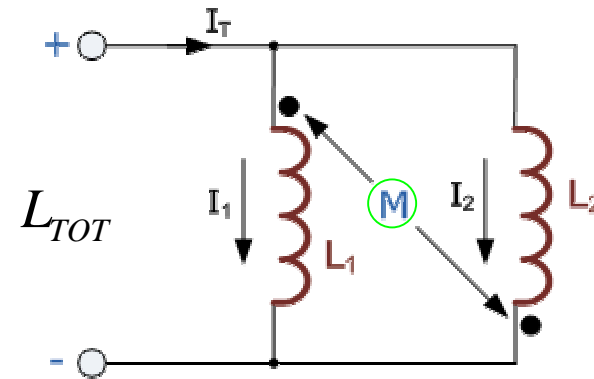


# In parallel with mutual inductance



Parallel connected coils

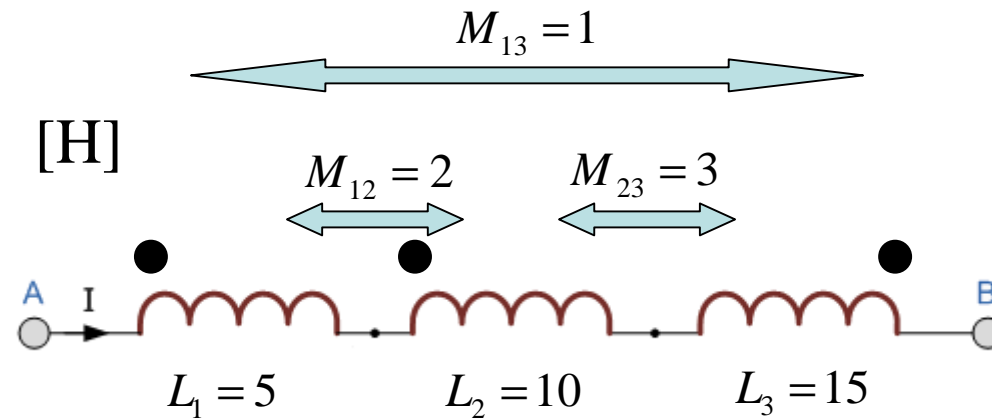
$$L_{TOT} = \frac{L_1 \cdot L_2 - M^2}{L_1 + L_2 \ominus 2M}$$



