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DH2323 DGI15

INTRODUCTION TO
COMPUTER GRAPHICS AND INTERACTION

RAYTRACING

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Based on DGI12 notes by Carl Henrik Ek

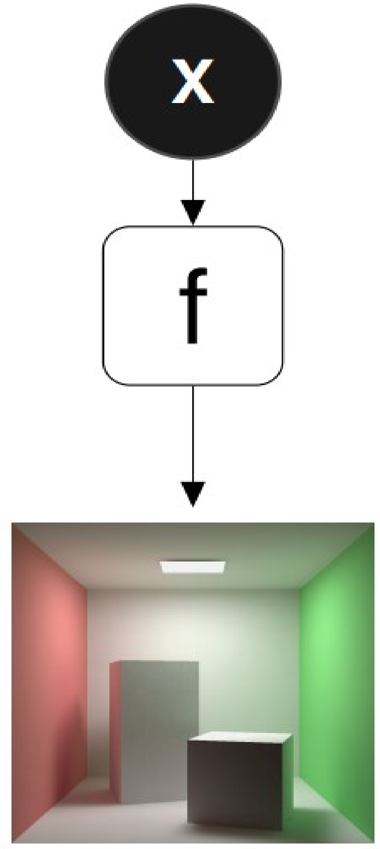
Image Synthesis

In computer graphics, create images based on a *model*

Recall:

An underlying process generates observations

Can control generation through parameters



Nice Results

"Pebbles" by Jonathan Hunt

"Bonsai Life" by Jeremy M. Praay

"Glasses" by Gilles Tran

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Nice Results

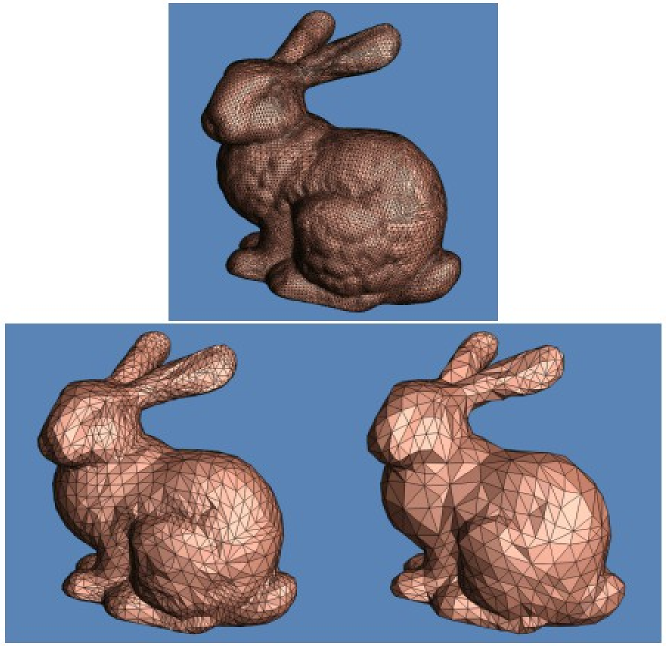
"Distant Shores" by Christoph Gerber

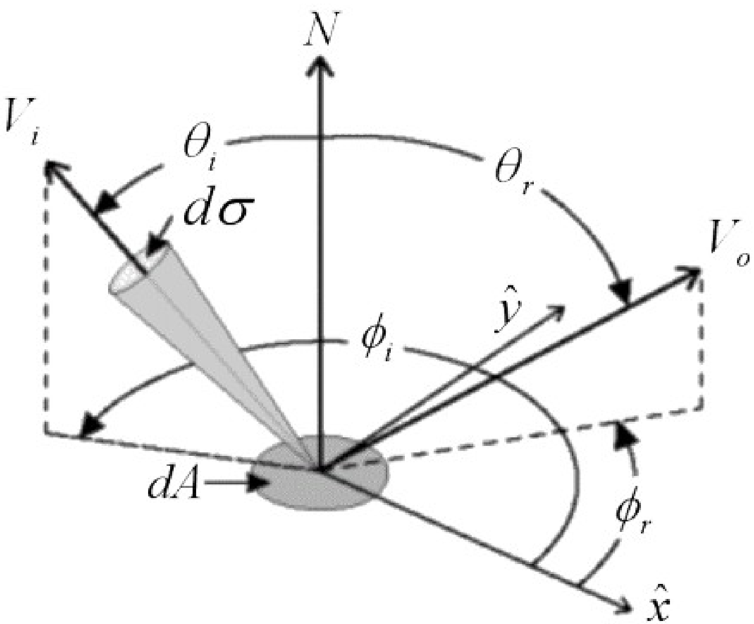
"Christmas Baubles" by Jaime Vives Piqueres

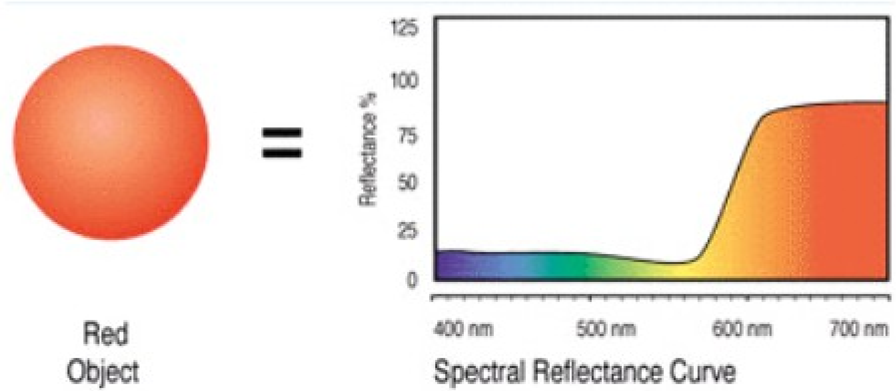
"Still with Bolts" by Jaime Vives Piqueres

Some Constituents I

- Light
- Geometry
- Surface properties
- Anything else?

