SF1625 Calculus in one variable
Exam
Tuesday 7th of april 2015

Time: 08:00-13:00
No calculators, formula sheets etc allowed
Examiner: Lars Filipsson

This exam consists of nine problems, each worth four points, hence the maximal score is 36. Part A consists of the three first problems. To the score on part A your bonus points are added, up to a maximum of 12. The score on part A is at most 12, bonus points included. The bonus points are added automatically. You can check how many bonus points you have on your results page.

The following three problems constitute part B and the last three problems part C. You need a certain amount of points from part C to obtain the highest grades.

The grading will be performed according to this table:

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

To obtain a maximal 4 for a solution to a problem on the exam, your solution must be well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained. Solutions that are clearly inadequate in these respects will be awarded no more than 2 points.

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PART A

1. Let $f(x) = \arcsin x + 2\sqrt{1-x^2}$.
   A. Find the domain of definition of f.
   B. Find the maximum and minimum values of f.

   (If you do not recall the derivative of $arcsinx$ you can find it by implicit differentiation of the relation $\sin(\arcsin x) = x$. If you recall the derivative you do not have to do this.)

2. Is it true that $\displaystyle\int_{-1}^{1} e^{-|x|}\,dx < 2$?

3. Simple harmonic motion is described by the differetial equation

$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$

   where $y(t)$ is the deviation from the equilibrium position at time t och ω is a constant.
   A. Solve the differential equation if $\omega = 4$.
   B. Find the solution to the differential equation (still with $\omega = 4$) that satisfies the conditions $y(0) = -6$ and $y'(0) = 32$.
   C. Find the period and the amplitude of your solution.

PART B

4. A. Compute the area of the domain between the curves $y = 1$ och $y = \dfrac{x^2 + 4x + 4}{x^2 + 4x + 3}$, for x n the i interval $0 \le x \le R$.

   B. Decide whether the domain between the curves $y = 1$ och $y = \dfrac{x^2 + 4x + 4}{x^2 + 4x + 3}$, for x in the interval $0 \le x < \infty$, has finite area. If the area is finite, compute it.

5. Let $f(x) = 1 - (x - 1)^2,\ \ 0 \le x \le 2$. Make a simple sketch of the function graph $y = f(x)$ and find the point (x_0, y_0) on the graph making the area of the triangle with corners at $(0, 0)$, $(x_0, 0)$ and (x_0, y_0) maximal.

6. Compute, using for instance substitution or integration by parts, the integral

$$\int_0^1 x\sqrt{1 - x}\, dx$$

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PART C

7. A. Formulate the mean-value theorem of differential calculus.
   B. Use it to show that a function with derivative equal to zero in an open interval must be constant on that interval.

8. Let f be three times differentiable on the interval $-1 < x < 2$, and suppose $f(0) = f'(0) = 0$, $f''(0) = 6$ and $|f'''(x)| \leq 1$ for all x in the interval. Show that
$$1 - \frac{1}{24} \leq \int_0^1 f(x)\,dx \leq 1 + \frac{1}{24}.$$

9. Is there a solution $y(t)$ to the differential equation $y''(t) + y(t) = e^t$ such that $y(t)/t^2$ is bounded as $t \to 0$? Find such a solution, if such a solution exists. Can there be more than one?