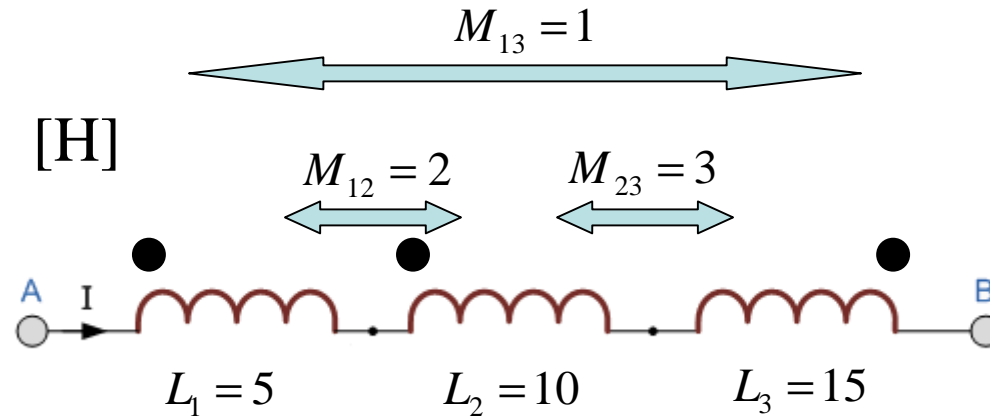
**Antiparal** conected coils

$$L_{TOT} = \frac{L_1 \cdot L_2 - M^2}{L_1 + L_2 \oplus 2M}$$

# Ex. 15.7 Series connection



# Ex. 15.7 Series connection



$$L_{TOT} =$$

$$L_1 + M_{12} - M_{13} +$$

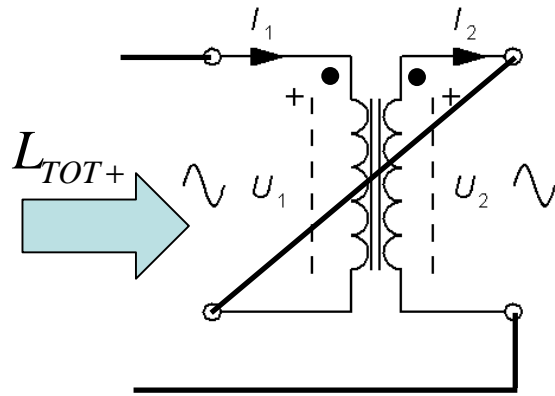
$$L_2 + M_{12} - M_{23} +$$

$$L_3 - M_{23} - M_{13} =$$

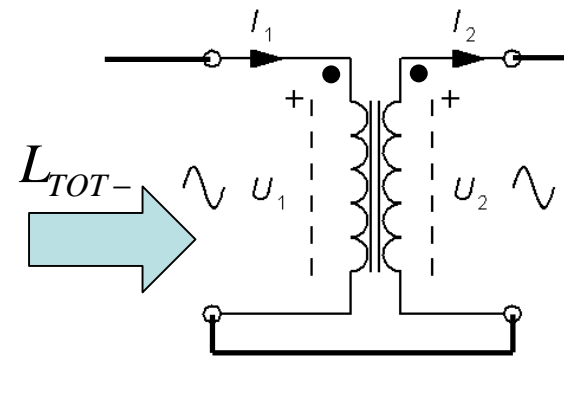
$$= 5 + 2 - 1 + 10 + 2 - 3 + 15 - 3 - 1 = 26 \text{ [H]}$$

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# Measuring the mutual inductance?