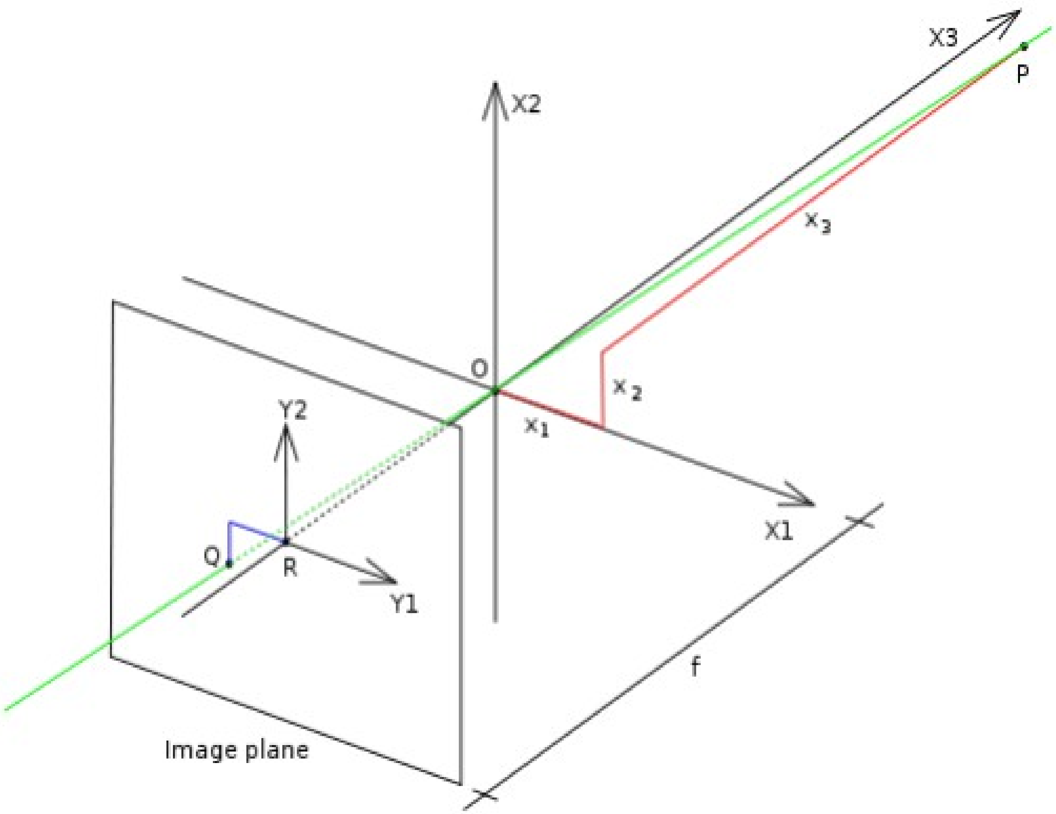


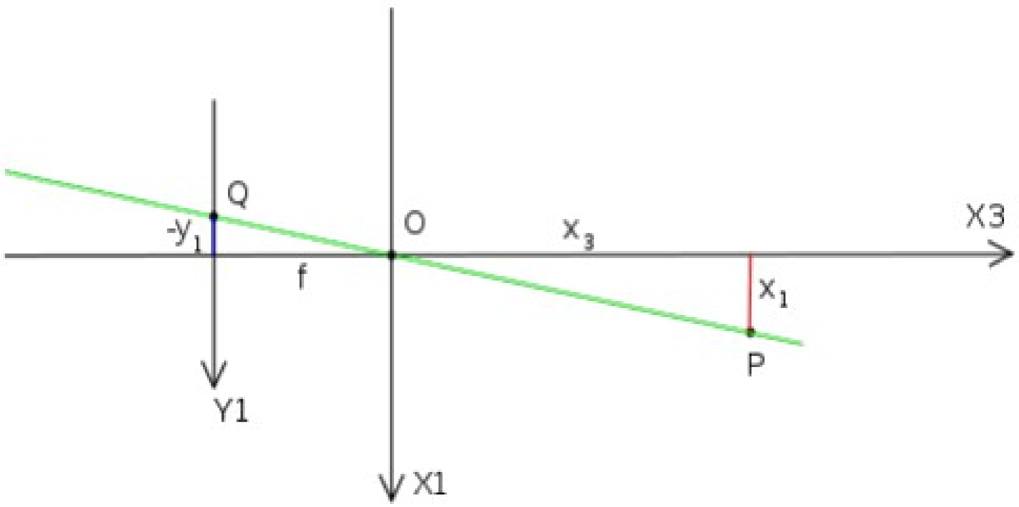




Some Constituents II

- Camera Model (pinhole)





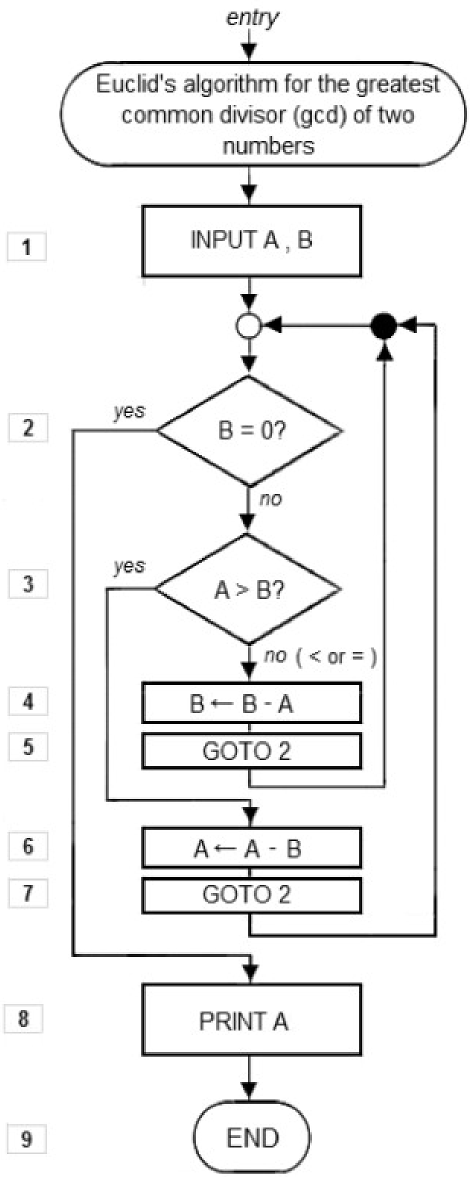
Idea 1

- Use the concept of *light rays* for modelling transport of light
- Define light sources that emit rays
- Test for intersections between rays and geometric shapes in the scene
- When a ray hits the surface of a shape
  - See how much light energy bounces i.e. is *reflected*
- When a ray hits the image plane of the camera
  - Render a colour to the screen

Question

How do we go from this *idea* to a computational model?

i.e. how to define an algorithm or step-by-step procedure



Answer: Remember this?

Interactive computer graphics is essentially:

Mathematics programming

“It's matrices all the way down!”

Quite possibly the most fun and rewarding maths programming you will ever do*

**disclaimer: you'll get from it what you put in*

Maths Programming Toolkit

- Define a toolkit of mathematical operations
- Each will be important for our final algorithm
- Important keywords from idea 1:
  - Light rays from light sources
  - Intersections with geometry
  - Bounces/reflects off surfaces
  - Render to screen

What is a 'Light Ray'?

- Concept
  - Idealised narrow beam of light (optics)
  - Discrete, particles
- Geometrically speaking:
  - Similar in some ways to a straight line
  - Has a starting point and direction
  - But extends infinitely in defined direction
- Mathematically:

$$\mathbf{r}_0 = [x_0, y_0, z_0]^{\mathrm{T}}$$
$$\mathbf{r}_d = [x_d, y_d, z_d]^{\mathrm{T}}, \ ||\mathbf{r}_d|| = 1$$
$$\mathbf{r}_t = \mathbf{r}_0 + t \cdot \mathbf{r}_d$$

One degree-of-freedom

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Where do they come from?

