



KTH Electrical Engineering

## Complementary test in EG2050 System Planning, 9 April 2015, 14:00-18:00, the seminar room

### Instructions

Only the problems indicated on the attached answer sheet have to be answered (the score of the remaining problems is kept from the exam). Motivations and calculations do not have to be presented.

The maximal score of the complementary test is 40 points including the points that are kept from the exam. You are guaranteed to pass if you get at least 33 points.

### Allowed aids

In this complementary test you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten, single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.



## Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

**a) (2 p)** Which players have the responsibility to maintain the frequency of the power system within nominal values (for example 49.9–50.1 Hz in the Nordel system)?

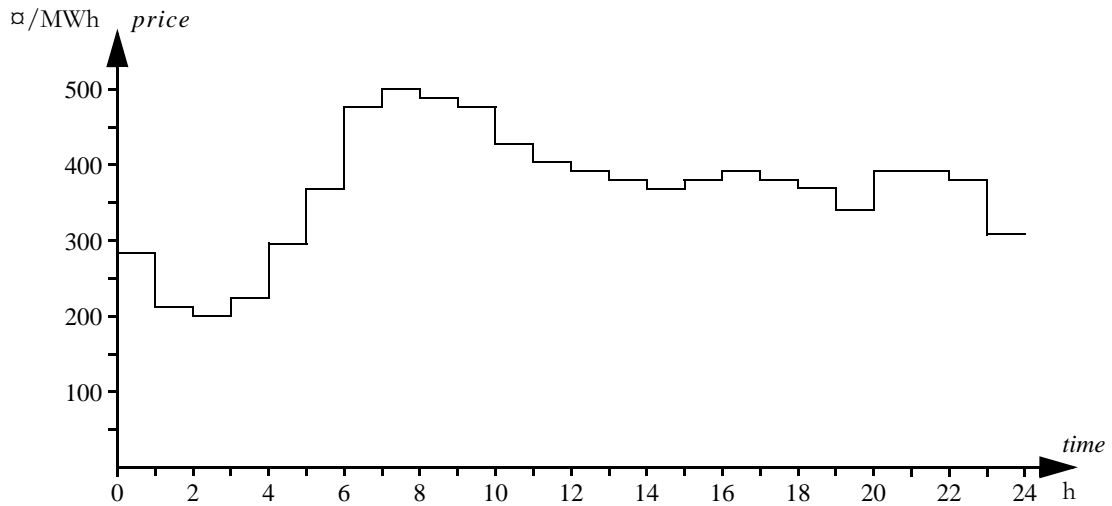
1. All producers have a shared responsibility for the frequency.
2. The largest power companies have a shared responsibility for the frequency.
3. All consumers have a shared responsibility for the frequency.
4. The balance responsible players have a shared responsibility for the frequency.
5. The system operator is responsible for the frequency.

**b) (2 p)** The following applies to a power exchange using a price cross: I) Players who are selling to the exchange state the lowest price for which they are willing to sell, II) Players who are buying from the exchange state the highest price they are willing to pay, III) All players whose bids are accepted will trade at the price submitted in the bid.

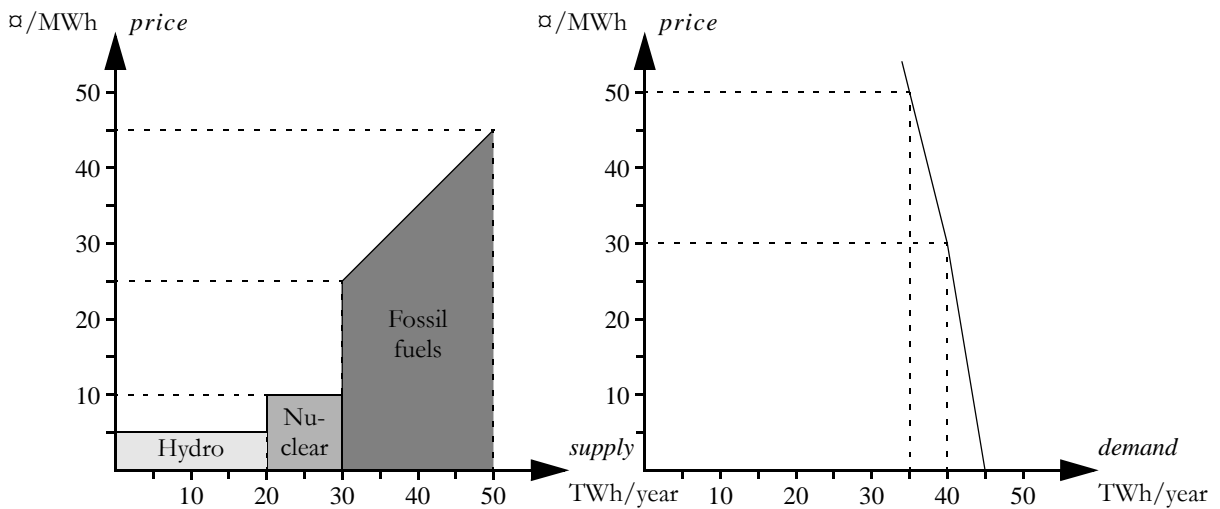
1. Only I is true.
3. Only II is true.
3. Only III is true.
4. I and II are true but not III.
5. All the statements are true.