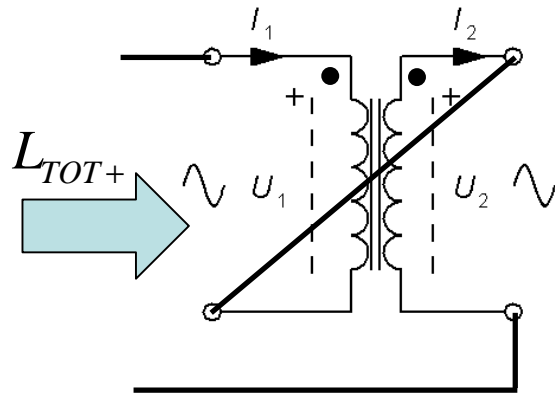


$$L_{TOT+} = L_1 + L_2 + 2M$$

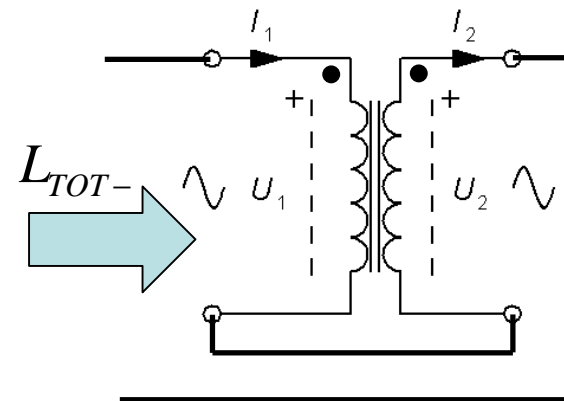


$$L_{TOT-} = L_1 + L_2 - 2M$$

# Measuring the mutual inductance?



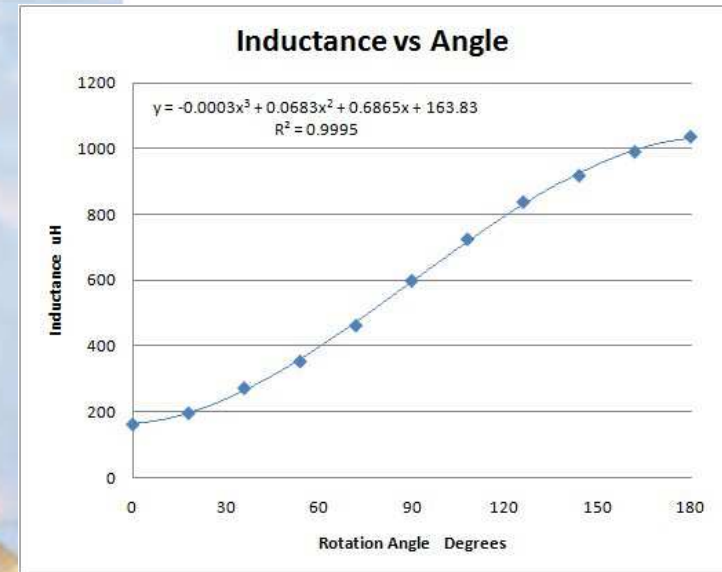
$$L_{TOT+} = L_1 + L_2 + 2M$$



$$L_{TOT-} = L_1 + L_2 - 2M$$

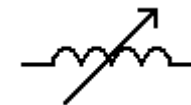
$$M = \frac{L_{TOT+} - L_{TOT-}}{4}$$