- Emitted from light sources
- Parameterised
  - Position
  - Colour
  - Intensity

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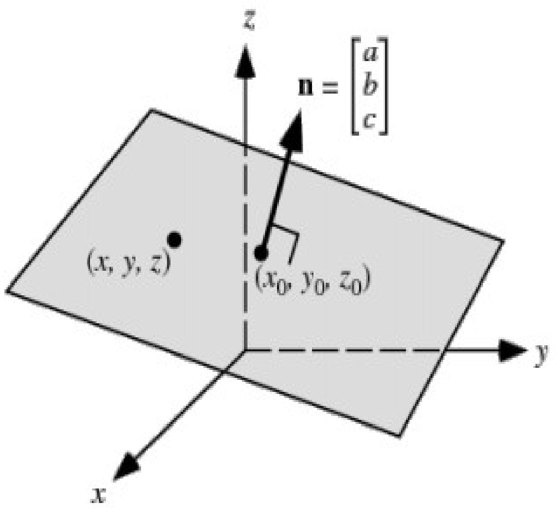
Intersections

- Need to be able to test for intersections between rays and scene geometry (objects)
- Different types of geometry:
  - Planes
  - Spheres
  - Triangles
- Resolving intersections involve solving equations

Ray-plane Intersection

- Plane defined as:

  - Plane normal $\mathbf{n} = [a, b, c]$
  - Unit normal $||n||_2 = 1$
  - d offset to origin
  - Equation $a \cdot x + b \cdot y + c \cdot z + d = 0$
  - *Two degrees-of-freedom*



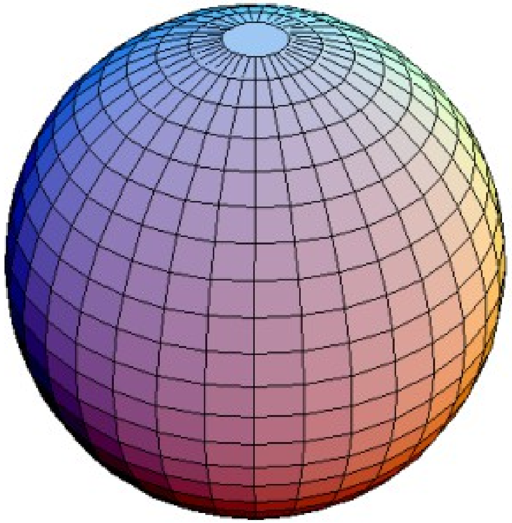
- Intersection point:

$$\mathbf{p}_i = \mathbf{r}_0 - \frac{\mathbf{n}^{\mathrm{T}}\mathbf{r}_0 + d}{\mathbf{n}^{\mathrm{T}}\mathbf{r}_d} \cdot \mathbf{r}_d$$

Ray-sphere Intersection

- Sphere defined as:

  - Center of sphere $\mathbf{x}_c = [x_c, y_c, z_c]^{\mathrm{T}}$
  - Radius r
  - $(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 = r^2$



- Intersections:

$$\mathbf{p}_i = -\mathbf{r}_d^{\mathrm{T}}(\mathbf{r}_0 - \mathbf{r}_c) \pm \sqrt{\mathbf{r}_d^{\mathrm{T}}(\mathbf{r}_0 - \mathbf{r})_c - (\mathbf{r}_0 - \mathbf{x}_c)^{\mathrm{T}}(\mathbf{r}_0 - \mathbf{x}_c) + \mathbf{r}^2}$$

Worked Example

(recommend to work this out later using a pen and paper)

Find the intersections, if any, between the Ray with $r_0 = (0,2,0)^T$, $r_d = (0,-1,0)^T$ and the

Sphere with $x_c = (0,0,0)^T$, $r = 1$

Apply the quadratic formula $t = -b +/- \text{sqrt}(b^2-4ac)/(2a)$ to find two solutions, where:

$a = r_d.r_d$

$b = 2r_d.(r_0-x_c)$

$c = (r_0-x_c).(r_0-x_c)-r^2$

The value of b^2-4ac indicates how many roots the equation has, where negative number indicates no intersections between the ray and sphere, a zero indicates a single intersection on the edge of the sphere and a positive number indicates two intersections where the ray enters and exits the sphere. In this example, b^2-4ac is positive indicating **two intersections**.

Apply formula; $r_d.r_dt^2+2r_d.(r_0-x_c)t+(r_0-x_c).(r_0-x_c)-r^2=0$

Entering the above value gives $\mathbf{t^2-4t+3=0}$

=> t = 3 and t = 1

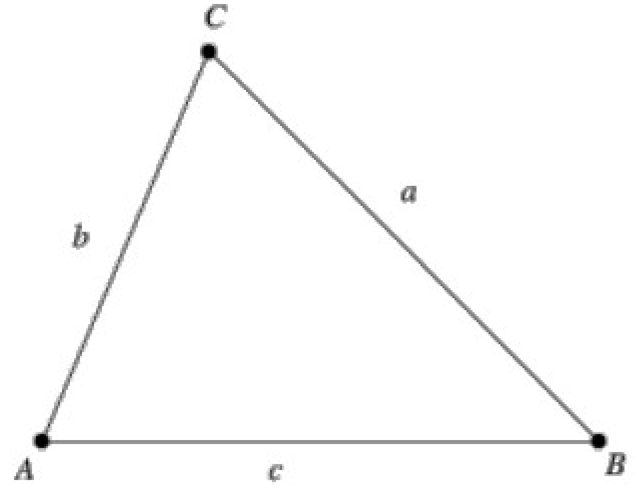
Recalling ray equation: $r_0+t.r_d$

t=1: (0,2,0)+(0,-1,0) = **(0,1,0) … first intersection point**

t=3: (0,2,0)+(0,-3,0) = **(0,-1,0) … second intersection point**

Ray-triangle Intersection

- Triangle defined as:
  - Three vertices
    - $\mathbf{t}_i = [x, y, z]^{\mathrm{T}}, i = 1 \ldots 3$



