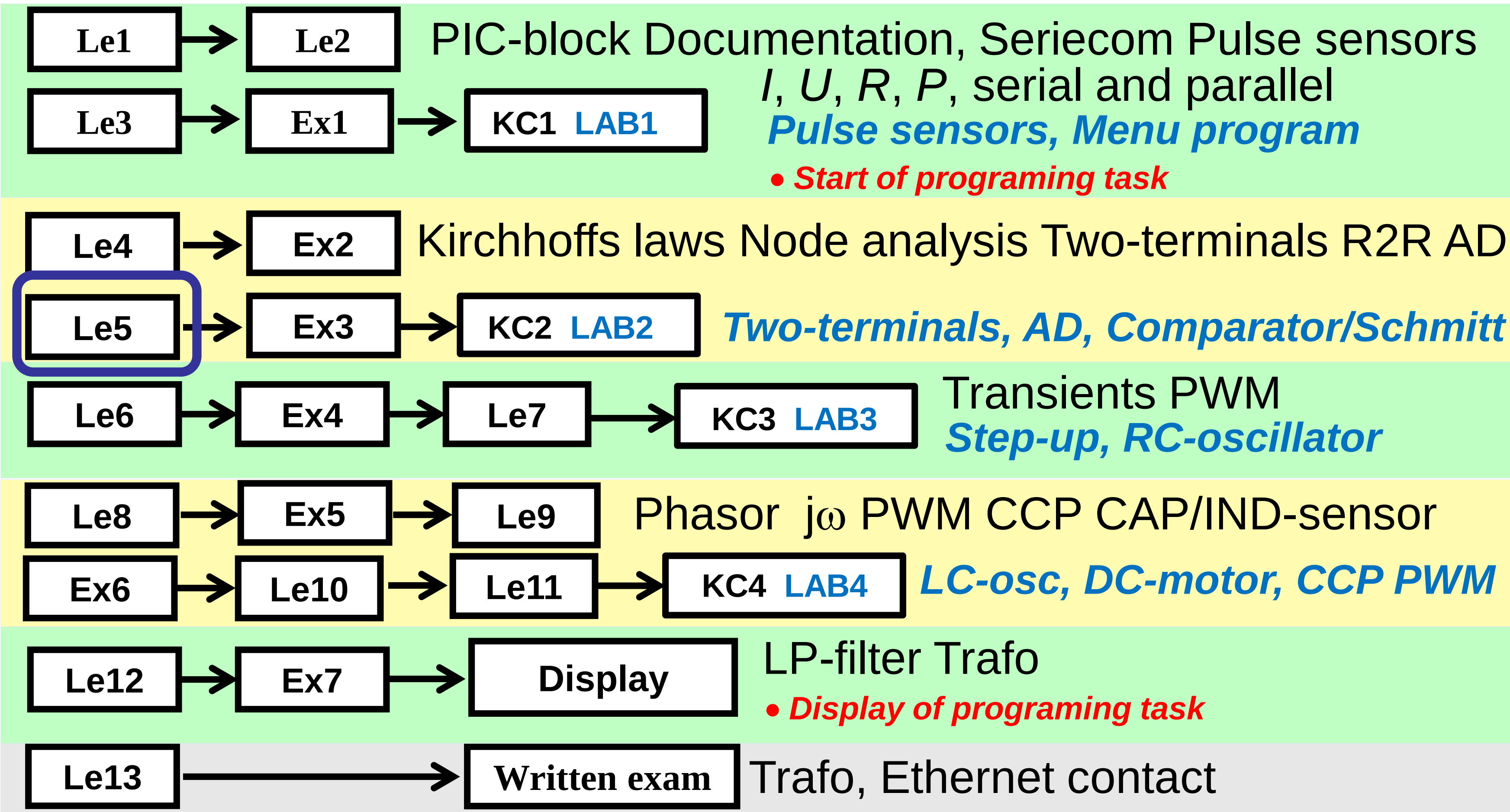
IE1206 Embedded Electronics



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