Turbulence
There are no “simple” turbulent flows

Turbulent boundary layer:

• Instantaneous velocity field (snapshot) $u_i(x, t)$
Prediction of turbulent flows

Approximation level

days
weeks
months
years

standard RANS

advanced RANS

hybrid RANS-LES

LES

DNS

Log(CPU)
Standard RANS models

Standard RANS models are available in commercial codes and include:
- $k-\varepsilon$
- $k-\omega$, e.g., Menter SST
- Spalart-Allmaras

Advanced RANS models are good (only) for:
- Attached boundary layers
- Thin shear flows

Hybrid RANS-LES models are more computationally intensive, requiring:
- Days
- Weeks
- Months
- Years

Direct Numerical Simulations (DNS) are the most accurate and computationally demanding.
Advanced RANS models

- Differential Reynolds stress models, DRSM, RST
- Algebraic Reynolds stress models, EARSM
- Structure-based modelling (Kassinos & Reynolds)
- Two-point correlations

Needed for:
- Rotation and swirl
- Strong 3D effects
- Body forces
Direct Numerical Simulation – DNS

- **standard RANS**
  - Valid (!)
  - no approximation
  - Universal
  - Full information

- **advanced RANS**

- **hybrid RANS-LES**

- **LES**

- **DNS**
  - But:
    - Extremely expensive
    - Only low Re
    - Full aircraft ~2080 ?
    - Research tool only

Approximation level vs. CPU time (days, weeks, months, years).
• Valid
  • Little approximation
• Universal (if correctly done)
• Large scale dynamics:
  • Acoustics
  • Dynamic loads

Also very expensive
• Full aircraft ~2050
• Affordable in internal flow (industrial use)
Hybrid RANS – LES methods

- **standard RANS**
  - RANS in attached BLs
  - LES in free turbulence, separation
  - Affordable, but still expensive

- **advanced RANS**

- **hybrid RANS-LES**

- **LES**

- **DNS**

Many different methods:

- DES, DDES, IDDES, VLES, XLES, PANS, PITM

Time scale:

- days
- weeks
- months
- years

Approximation level:

- Log(CPU)
Prediction of turbulent flows

Approximation level

standard RANS

advanced RANS

hybrid RANS-LES

DNS

days

weeks

months

years

Log(CPU)
Basic concepts

- Turbulence is:
  - random fluctuations
  - 3D
  - time dependent
  - present in most flows of engineering interest

- Energy cascade
  - generated at the largest scales \((L \text{ and } U)\)
  - large scale vortices break down to smaller vortices
  - dissipates to heat at the smallest viscous scales, \(\varepsilon\)
  - balanced cascade

\[
\varepsilon \sim \frac{U^3}{L}
\]

- turbulent kinetic energy

\[
K \sim U^2
\]
DNS – RANS

DNS

Instantaneous

low velocity regions

RANS

Time averaged

Length of the recirculation region is of engineering interest
DNS (and also LES):
- 3D
- Time dependent
  Full information of turbulence scales
  Acoustics and dynamic loads
- Huge Reynolds number dependency

Expensive!
DNS – RANS

RANS:
- Reduction of dimensions \(\rightarrow\) cheap
  Here: 2D and steady
- Only statistical information of turbulence scales:
  Time and length scales
  rms values

Time averaged

Low velocity regions

Length of the recirculation region is of engineering interest
Scale separation

- Large scales $L$ and $U$ related to geometrical scales
- Small viscous scales (related to $\nu$ and $\varepsilon$)
  \[ l_K = \eta \sim \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}, \quad t_K \sim \sqrt[4]{\frac{\nu}{\varepsilon}} \]
- Scale separation
  \[ \frac{L}{\eta} \sim Re^{3/4}, \quad \frac{t}{t_K} \sim Re^{1/2} \]
- Reynolds number
  \[ Re = \frac{LU}{\nu} \]
Different Reynolds numbers