# Variometer (to an antique radio)



$$L_{TOT} = L_1 + L_2 \pm 2M$$

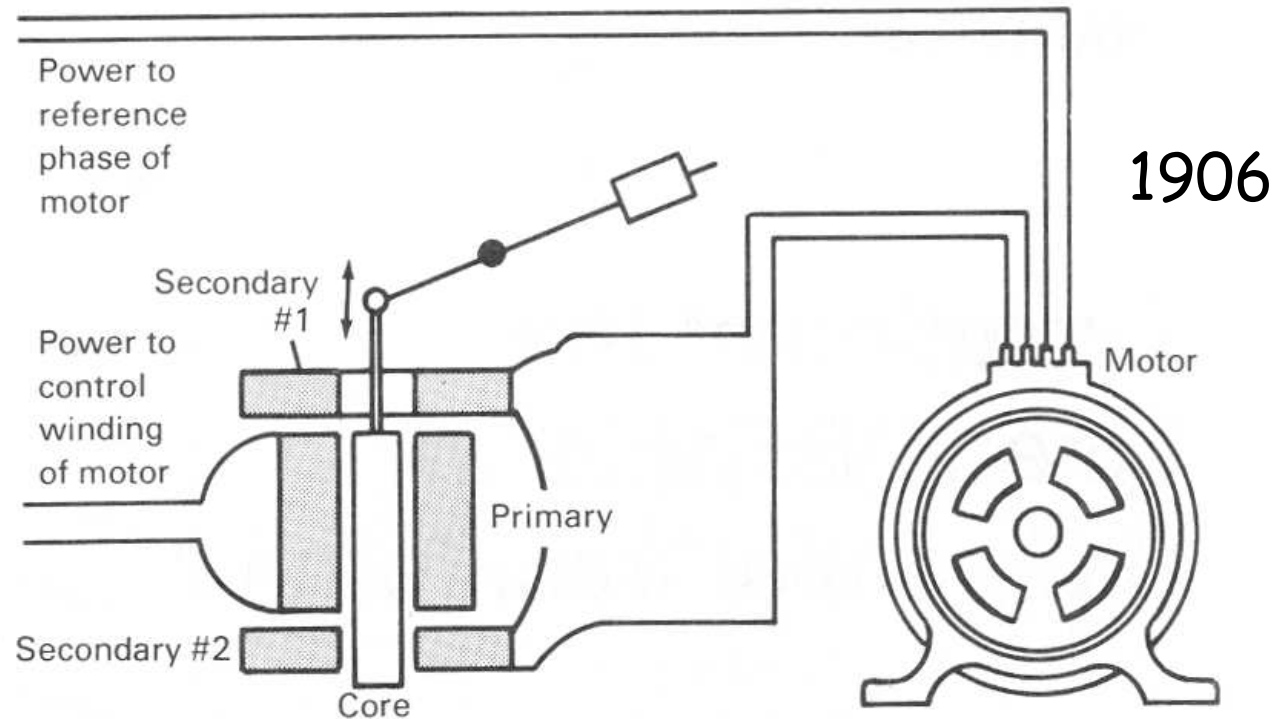
$$M = f(\alpha)$$



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# A bad actuator can become a good sensor



*Porter & Currier patent (simplified), the earliest variable differential transformer.*

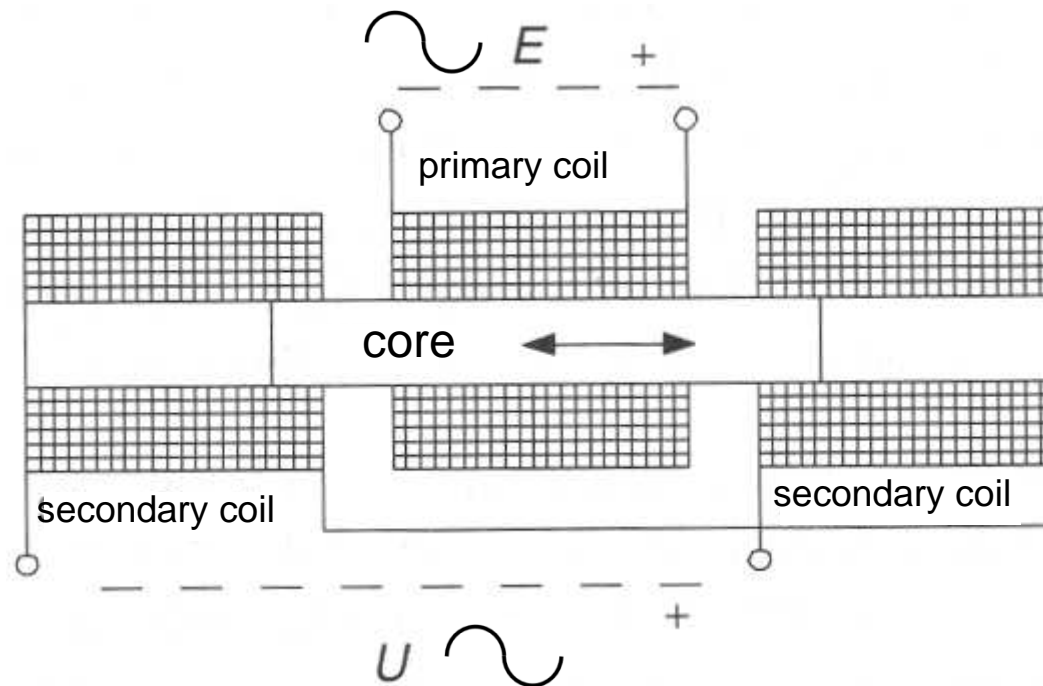
# The industry's "rugged" position sensor