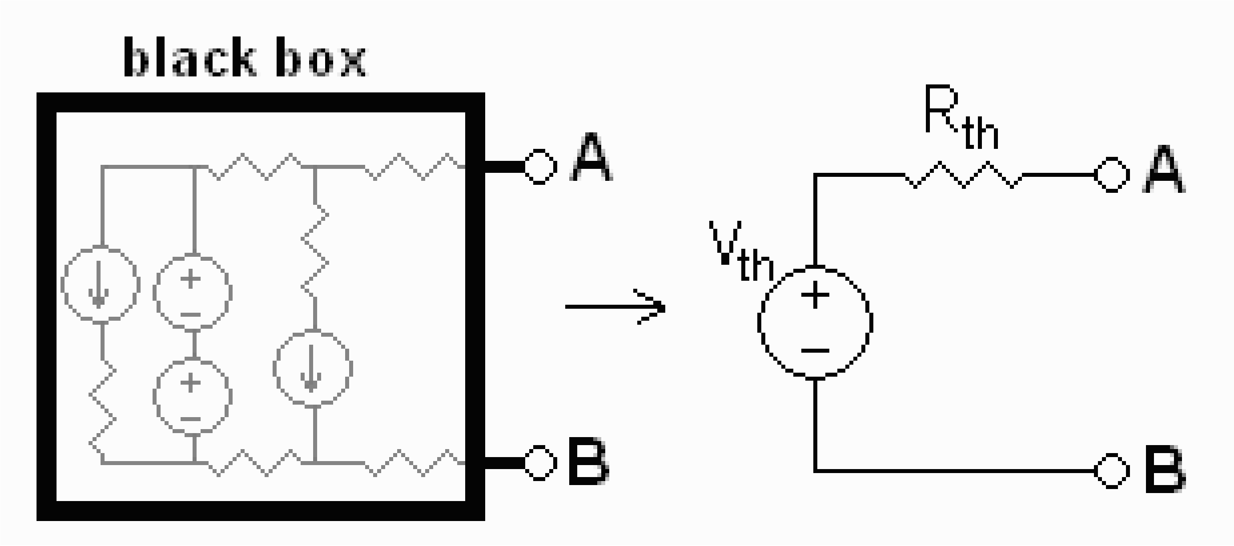
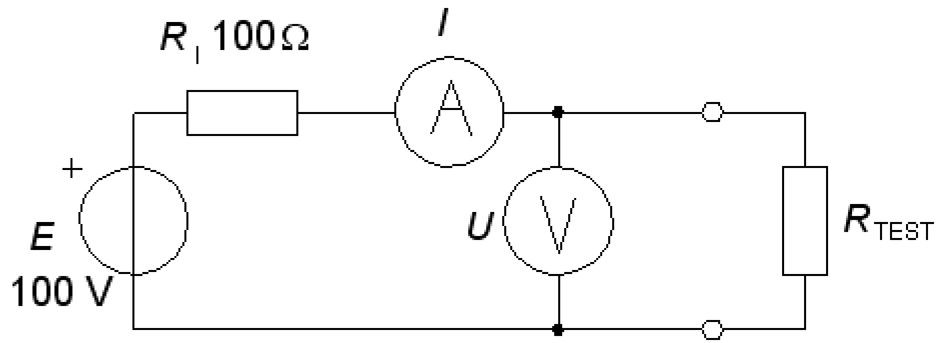
Equvivalent circuits – Black box