- Based on global scales
  \[ Re_L = \frac{LU}{v} \]

- Based on distance \( x \) from leading edge (flat plate)
  \[ Re_x = \frac{xU}{v} \]

- Based on boundary layer thickness (\( \delta \))
  \[ Re_\delta = \frac{\delta U}{v} \]

- Based on wall skin friction
  \[ Re_\tau = \frac{\delta u_\tau}{v} \]
Viscosity

- Kinematic viscosity, $\nu$
- Dynamic viscosity, $\mu$
- Density, $\rho$

\[ \mu = \rho \nu \]
Boundary layers (BL)

- Thin layers
  - Thickness Reynolds number dependent
- Laminar boundary layers
  - Thickness related to wall skin friction
- Turbulent boundary layers
  - Inner and outer scales separated
  - Scale separation Reynolds number dependent

- Figures (Wing+bl – bl – near-wall)
BL on the ONERA A-profile
Approximation of BLs

- Slip wall boundary condition
  - Boundary layer completely neglected
  - Euler (non-viscous) computations possible
  - Slip BC can also be applied to viscous & turbulent CFD

- No slip boundary condition
  - Boundary layer completely resolved \( (y^+ = 1) \)
  - Extreme resolution needed \( (\Delta y = 1-100 \mu m) \)
  - 40-80 grid points within the boundary layer

- Log-law boundary condition (turbulence)
  - First grid point within log layer \( (y^+ > 20 \ AND \ y < 0.1\delta) \)
  - 10-20 grid points within the boundary layer
  - Warning: standard log-law BCs inconsistent with too small grid size. READ SOLVER DOCUMENTATION !!!
What’s in Fluent?

- **Standard and Non-Equilibrium Wall Functions:**
  - “Wall adjacent cells should have y+ values between 30 and 300–500” — (remember $y < 0.1\delta$!)
  - “The mesh expansion ratio should be small (no larger than around 1.2)”
  - “Non-equilibrium wall function method attempts to improve the results for flows with higher pressure gradients, separations, reattachment and stagnation”

- **Scalable Wall Functions:**
  - Consistent for all y+ values
What’s in Fluent? ...

- Enhanced Wall Treatment Option
  - Combines a blended law-of-the wall and a two-layer zonal model.
  - Suitable for low-Re flows or flows with complex near-wall phenomena.
  - Generally requires a fine near-wall mesh capable of resolving the viscous sublayer
  - $y^+ < 5$, and a minimum of 10–15 cells across the “inner layer” for best results
  - Valid for all $y^+$
  - Available for all k-e and k-w models
  - Not yet for Spalart-Allmaras ($y^+ < 3$ OR $y^+ > 15$)
Recommendations for Fluent

- For $K-\varepsilon$ models
  - use Enhanced Wall Treatment: EWT-$\varepsilon$
- If wall functions are favored with $K-\varepsilon$ models
  - use scalable wall functions
- For $K-\omega$ models
  - use the default: EWT-$\omega$
Wall bounded turbulence

- Viscous wall scales
  \[ l_\ast = \frac{v}{u_\tau}, \quad t_\ast = \frac{v}{u_\tau^2} \]

- Wall friction velocity
  \[ u_\tau = \sqrt{\frac{\tau_w}{\rho}} = \sqrt{v \frac{\partial U}{\partial y}} \bigg|_{\text{wall}} = U \sqrt{\frac{1}{2} C_f} \]

- Friction coefficient
  \[ C_f = \frac{\tau_w}{\frac{1}{2} \rho U^2} = 2 \left( \frac{u_\tau}{U} \right)^2 \]

- Viscous wall distance
  \[ y^+ = \frac{y}{l_\ast} = \frac{yu_\tau}{v} \]
Empirical relations for BLs

- Friction coefficient
  - Turbulent: \( \frac{C_f}{2} \approx 0.0296 \frac{1}{Re_x} \)
  - Laminar: \( \frac{C_f}{2} = 0.332 \frac{1}{Re_x} \)