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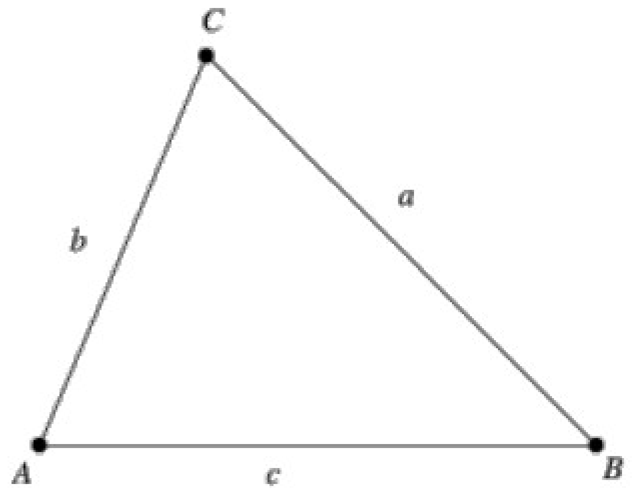
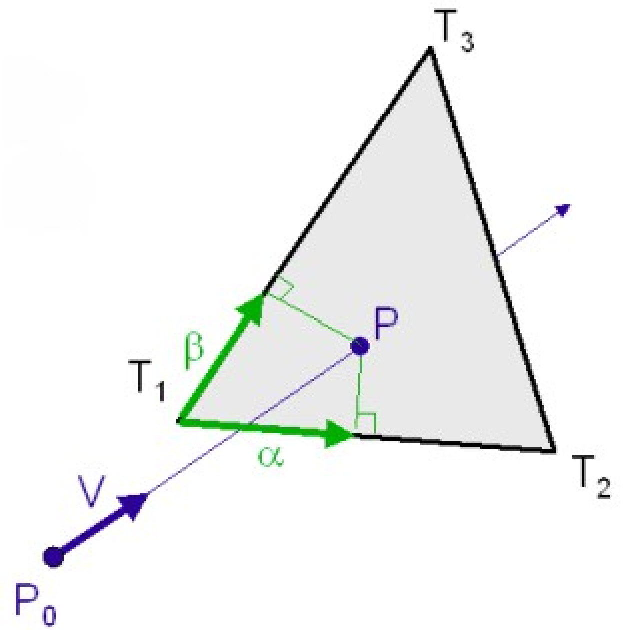


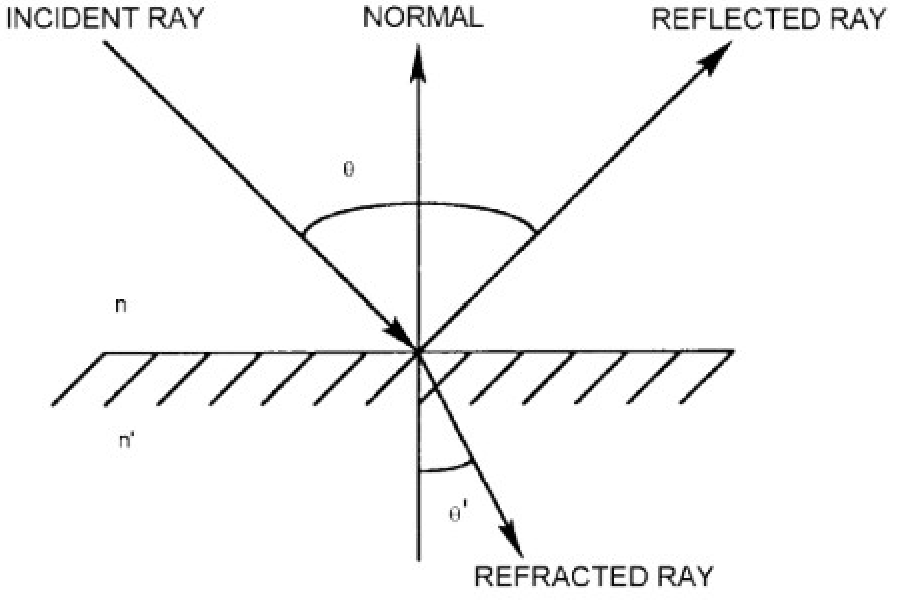
- Intersection:
  1. Check collision with plane
  2. Check if inside triangle
     - $0 \leq \alpha, \beta \leq 1$
     - $\alpha + \beta \leq 1$



Surfaces

Must also consider rays hitting and bouncing off surfaces

1. Incoming ray hits surface
2. Divided into,
   - reflected component
   - refracted component



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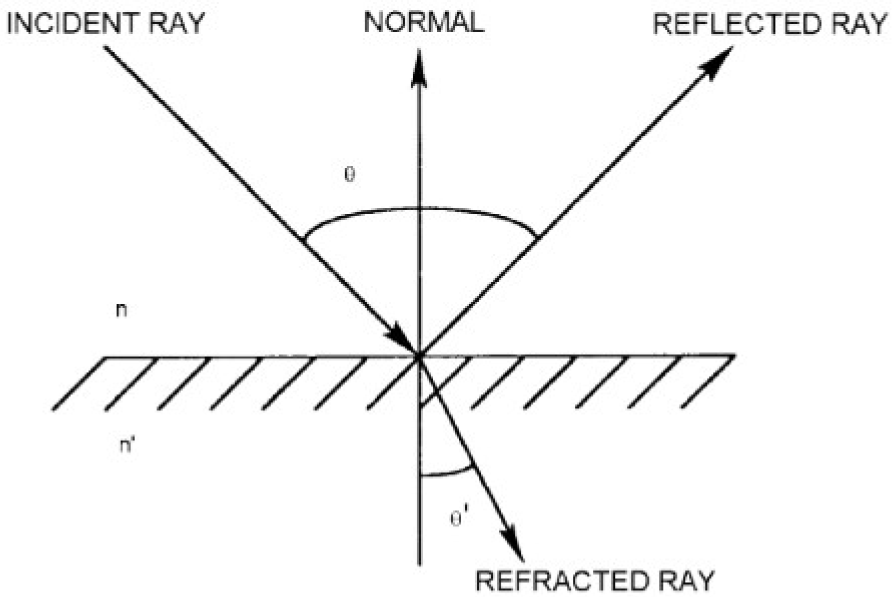
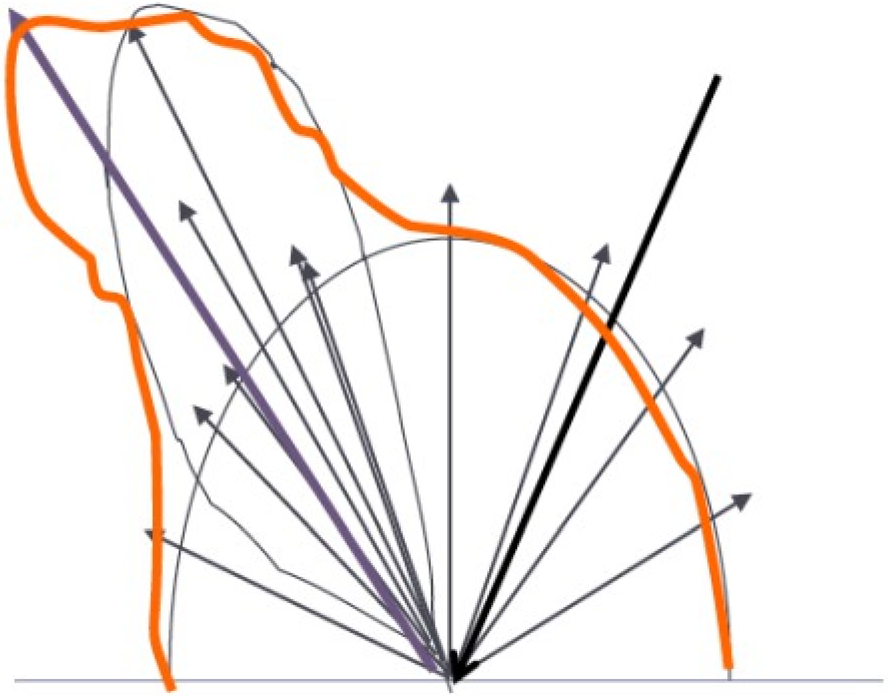