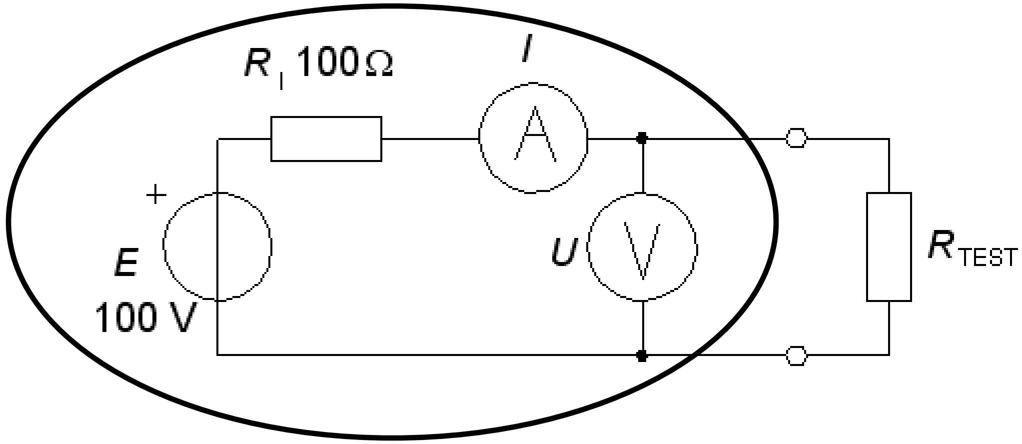
? **=** **!**

Voltage- and Current-sources



An unknown voltage source is to be tested with some test resistors.
(But we happens to know that the voltage source has emf 100 V and the internal resistance 100 Ω).

Voltage- and Current-sources



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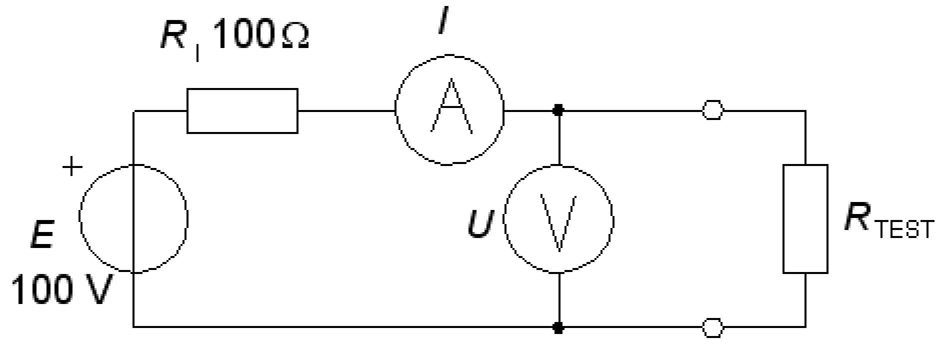
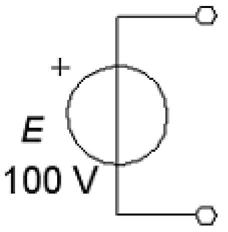
It appears to be an ideal 100 V emf?



R_{TEST} = 10 kΩ $I \approx$ 10 mA $U \approx$ 100 V (= 99,0099 V)

R_{TEST} = 20 kΩ $I \approx$ 5 mA $U \approx$ 100 V   (= 99,5025 V)

There seems to be an "ideal" 100 V voltage source because we can **double the current** without the terminal voltage is affected (noticeably)?



Ideal voltage source without resistance

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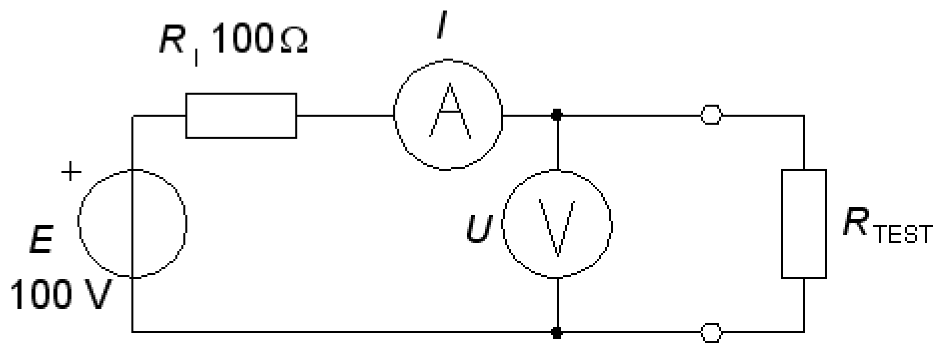
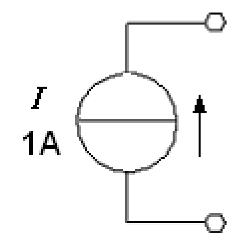
It appears to be an ideal current generator 1A?



