SF1625 Calculus in one variable
Exam
Monday 12th of january 2015

Time: 08:00-13:00
No calculators, formula sheets etc allowed
Examiner: Lars Filipsson

This exam consists of nine problems, each worth four points, hence the maximal score is 36. Part A consists of the three first problems. To the score on part A your bonus points are added, up to a maximum of 12. The score on part A is at most 12, bonus points included. The bonus points are added automatically. You can check how many bonus points you have on your results page.

The following three problems constitute part B and the last three problems part C. You need a certain amount of points from part C to obtain the highest grades.

The grading will be performed according to this table:

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

To obtain a maximal 4 for a solution to a problem on the exam, your solution must be well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained. Solutions that are clearly inadequate in these respects will be awarded no more than 2 points.

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Part A

1. We will investigate the function f given by $f(x) = xe^{1/x}$.
   A. Find the domain of definition of f.
   B. Compute the four limits $\lim_{x\to\pm\infty} f(x)$ and $\lim_{x\to 0^\pm} f(x)$
   C. Find all local extreme values of f.
   D. Sketch, using the above, the graph $y = f(x)$

2. Compute the integral
$$\int_{\pi^2/4}^{\pi^2} \cos\sqrt{x}\,dx$$
   by doing the following:
   A. Perform the substitution $\sqrt{x} = t$ (don't forget to change the interval of integration).
   B. Compute, using integration by parts, the integral you obtain in A.

3. A tin can in the shape of a cylinder, with lid and bottom plate, containing 1 litre, is to be manufactured. Compute the height of the cylinder and the radius of its circular bottom plate in order to minimize the total surface area of the can.

PART B

4. We study the differential equation $y''(t) + y(t) = \sin t$.
   A. Solve the differential equation.
   B. Does there exist a bounded solution to the differential equation?

5. Find the Taylor polynomial of degree 2 about the point $x = 100$ to the function $f(x) = \sqrt{x}$ and use this Taylor polynomial to compute an approximate value of $\sqrt{104}$. Also, decide wether the error of your approximation is less than 10^{-4} in absolute value.

6. Is the improper integral
$$\int_1^\infty \frac{dx}{x^2 + x}$$
convergent or divergent? If it is convergent, compute the integral.
Hint: For $x \geq 1$ we have $\dfrac{1}{x^2} \geq \dfrac{1}{x^2 + x} = \dfrac{1}{x} - \dfrac{1}{x+1}$.

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Part C

7. A. Give the definition of what it means for a function f to be continuous at a point a.
   B. Give the definition of what it means for a function f to be differentiable at a point a.
   C. Show that a function f differentiable at a point a also must be continuous at a.
   D. Give an example showing that a function that is continuous at a point does not have to be differentiable at that point.

8. A hole with radius 1 is drilled through the center of a ball with radius 2. How great a part (in percent) of the volume of the ball remains?

9. Show that the function
$$f(x) = x\left(\frac{\pi}{2} - \arctan x\right)$$
is increasing.