Surface types can reflect/refract rays in different ways

Accurate Reflection

Complex opaque surfaces scatter incoming light in many different directions

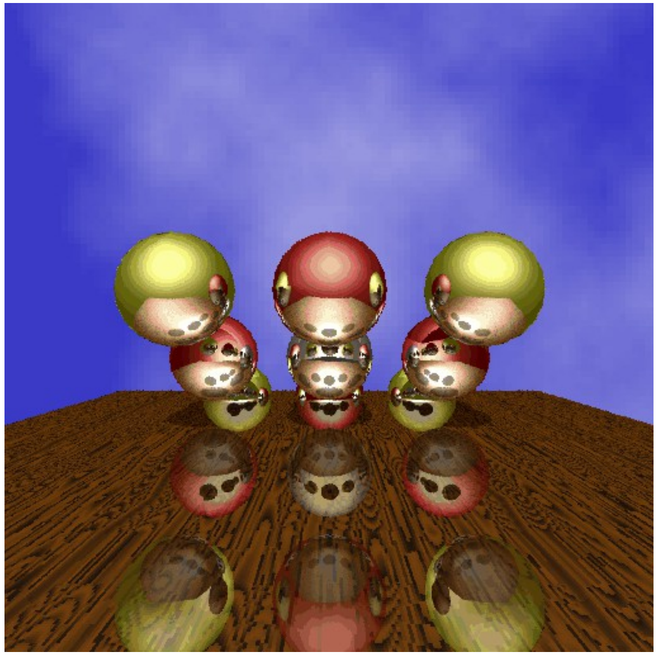
- BRDF - **B**idirectional **R**eflectance **D**istribution **F**unction



Surfaces

Specular surfaces are like mirrors

Light from a single incoming direction reflected into a single outgoing direction

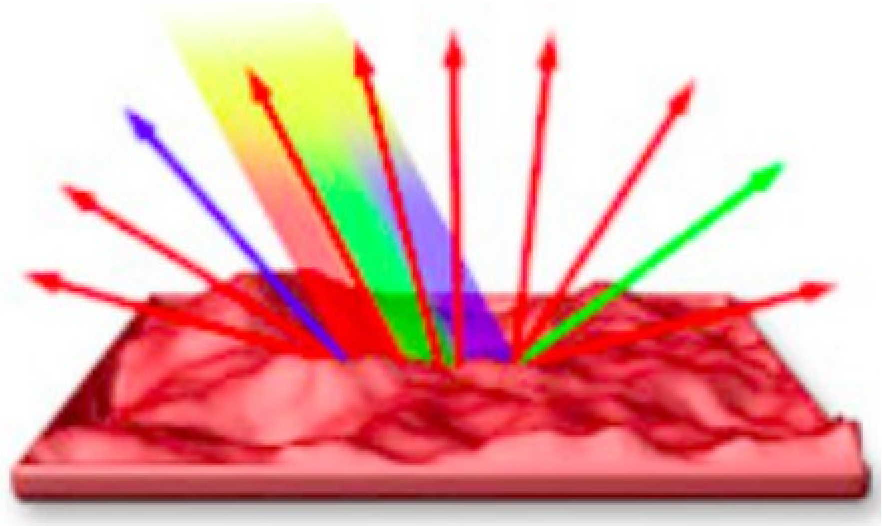


Surfaces

Diffuse (matte) surfaces are more rough

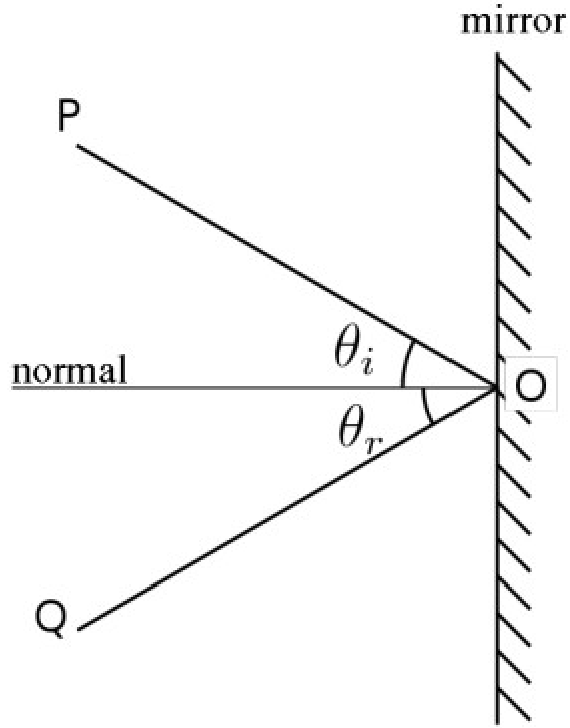
Light from a single incoming direction reflected in multitude of outgoing directions

Lambertian surface



Modelling Specular Surfaces

- Incoming ray *P*, outgoing ray *Q* and face normal are in the same plane
- Angle from normal the same between incoming and outgoing ray
- Rays *P* and *Q* are on opposite sides of the face normal



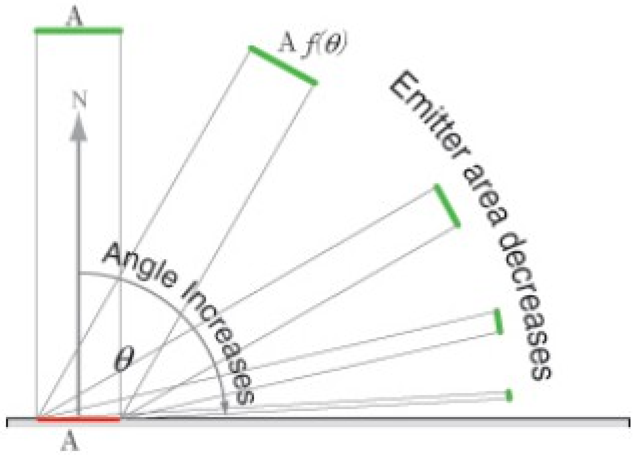
$$\mathbf{d}_o = \mathbf{d}_i - 2(\mathbf{d}_i^{\mathrm{T}} \cdot \mathbf{n})\mathbf{n}$$