## Problem 2 (6 p)

**a) (3 p)** The figure below shows the electricity price in a certain electricity market during one day. Assume that there is perfect competition in this electricity market and that all players have access to perfect information. How much will be generated during this day in a power plant with the variable operation cost 350  $\text{€}/\text{MWh}$  and the installed capacity 200 MW?



**b) (3 p)** The figures below shows the supply and demand curves of a certain electricity market. What will the electricity price become in this electricity market if we assume perfect competition, perfect information and that there are neither transmission, reservoir nor capacity limitations?



### Problem 3 (6 p)

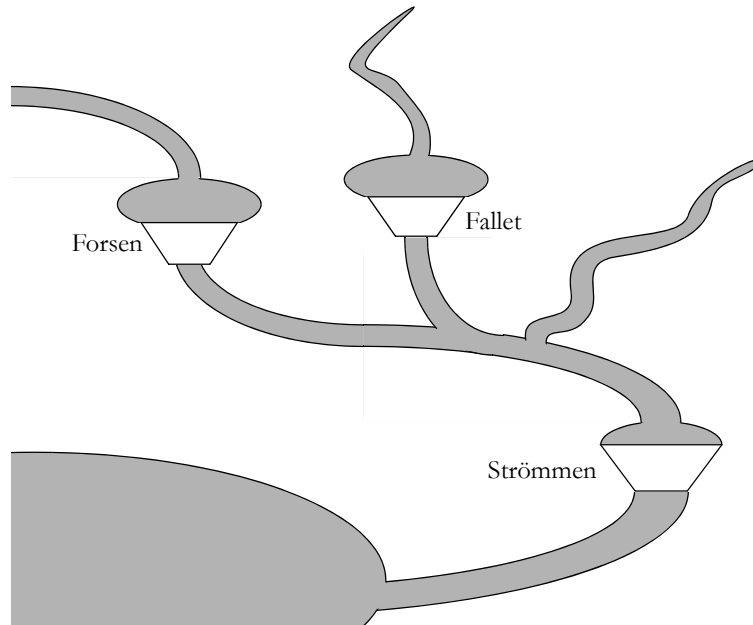
Consider a power system where the primary control is divided in a normal operation reserve and a disturbance reserve. The normal operation reserve has the gain 5 000 MW/Hz and is mainly designed to manage normal variations in for example load and wind power generation. The disturbance reserve has the gain 2 250 MW/Hz and is mainly designed to manage outages in larger power plants. The normal operation reserve is available in the frequency range 49.9–50.1 Hz and the disturbance reserve is available in the frequency range 49.5–49.9 Hz.

**a) (2 p)** At 14:05 there is balance between production and consumption in the system and the frequency is 50.02 Hz. At this time a lightning strike in a substation causes 250 MW of generation to be lost. The concerned power plants were not part of the primary control. What will the frequency be once the primary control has stabilised the frequency in the system again?

**b) (2 p)** At 14:10 there is balance between production and consumption in the system and the frequency is 49.98 Hz. At this time the generation in an off-shore wind farm increased by 50 MW. The wind farm is not participating in the primary control. What will the frequency be once the primary control has stabilised the frequency in the system again?

**c) (2 p)** At 14:15 there is balance between production and consumption in the system and the frequency is 49.96 Hz. At this time a lightning strike in a substation causes 750 MW of generation to be lost. The concerned power plants were not part of the primary control. What will the frequency be once the primary control has stabilised the frequency in the system again?