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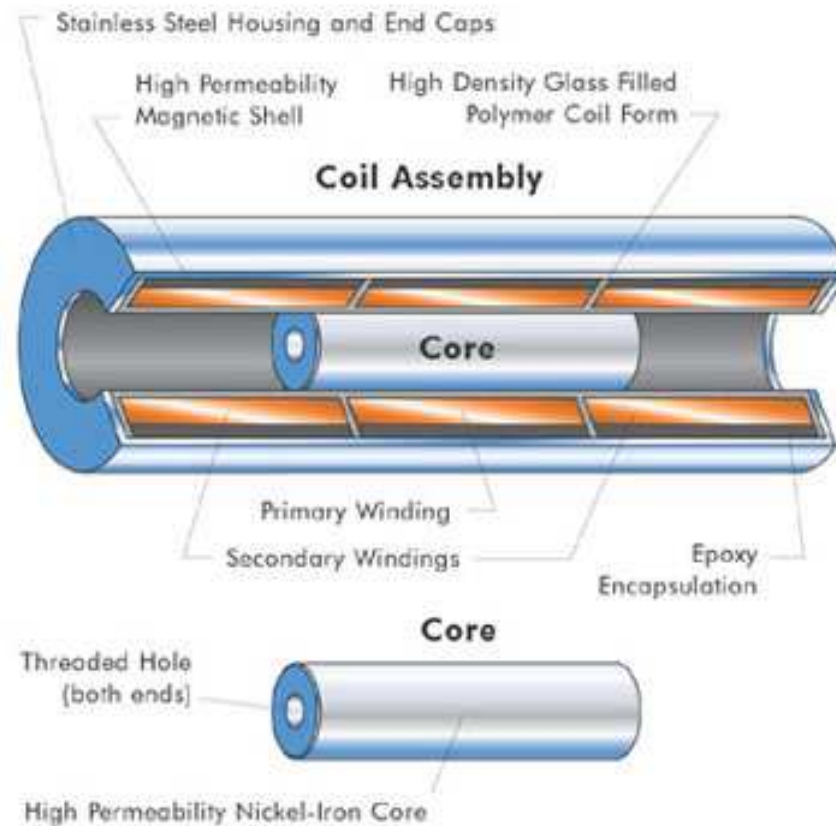
# Differential transformer