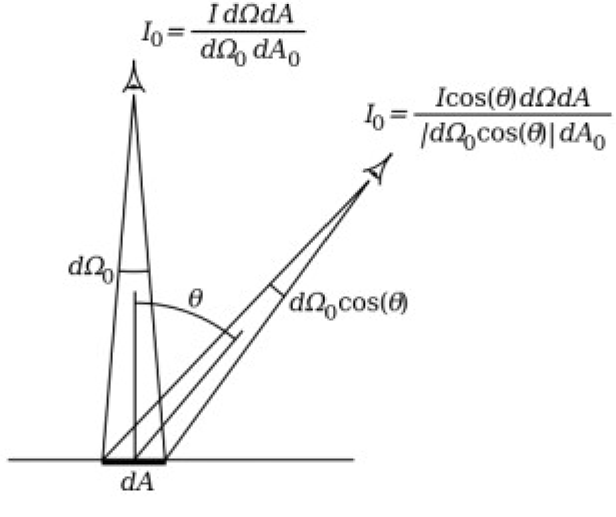
Modelling Diffuse Surfaces

Lambertian reflection

Surface is equally bright independent of viewing angle

Isotropic – uniform in all directions





Lambert's Cosine Law

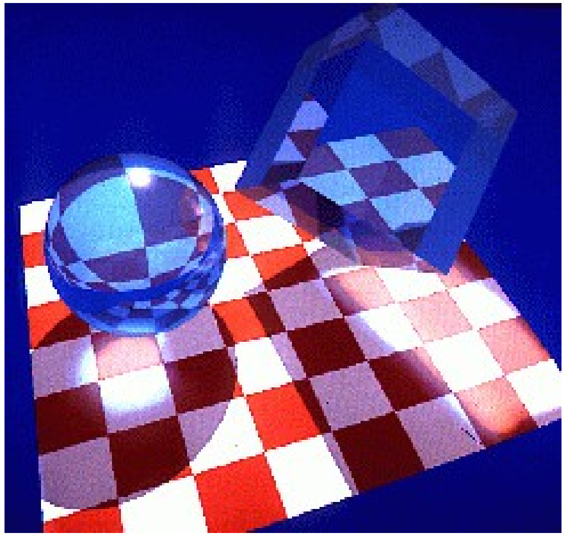
$$I_0 = I\frac{\cos(\theta)d\Omega dA}{\cos(\theta)d\Omega_0 dA_0}$$

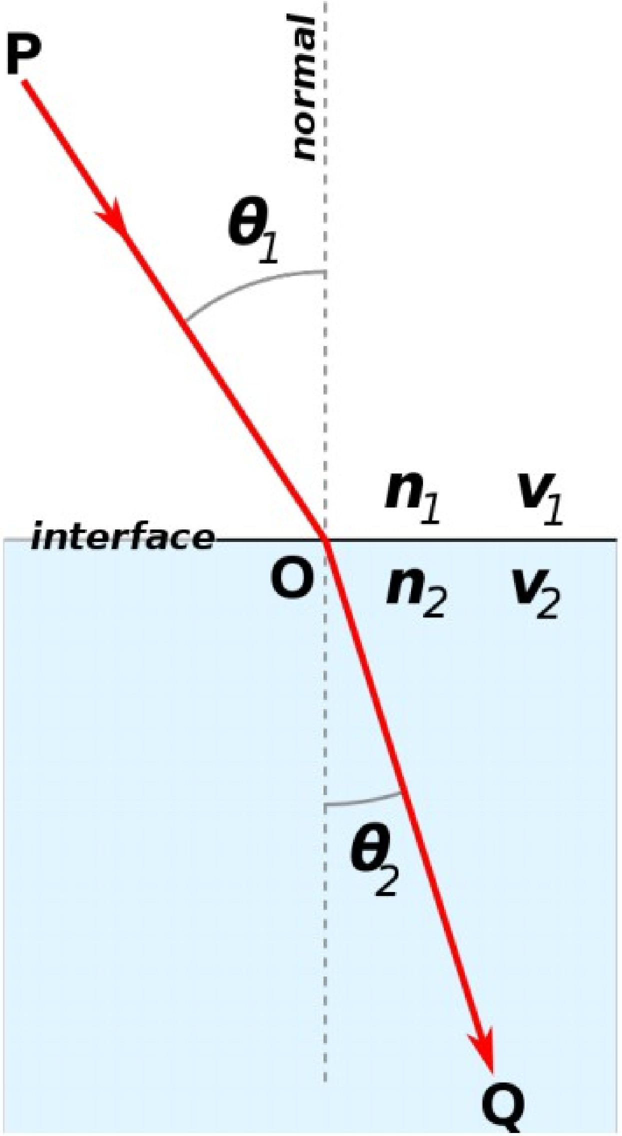
Modelling Refraction

- Snell's Law

$$\frac{\sin(\theta_1)}{\sin(\theta_2)} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

  - Refractive Index of Material





Some Light Types

Ambient Lighting

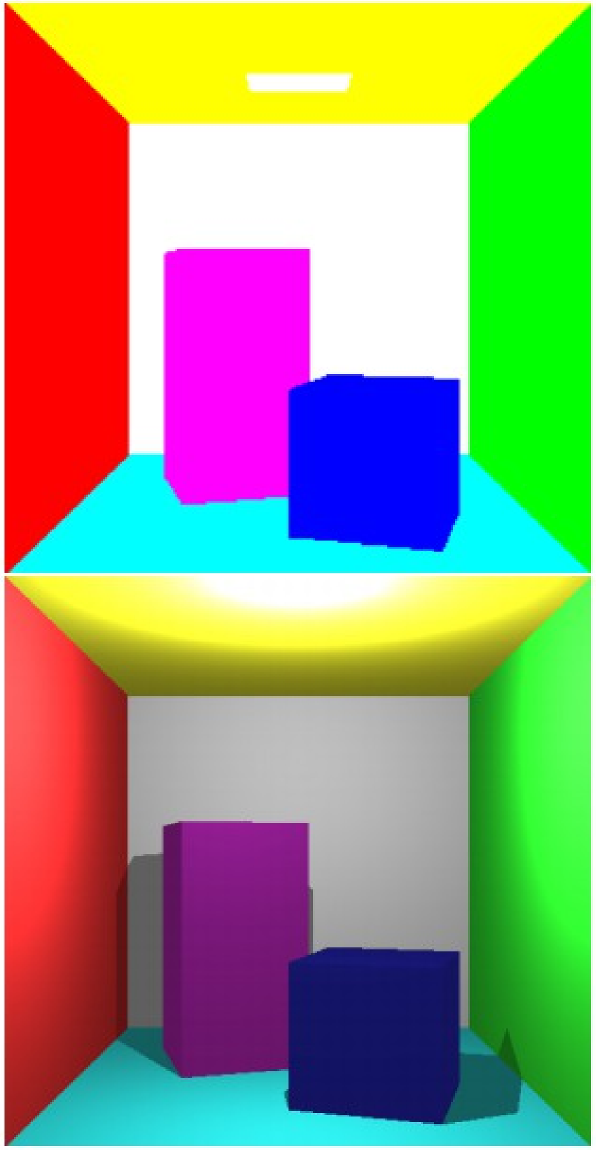
Constant light affecting each part of the scene equally