- Boundary layer thickness
  - Turbulent: \( \frac{\delta}{x} \approx 0.37 \frac{1}{Re_x} \)
  - Laminar: \( \frac{\delta}{x} = 5.0 \frac{1}{Re_x} \)
Empirical relations plotted

Laminar Turbulent

Laminar Turbulent
Empirical relations plotted
Turbulence modelling

• Reynolds decomposition

\[ \tilde{u}_i(x, t) = U_i(x) + u_i(x, t) \]

where \( U_i(x) = \overline{\tilde{u}_i(x, t)} \) and \( u_i(x, t) = 0 \)

• The “mean” is time average, ensemble average or averaging in homogeneous directions. \( U_i(x) \) may actually vary in time with a time scale much longer than the turbulent time scale.

• Take the mean of the Navier-Stokes equations -> RANS

\[ \frac{\partial U_i}{\partial x_i} = 0 \]

\[ \frac{\partial U_i}{\partial t} + U_k \frac{\partial U_i}{\partial x_k} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_k} \left( \nu \frac{\partial U_i}{\partial x_k} - u_i u_k \right) \]
Reynolds stresses

- Not “small”
- Significant effects on the flow
- Needs to be modelled in terms of mean flow quantities
- Reduces the problem to steady (or slowly varying)
- 2D assumptions possible

- Equation can be derived from Navier-Stokes equations
- Need modelling
Eddy-viscosity models (EVM)

- Assume: Reynolds stresses related to an “eddy viscosity”, $\nu_T$
  \[
  \overline{u_i u_j} = -2\nu_T S_{ij} \left( + \frac{1}{3} \overline{u_k u_k} \delta_{ij} \right)
  \]
  where $S_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)$

Based on Boussinesq (1877)
- Eddy viscosity $\sim$ turbulence velocity $V$ and length $L$ scales: $\nu_T \sim V L$
One-equation models

- One transport equation for $K$ (turbulent kinetic energy) or $\nu_T$.
- Additional information from global conditions (typically wall distance)
- Works well for attached boundary layers
- Not very general, but more than algebraic models

Example: Spalart-Allmaras (1992)
  - reasonable and robust model for external aerodynamics
  - Boeing’s “standard model”
Two-equation models

- Two transport equations for the turbulence scales \((K-\varepsilon)\) or \((K-\omega)\)
- Completely determined in terms of local quantities (except near-wall corrections which may be dependent on wall distance)
- Works well for attached boundary layers
- Somewhat more general than zero-, one-equation models
- Model transport equations loosely connected to the exact equations.

Examples:
- Standard \(K-\varepsilon\) model (Launder & Spalding 1974)
- Wilcox \(K-\omega\) (1988, ...) models
- Menter (1994) SST \(K-\omega\) model (performing reasonable well also in separated flows)
  - Airbus’ “standard model”
Eddy-viscosity models ...

• Problems:
  - No dependency on rotation or curvature. Real turbulence strongly dependent.
  - Modelled production proportional to strain rate squared $\sim S^2$. Exact production $\sim S$. Results in an overestimated production of $K$ in highly sheared flows (around stagnation points, impinging jets, pressure gradient BLs, separated flows).

• Fixes
  - Rotation & curvature corrections
  - Yap correction (limit excessive turbulent lengthscale)
  - Menter SST correction (limit excessive $v_T$).
LES and LES/RANS hybrids

- Simulation of only the large scale turbulence (compare with DNS, simulation of all scales)
  - Always time dependent and 3D -> expensive
- Wall free turbulence simulations almost Re independent
- Wall bounded turbulence largely Re dependent
  - fully resolved near-wall region very expensive (almost as DNS)
  - wall-function or near-wall RANS coupling saves computational cost
  - hybrid RANS-LES (RANS in attached BLs and LES in wall-free separated regions) a very active research field, eg DES
LES and LES/RANS hybrids ...