LVDT Linear Variable Differential Transformer

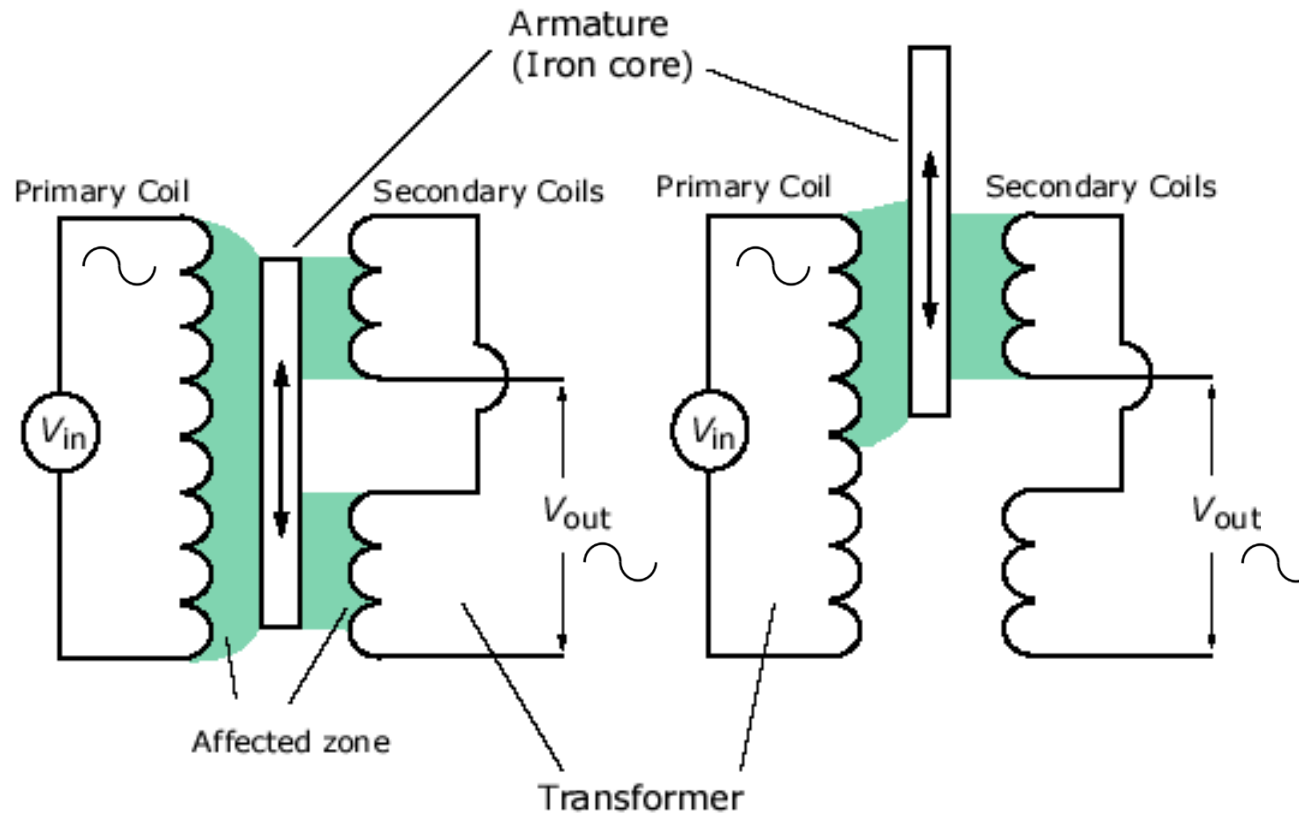


The secondary coils are connected in series but with opposite polarity – when the core is in the middle  $U = 0$ .

# LVDT design

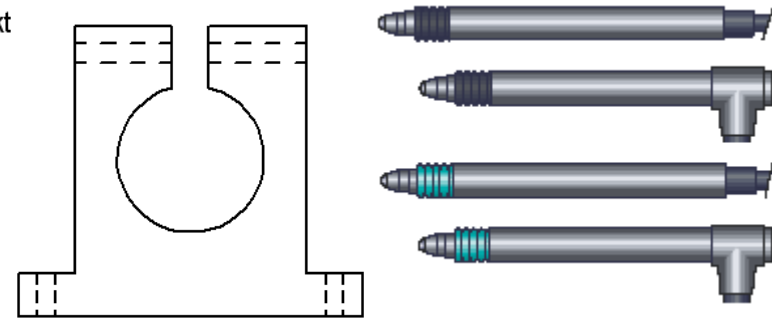
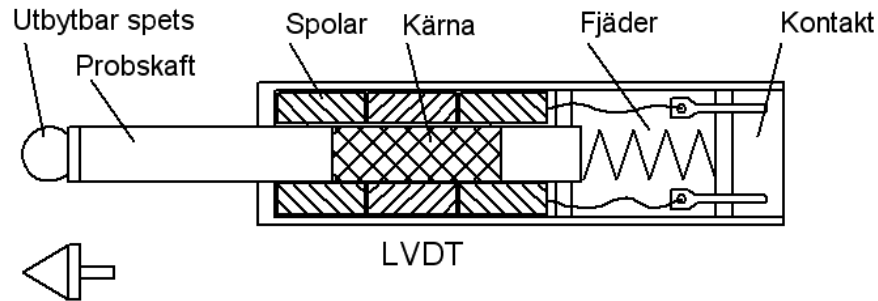