$R_{\text{TEST}} = 1\Omega$ $I \approx 1\text{A}$ $U \approx 1\text{ V}$ (= 0,9901 A)

$R_{\text{TEST}} = 2\Omega$ $I \approx 1\text{A}$ $U \approx 2\text{ V}$ (= 0,9804 A)

There seems to be an "ideal" 1A current generator because we can ***double the load resistance*** without the current is affected (noticeably)?



Ideal current generator, the internal resistance is infinite

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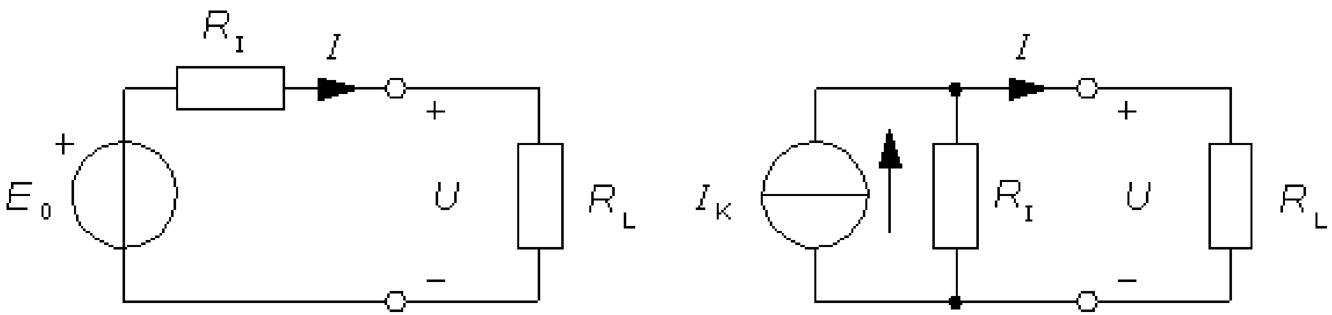
Ideal Voltage/Current sources

A voltage source behaves as an **ideal emf** if the internal resistance $\boldsymbol{R}_{\mathbf{I}}$ is **small** in relation to the use of external resistors.

A voltage source behaves as an **ideal current generator** if the internal resistance $\boldsymbol{R}_{\mathbf{I}}$ is large in relation to the use of external resistors.

Two-terminal Equivalent circuit

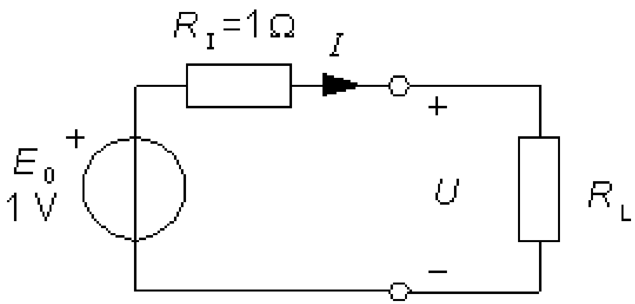
Voltage sources and current sources, can be described either by emf models or current generator models. This applies to any two-terminal circuit, ie two wires leading out from a "general network" consisting of emf’s, resistors, current generators.