- LES in academic research for:
  - low Re generic flows
  - complement to DNS for higher Re
  - gives detailed knowledge about turbulence

- LES in industrial use in:
  - internal flow with complex geometries
  - flows around blunt bodies (with large separated regions)
  - atmospheric boundary layers (e.g. weather forecasts)
  - combustion simulation
  - other complex flow physics at moderate Re

- Warning: LES is extremely expensive in high attached and slightly separated wall-bounded flows, if properly resolved.
How expensive is DNS?

• DNS of flat plate turbulent boundary layer
  - Schlatter, et al., KTH, Dept. of Mechanics
  - APS meeting 2010: http://arxiv.org/abs/1010.4000
  - http://www.youtube.com/watch?v=4KeaAhVoPlw
  - http://www.youtube.com/watch?v=zm9-hSP4s3w
  - $Re_\theta = 4300$
  - $8192 \times 513 \times 768 = 3.2 \times 10^9$ spectral modes ($7.5 \times 10^9$ nodes)
  - $\Delta x^+ = 9$, $\Delta z^+ = 4$  $\Rightarrow$ box: $L^+ = 70000$, $H^+ = W^+ = 3000$
  - BL relations: $Re_x = 1.4 \times 10^6$
  - CPU time: 3 months @ 4000 CPU cores = 1 unit

• DNS of model airplane, same Reynolds number ($Re_x = 1.4 \times 10^6$)
  - Only a narrow stripe – wing requires about 1 000 stripes
  - $N_{\text{nodes}} = 10^{13}$
  - $\text{CPU} = 10^3$ units
Empirical turbulent BL relations

- Skin friction coefficient:
  \[ \frac{C_f}{2} = \frac{\tau_w}{\rho U_\infty^2} = \left( \frac{Re_\tau}{Re_\delta} \right)^2 \approx 0.0296 Re_x^{-1/5} \]

- Boundary layer thickness:
  \[ \frac{\delta}{x} = \frac{Re_\delta}{Re_x} \approx 0.37 Re_x^{-1/5} \]

- Boundary layer momentum thickness:
  \[ Re_\tau \approx 1.13 Re_\theta^{0.843} \]

- Reynolds numbers:
  \[ Re_\tau \equiv \frac{\delta u_\tau}{\nu} \quad Re_\theta \equiv \frac{\theta U_\infty}{\nu} \quad Re_\delta \equiv \frac{\delta U_\infty}{\nu} \quad Re_x \equiv \frac{x U_\infty}{\nu} \]
DNS – full scale airplane

• Re scaling – wall bounded flow

  – Nodes: \[ N_{\text{nodes}} \sim \frac{L \times B \times H}{\Delta x \Delta z \Delta y} \sim L^+ H^+ \sim \text{Re}_x^{5/2} \]

  – Time steps: \[ N_{\Delta T} \sim \frac{T}{\Delta T} \sim T^+ \sim \text{Re}_x^{4/5} \]

  – CPU time: \[ N_{\text{CPU}} \sim N_{\text{nodes}} \times N_{\Delta T} \sim \text{Re}_x^{33/10} \]

• DNS of Airplane (\( \text{Re}_x = 70 \times 10^6 \)) (factor of 50)
  – \( N_{\text{nodes}} = 10^{17} \)
  – CPU = \( 10^9 \) units
Supercomputer development

- Doubling of CPU every 18 months
- Present
- 45 year?
- $10^9$ units
- DNS of full airplane year 2055?
### Computational effort – different approaches

From Spalart, *Int. J. Heat and Fluid Flow, 2000*

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- **RANS**: Reynolds Averaged Navier-Stokes
- **URANS**: Unsteady RANS – slowly in time
- **DES**: Detached Eddy Simulation
- **LES**: Large eddy simulation
- **QDNS**: Quasi DNS, or wall resolved LES
- **DNS**: Direct Numerical Simulation (of the Navier-Stokes eq’s)
- “Ready”: When first results can be expected