# LVDT principle

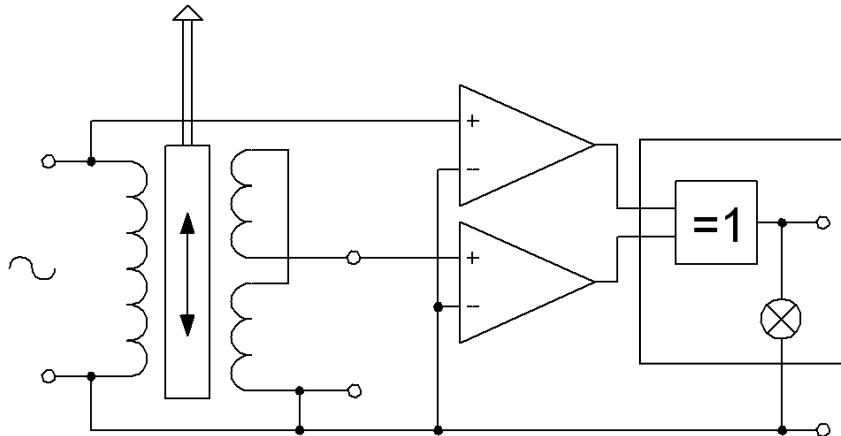


The output voltage is relatively high – it makes this a popular sensor ...

# LVDT probe

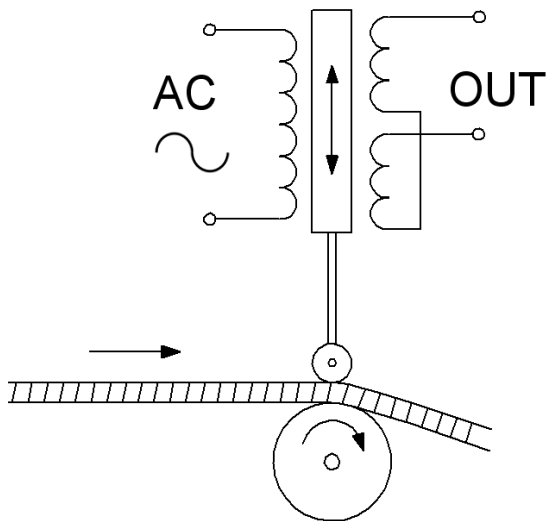


Monteringsblock



Output signal changes **phase 180°** exactly when the core pass the middle point.

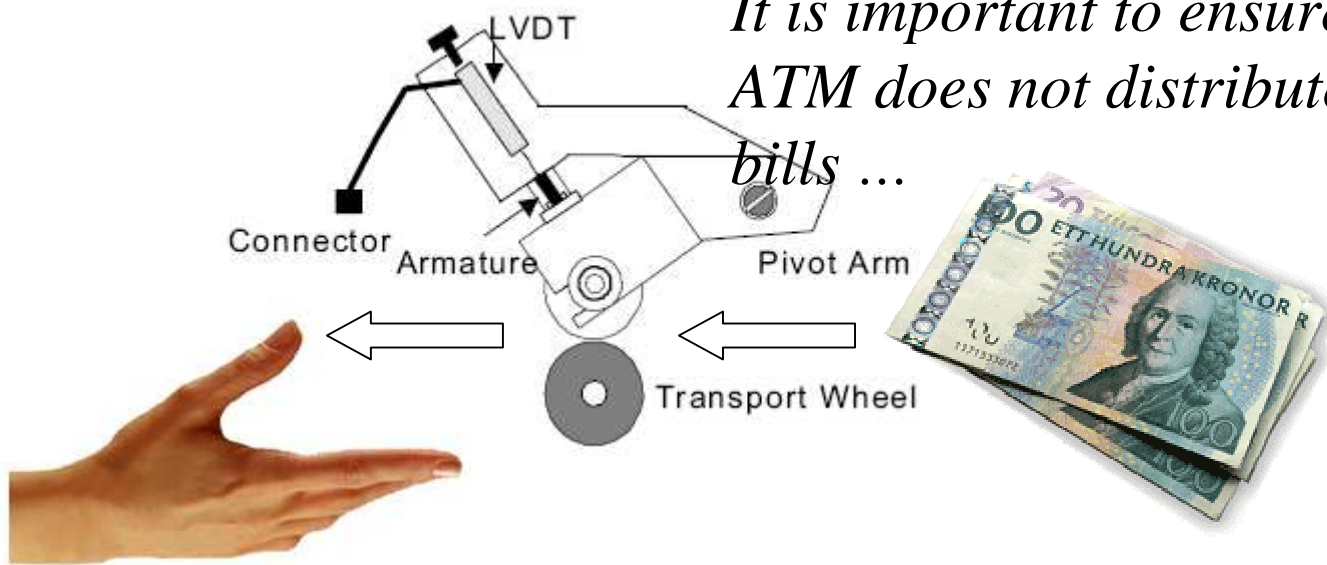
A **XOR-gate** kan **indicate** this change.