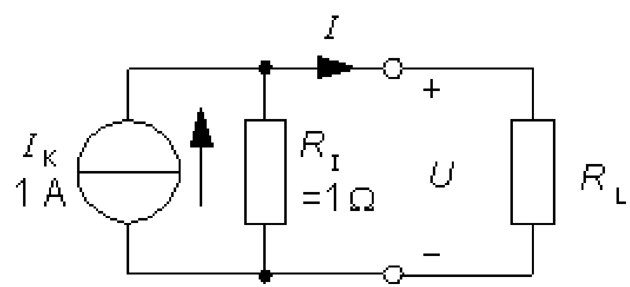


Thévenin voltage source model, and **Norton** current source-model for two-terminal circuits.

Thévenin and Norton

Thévenin and Norton-models are equivalent. Regardless of the external resistor one is connecting to the models they provide the same *U* and *I* !





We compare the two models with external resistor $R_L = 1\ \Omega$

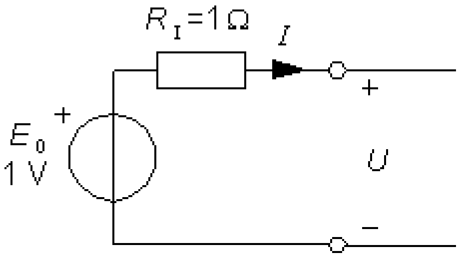
$$U = E \cdot \frac{R_L}{R_I + R_L} = 1 \cdot \frac{1}{1+1} = \boxed{0{,}5\ \text{V}}$$

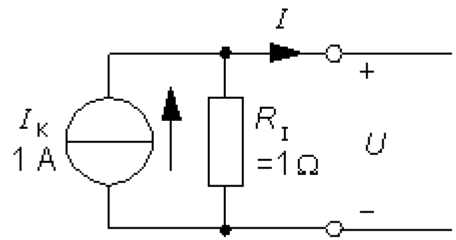
$$U = I_K \cdot \frac{R_I \cdot R_L}{R_I + R_L} = 1 \cdot \frac{1 \cdot 1}{1+1} = \boxed{0{,}5\ \text{V}}$$

$$I = \frac{E}{R_I + R_L} = \frac{1}{1+1} = \boxed{0{,}5\ \text{A}}$$

$$I = I_K \cdot \frac{R_I}{R_I + R_L} = 1 \cdot \frac{1 \cdot 1}{1+1} = \boxed{0{,}5\ \text{A}}$$

Open circuit voltage and short circuit current



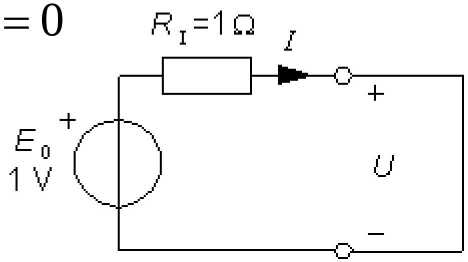


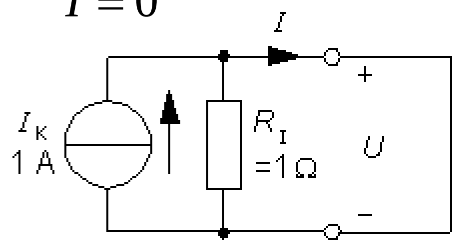
$$U = E_0 - R_I \cdot I = 1 - 1 \cdot 0 = 1\ \mathrm{V}$$

$$I = 0$$

$$U = I_K \cdot R_I = 1 \cdot 1 = 1\ \mathrm{V}$$

$$I = 0$$





$$U = 0$$

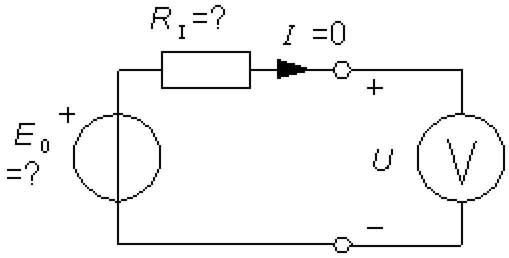
$$I = \frac{E_0}{R_I} = \frac{1}{1} = 1\ \mathrm{A}$$