Point Light Sources

Rays travelling from a point in all directions

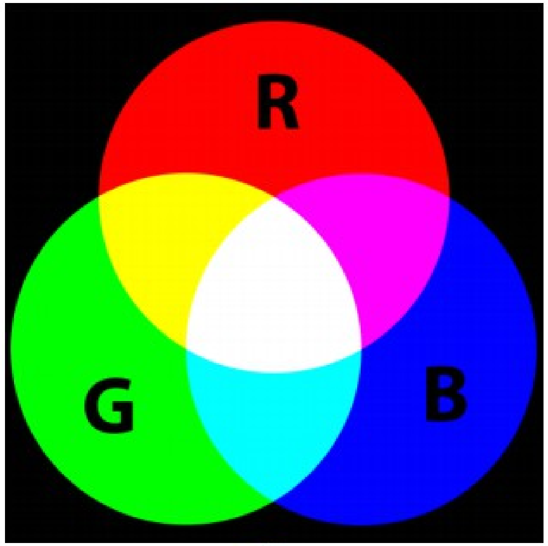
Spotlight Sources

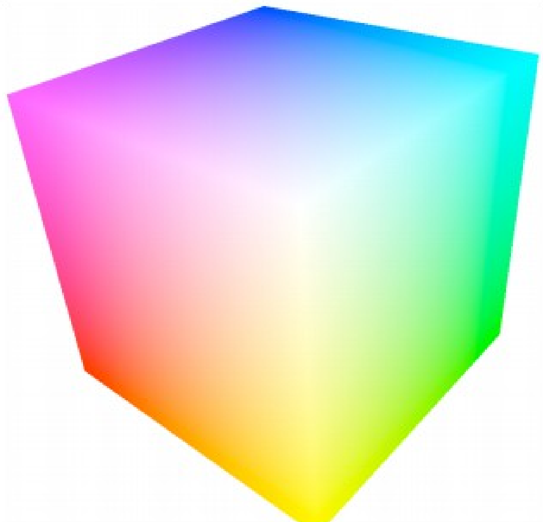
Rays travelling from a point in limited directions



Colour Representation

- $\mathbf{c} = [r, g, b]^{\mathrm{T}}$
- Combine rays,
  - $\mathbf{c}_1 + \mathbf{c}_2 = [r_1 + r_2, g_1 + g_2, b_1 + b_2]^{\mathrm{T}}$
  - $||\mathbf{c}||_\infty \leq 1$

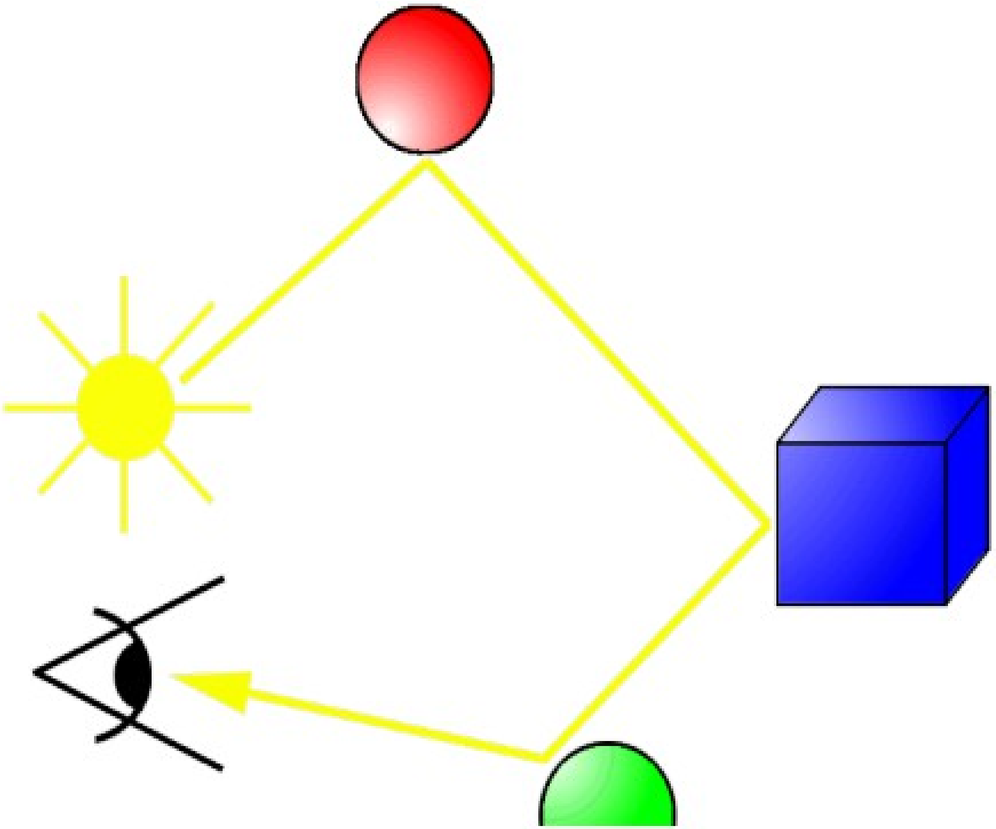




Our Initial Idea

We have now defined some modelling elements

Remember our initial idea:



Question

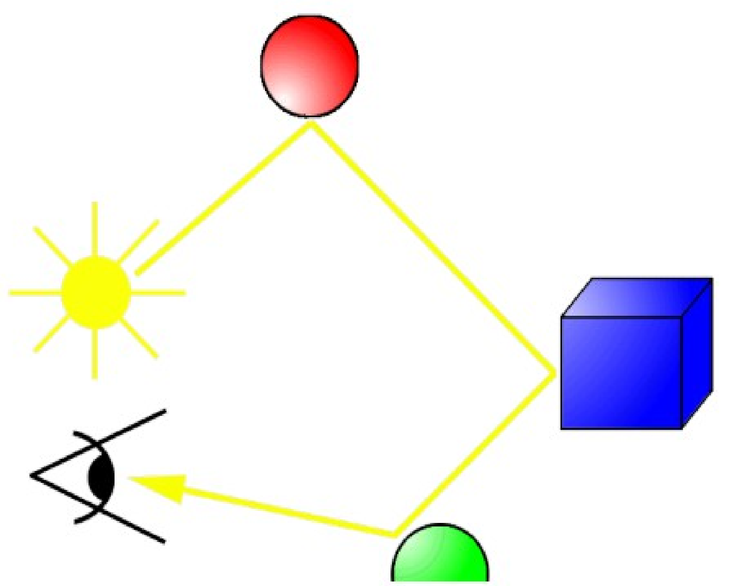
Is this feasible??