## Problem 4 (12 p)



AB Vattenkraft owns three hydro power plants located as in the figure above. The company sells power to customers with firm power contracts, but they also expect to be able to purchase or sell a certain volume per hour in the power exchange ElKräng. Assume that the company has formulated its short-term planning problem as an LP problem and that the following symbols have been introduced:

Indices for the power plants: Forsen 1, Fallet 2, Strömmen 3.

- $\gamma_i$  = expected future production equivalent for water stored in reservoir  $i$ ,  
 $i = 1, 2, 3$ ,
- $D_t$  = contracted load hour  $t$ ,  $t = 1, \dots, 24$ ,
- $\lambda_t$  = expected electricity price at ElKräng hour  $t$ ,  $t = 1, \dots, 24$ ,
- $\lambda_{25}$  = expected electricity price at ElKräng after the end of the planning period,
- $\bar{M}_i$  = maximal contents of reservoir  $i$ ,  $i = 1, 2, 3$ ,
- $M_{i,t}$  = contents of reservoir  $i$  at the end of hour  $t$ ,  $i = 1, 2, 3$ ,  $t = 1, \dots, 24$ ,
- $\mu_{i,j}$  = marginal production equivalent in power plant  $i$ , segment  $j$ ,  
 $i = 1, 2, 3$ ,  $j = 1, 2$ ,
- $p_t$  = purchase from ElKräng hour  $t$ ,  $t = 1, \dots, 24$ ,
- $\bar{p}_t$  = maximal purchase from ElKräng hour  $t$ ,  $t = 1, \dots, 24$ ,
- $Q_{i,j,t}$  = discharge in power plant  $i$ , segment  $j$ , during hour  $t$ ,  
 $i = 1, 2, 3$ ,  $j = 1, 2$ ,  $t = 1, \dots, 24$ ,
- $\bar{Q}_{i,j}$  = maximal discharge in power plant  $i$ , segment  $j$ ,  $i = 1, 2, 3$ ,  $j = 1, 2$ ,
- $r_t$  = sales to ElKräng hour  $t$ ,  $t = 1, \dots, 24$ ,
- $\bar{r}_t$  = maximal sales to ElKräng hour  $t$ ,  $t = 1, \dots, 24$ ,
- $S_{i,t}$  = spillage from reservoir  $i$  during hour  $t$ ,  $i = 1, 2, 3$ ,  $t = 1, \dots, 24$ ,
- $\bar{S}_i$  = maximal spillage from reservoir  $i$ ,  $i = 1, 2, 3$ .

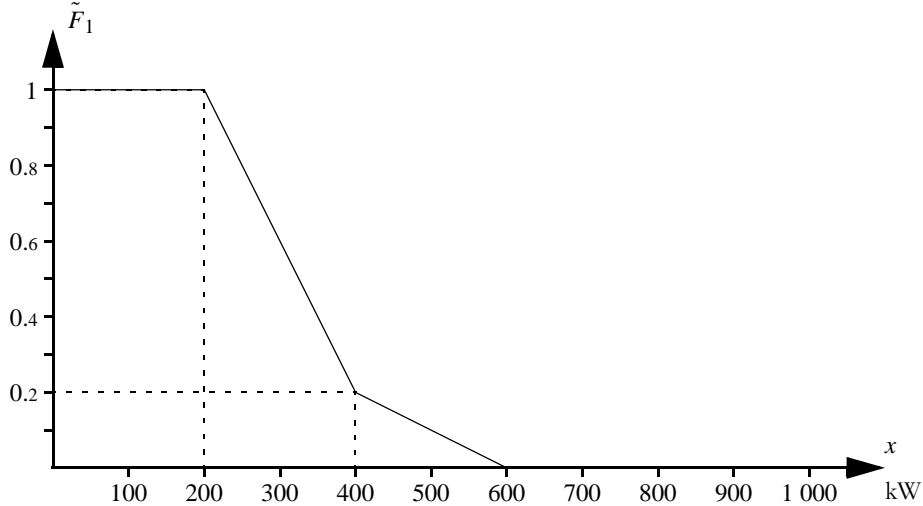
**a) (4 p)** Formulate the objective function if the aim of the planning problem is to maximise the income of sold electricity at ElKräng plus the value of stored water minus the costs of purchasing electricity from ElKräng. Use the symbols defined above.

**b) (4 p)** Formulate the load balance constraint for hour  $t$  in the short-term planning problem of the company. Use the symbols defined above.

**c) (4 p)** Formulate the limits for the optimisation variables defined above for the planning problem of Vattenkraft AB. Please also state the possible index values for each limit!