$$U = 0$$

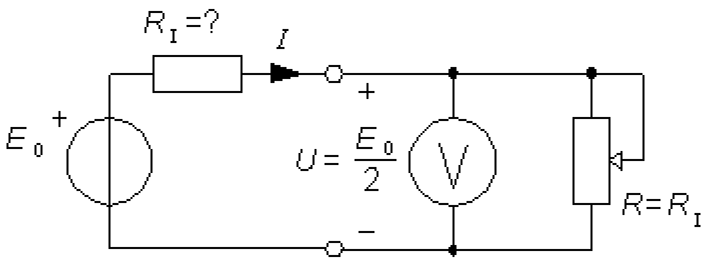
$$I = I_K = 1\ \mathrm{A}$$

Experimental measurement of E_0 and R_I

- E_0 can be measured directly with a good voltmeter. If the test current is ≈0 we get $U = E_0$.



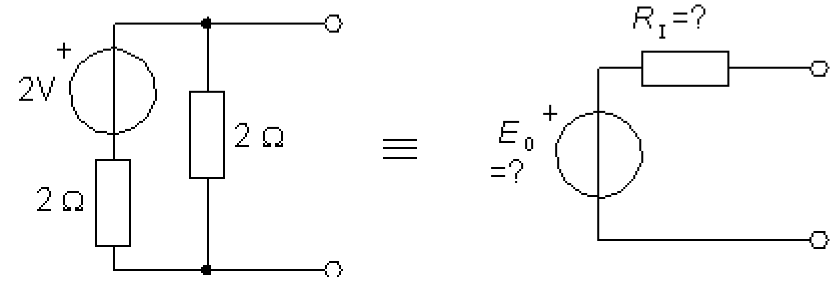
R_I is then determined by loading the circuit with an adjustable resistor so that U drops to $E_0/2$. Then the adjustable resistance has the same value as the internal resistance. $R = R_I$.



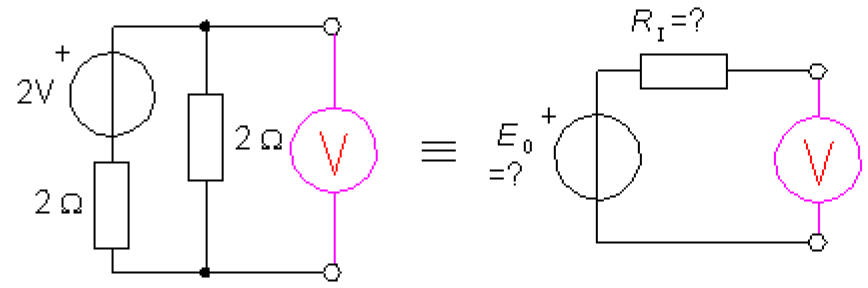
- The adjustable resistance can then be measured with a Ω-meter (= R_I).

Equivalent circuit E_0 (8.3)

Replace a given two-terminal circuit with a simpler circuit having a voltage source in series with a resistor.

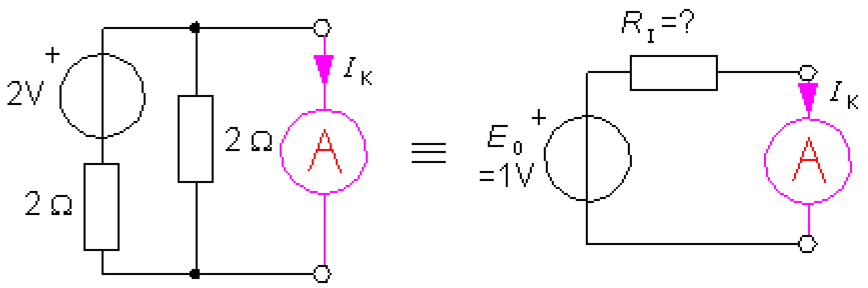


E_0 becomes the same as the given two-terminal open circuit voltage.



$$E_0 = 2\frac{2}{2+2} = 1\,\text{V}$$

Equivalent circuit R_I



After an imagined short circuit, the internal resistance R_I can be calculated with help of the imagined short circuit current I_K.