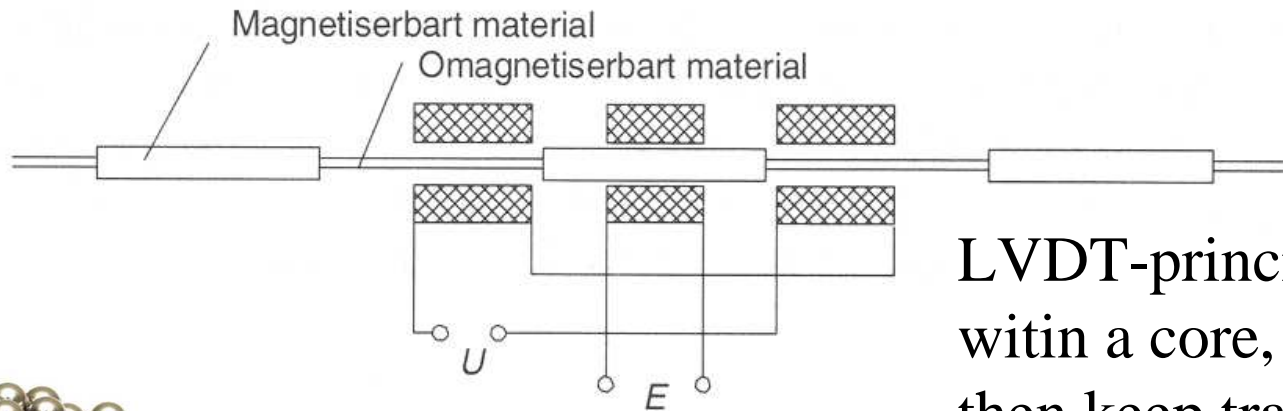
A LVDT probe can keep track on that the thickness is correct.

*Guess application?*

*It is important to ensure that the ATM does not distribute "double" bills ...*



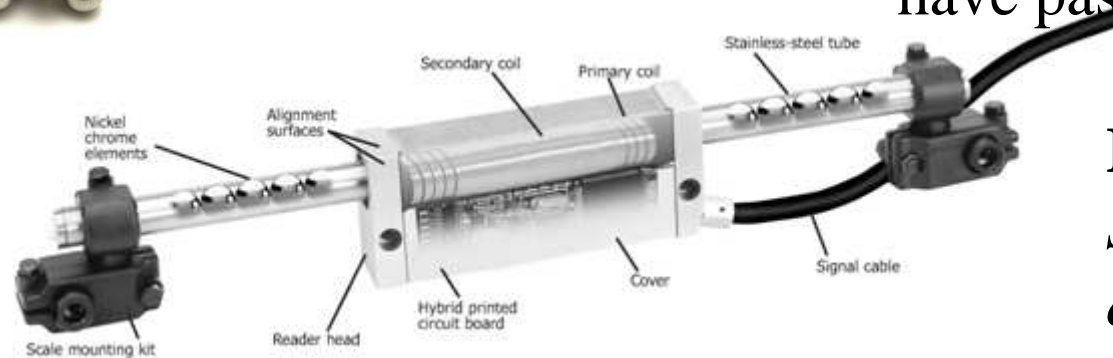
# Periodic differential transformer



LVDT-principel  
witin a core, and  
then keep track on  
how many cores that  
have passed.



A similar sensor?



**Renywell**  
*Spherical*  
*encoder*



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