Question

Is this feasible??

Think of what the procedure may look like:

```
for(i=0;i<nr_rays,i++)
{
  while(!hit image&&!infinity)
  {
    /*
    compute new direction
    compute new colour
    */
  }
}
```



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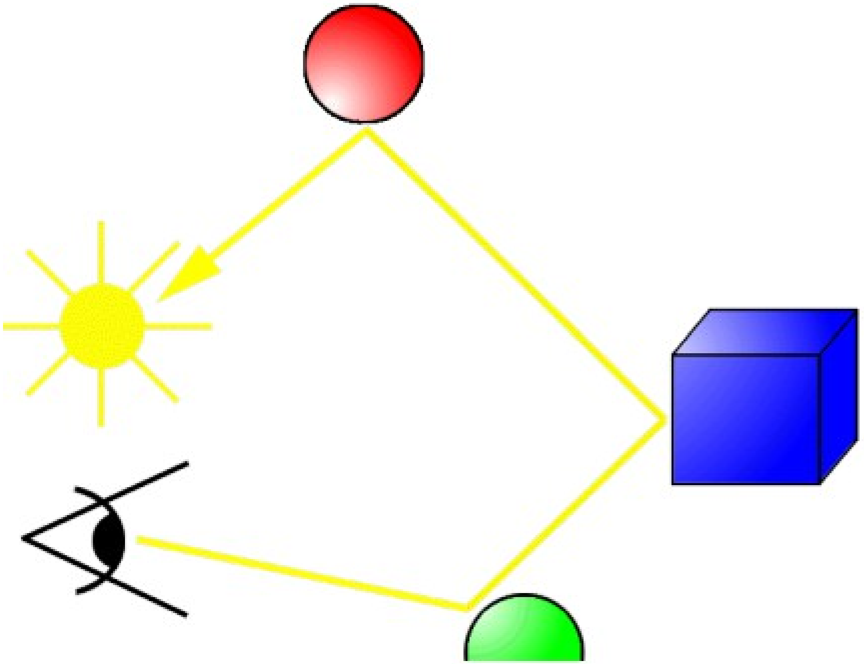
Idea 2

- Cast rays of light backwards through the viewing plane, into the scene
- Test for intersections between each ray and virtual objects in the scene
- Ray fails to hit anything
  - Background or default colour returned
- When ray hits (i.e. 'intersects') first object
  - Cast ray from intersection point on object to light sources
  - Test material properties of object

Backwards Raytracing

```
for(v=0;v<height,v++)
{
  for(u=0;u<width;v++)
  {
    for(i=0;i<max_nr_bounce;i↩
        ++)
    {
      /*
      compute new direction
      compute new colour
      */
    }
  }
}
```

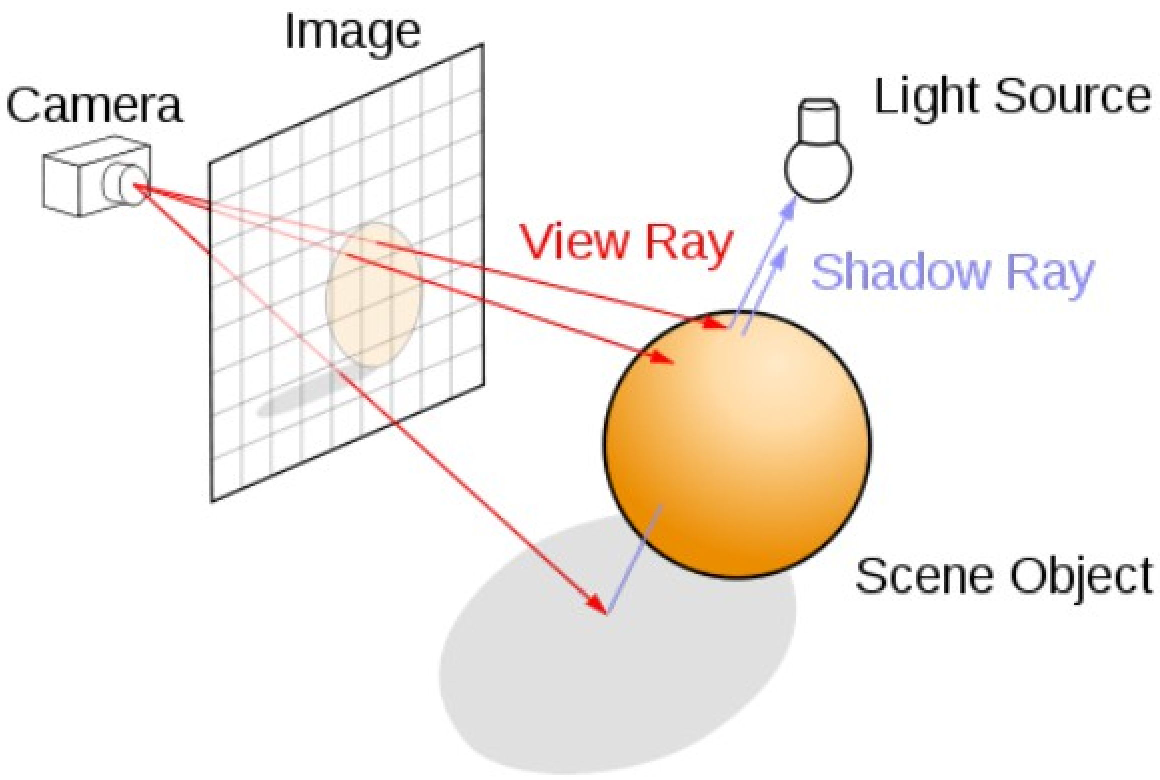


Termination Criteria

1. No intersection
2. Reach maximal depth
   - Number of bounces
1. Contribution of secondary ray attenuated below threshold
   - Each reflection/refraction bounce attenuates ray

Shadows

1. Shoot ray
2. Intersection
   - Shoot ray to light
   - Free/Blocked



Raytracing in General

Simple and basic algorithm
Capable of simulating complex light interactions
Well-suited for software rendering
Very flexible
Easy to incorporate new effects
Not always viewed as so interactive
Attempts have been made to change this
But also see: State of raytracing in games

Overall

Ray-tracing just one of a number of *global illumination* models

Others include:

Radiosity

Photon mapping

Path tracing

Ambient occlusion

Global illumination models

- More realism
- More computation

Links

Persistence of Vision Raytracer (POV-Ray)

A great way to try raytracing at the high level

Raytracing and Gaming Article

Interesting article written in 2008 on raytracing in computer games

Next lecture

- More details about project themes
- Next Wednesday (1st April)
- 13:00 – 15:00 L1

- Labs:
  - Download and start to look at the first lab
  - You are doing well at this stage if you have a basic version building