## Problem 5 (12 p)

Ebbuga is a town in East Africa. The town is not connected to a national grid, but has a local system of its own. The local grid is supplied by a hydro power plant and a diesel generator set. The hydro power plant does not have a reservoir, but the water flow is always sufficient to generate the installed capacity (450 kW) and the risk for outages in the power plant is negligible. The diesel generator set has a capacity of 150 kW, the availability is 85% and the operation cost is 10  $\text{€}/\text{kWh}$ .



**a) (1 p)** What is the expectation value of the load?

*Hint: Study  $EENS_0$ !*

**b) (3 p)** The expected energy not served when considering both the hydro power plant and the diesel generator is 9.5625 kWh/h. Calculate the expected total operation cost per hour for the system.

**c) (2 p)** Use probabilistic production cost simulation to compute the risk of power deficit in Ebbuga.

**d) (3 p)** To consider the losses in the grid, a Monte Carlo simulation has been performed of the power system in Ebbuga. The simulation is using control variates. The simplified model corresponds to the model used in probabilistic production cost simulation, whereas the detailed model considers such factors as how the losses depends on which power plants that are operated, and how the load is distributed within the system. The results are shown in table 1. Which estimate of  $ETOC$  is obtained for the detailed model when using the control variate method?

**Table 1** Results from a Monte Carlo simulation of the power system in Ebbuga.

Number of scenarios, $n$	Results from the detailed model, $\sum_{i=1}^n toc_i$	Results from the simplified model, $\sum_{i=1}^n \tilde{toc}_i$
1 000	85 139	17 028

**e) (2 p)** Assume that complementary random numbers are used in a simulation of the power system in Ebbuga. What is the value of the complementary random number,  $D^*$ , if the total load of the system is randomised to  $D = 363$  kW?



**f) (1 p)** The expectation value  $E[X]$  is to be determined using stratified sampling. Assume that  $L$  strata have been defined. Let  $x_{h,i}$  denote the  $i$ :th observation of  $X$  from stratum  $h$  and let  $\omega_h$  denote the stratum weight of stratum  $h$ . The total number of observations is  $n$ , and we use the symbol  $n_h$  to denote the number of observations from  $h$ . How is the estimate  $m_X$  calculated?

$$1. m_X = \frac{1}{n} \sum_{h=1}^L \sum_{i=1}^{n_h} \omega_h x_{h,i}.$$

$$2. m_X = \frac{1}{n} \sum_{h=1}^L \omega_h \sum_{i=1}^{n_h} x_{h,i}.$$

$$3. m_X = \sum_{h=1}^L \frac{\omega_h}{n_h} \sum_{i=1}^{n_h} x_{h,i}.$$



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## Answer sheet

Name: .....

Personal number: .....

### Problem 1

a) Alternative ..... is correct.

b) Alternative ..... is correct.

### Problem 2

a) ..... MWh      b) .....  $\alpha$ /MWh

### Problem 3

a) ..... Hz      b) ..... Hz

c) ..... Hz

### Problem 4

a) .....

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b) .....

c) .....

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### Problem 5

a) ..... kWh/h      b) .....  $\alpha$ /h

c) ..... %      d) .....  $\alpha$ /h

e) ..... kW

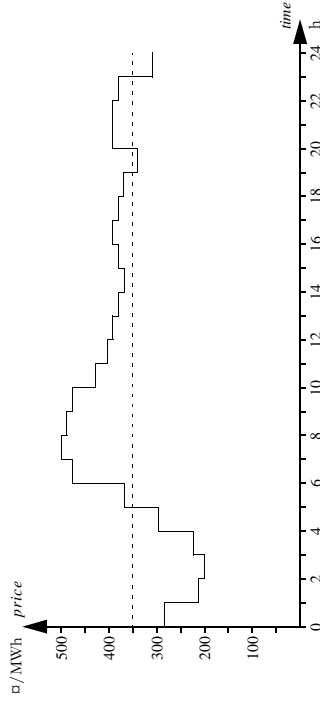
f) Alternative ..... is correct.

### Problem 1

- a) 5, b) 4.

### Problem 2

- a) The power plant will generate its installed capacity during those hours when the electricity price is higher than 350 €/MWh. Drawing a line at the level 350 €/MWh shows that the power plant will generate 200 MW during 17 hours, which results in a total generation of 3 400 MWh.



- b) The electricity price is determined by the intersection of the supply and demand curves. The intersection can be found graphically by drawing both curves in the same figure. An alternative method of solution is to assume an electricity price,  $\lambda$ , between 30 and 45 €/MWh. The supply at these price levels can be written as 30 (hydro & nuclear) +  $(\lambda - 25)$  (fossil fuels) and the demand can be written  $40 - (\lambda - 30)/4$ . These two expressions should be equal, which results in the electricity price  $\lambda = 34$  €/MWh.

### Problem 3

- a) The decrease in electricity generation results in a frequency decrease  $\Delta f = \Delta G/R = 250/5\ 000 = 0.05$  Hz, i.e., the new frequency is  $50.02 - 0.05 = 49.97$  Hz.  
 b) The increase in electricity generation results in a frequency increase  $\Delta f = \Delta G/R = 50/5\ 000 = 0.01$  Hz, i.e., the new frequency is  $49.98 + 0.01 = 49.99$  Hz.  
 c) The normal operation reserve can increase the generation by  $0.06 \cdot 5\ 000 = 300$  MW; if the frequency is 49.96 Hz. The frequency is then 49.9 Hz. The remaining 450 MW generation increase must be managed by the disturbance reserve, which results in a frequency decrease  $\Delta f = \Delta G/R = 450/2\ 250 = 0.2$  Hz Hz, i.e., the new frequency is  $49.9 - 0.2 = 49.7$  Hz.

### Problem 4

- a) maximise  $\sum_{t=1}^{24} \lambda_t(r_t - p_t) + \lambda_{25}(\gamma_1 + \gamma_2)M_{1,24} + (\gamma_2 + \gamma_3)M_{2,24} + \gamma_3 M_{3,24}$ .  
 b)  $\sum_{i=1}^3 \sum_{j=1}^2 \mu_{i,j} Q_{i,j,t} + p_t = D_t + r_t$   
 c) The optimisation variables involved in the problem are trading at ElKrång, reservoir contents, discharge and spillage, which yields the following limits:

$$\begin{aligned} 0 \leq p_t \leq \bar{p}_t, & \quad t = 1, \dots, 24, \\ 0 \leq r_t \leq \bar{r}_t, & \quad t = 1, \dots, 24, \\ 0 \leq M_{i,t} \leq \bar{M}_i, & \quad i = 1, 2, 3, t = 1, \dots, 24, \\ 0 \leq Q_{i,j,t} \leq \bar{Q}_{i,j}, & \quad i = 1, 2, 3, j = 1, 2, t = 1, \dots, 24, \\ 0 \leq S_{i,t} & \quad i = 1, 2, 3, t = 1, \dots, 24. \end{aligned}$$

### Problem 5

- a)  $E[D] = EENS_0 = \int_0^{\infty} \bar{F}_0(x) dx = 200 \cdot 1 + 200 \cdot (1 + 0.2)/2 + 200 \cdot 0.2/2 = 340$  kWh/h.  
 b) The expected generation in the diesel generator sets is  
 $EG_2 = EENS_1 - EENS_2 = \int_0^{\infty} \bar{F}_1(x) dx - EENS_2 =$  (since the hydro power plant is 100% we get  $\bar{F}_1(x) = \bar{F}_0(x)$ )  $= 0.15 \cdot 150/2 - 9.5625 = 1.6875$  kWh/h.  
 Hence, the expected operation cost is  $ETOC = 10EG_2 = 16.875$  €/h.  
 c) The risk of power deficit is given by

$$LOLP = \bar{F}_2(600) = 0.85 \bar{F}_1(600) + 0.15 \bar{F}_1(450) = 0 + 0.15 \cdot 0.15 = 2.25\%$$

$$m_{TOC} = m_{TOC} - \bar{TOC} + \mu_{TOC} = \frac{1}{n} \left( \sum_{i=1}^n to c_i - \sum_{i=1}^n \bar{to c}_i \right) + 16.875 = \frac{1}{1\ 000} (85\ 139 - 17\ 028) + 16.875 = 84.986 \text{ €/h}.$$

- e) The inverse transform method states that  $D = \bar{F}_D^{-1}(U)$ , where  $U$  is a  $U(0, 1)$ -distributed random number. Since it is the duration curve that is given in the problem, we may as well use the transform  $D = \bar{F}_D^{-1}(U)$ . The original random number must then have been  $U = \bar{F}_D(363) = 0.348$ . Hence,  $U^* = 1 - U = 0.652$ , which results in  $D^* = \bar{F}_D^{-1}(U^*) = 287$  kW.  
 f) 3.