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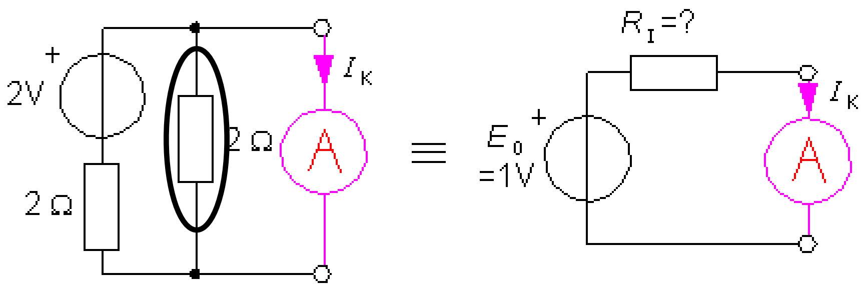
Equivalent circuit R_I

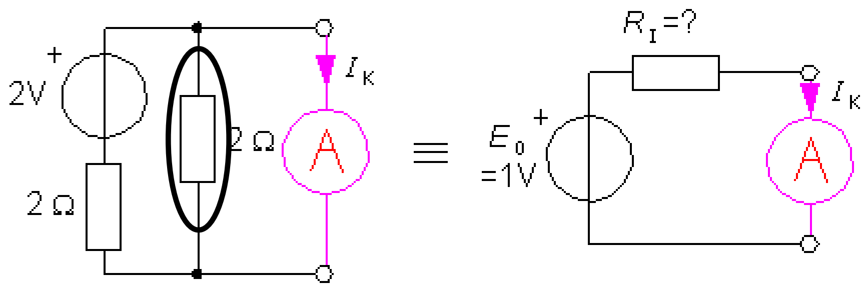


After an imagined short circuit, the internal resistance R_I can be calculated with help of the imagined short circuit current I_K.

When short-circuiting the original circuit the parallel 2Ω-resistor gets no current:

$$I_K = \frac{2}{2} = 1\,\text{A}$$

Equivalent circuit R_I



After an imagined short circuit, the internal resistance R_I can be calculated with help of the imagined short circuit current I_K.

When short-circuiting the original circuit the parallel 2Ω-resistor gets no current:

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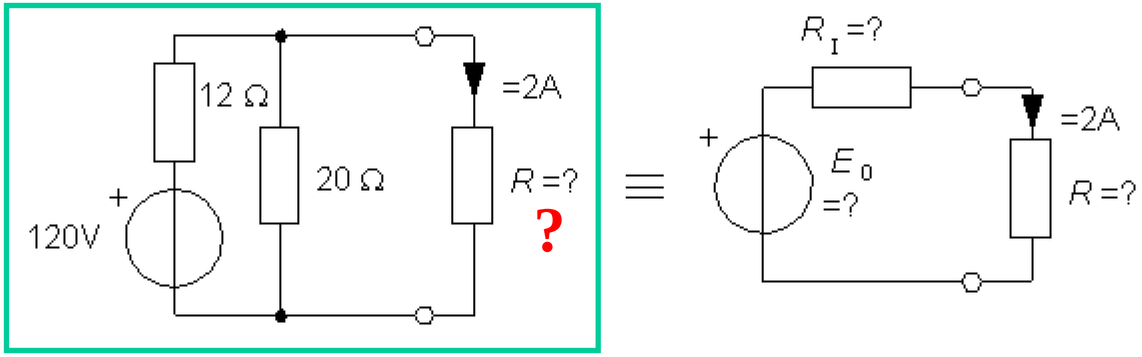
The equivalent circuit's R_I calculates so the short circuit current will be the same:

$$R_I = \frac{E_0}{I_K} = \frac{1}{1} = 1\,\Omega$$

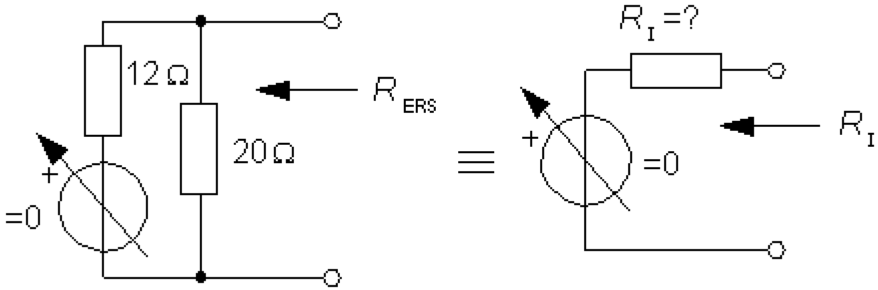
Example with equivalents



What value should *R* have for the current through the resistor will be 2A? If *R* was connected to a two-terminal equivalent the problem would be elemental.

• Let us use two-terminal equivalents as a trick to simpify calculations.

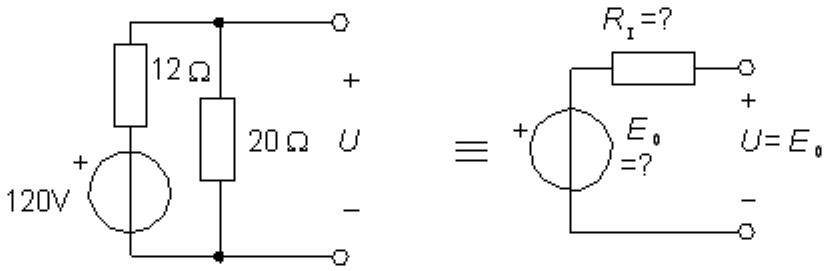
$R_\mathrm{I} = R_\mathrm{ERS}$



If all voltage sources would be halved in the original circuit then of course E_0 in the two-terminal equivalent is also halved. Therefore, if one "turn down" all voltage sources all the way to (almost) "0" in both circuits, there will only be resistors left, then one can see that R_I is equal to the replacement resistance in the original circuit R_ERS:

$$R_\mathrm{I} = R_\mathrm{ERS} = \frac{12 \cdot 20}{12 + 20} = 7{,}5\,\Omega$$

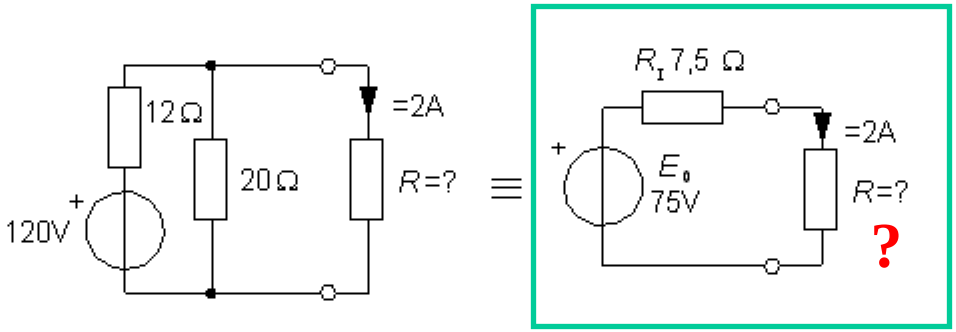
$E_0 = U$ open circuit voltage



If you do not turn down the voltage sources we see that $E_0 = U$ = open circuit voltage:

$$E_0 = U = 120 \cdot \frac{20}{12+20} = 75 \text{ V}$$

Now it is easier to calculate the resistor



$$I = \frac{E_0}{R_\mathrm{I} + R} = 2 \quad \Leftrightarrow \quad \frac{75}{7{,}5 + R} = 2 \quad \Rightarrow \quad R = 30\,\Omega$$

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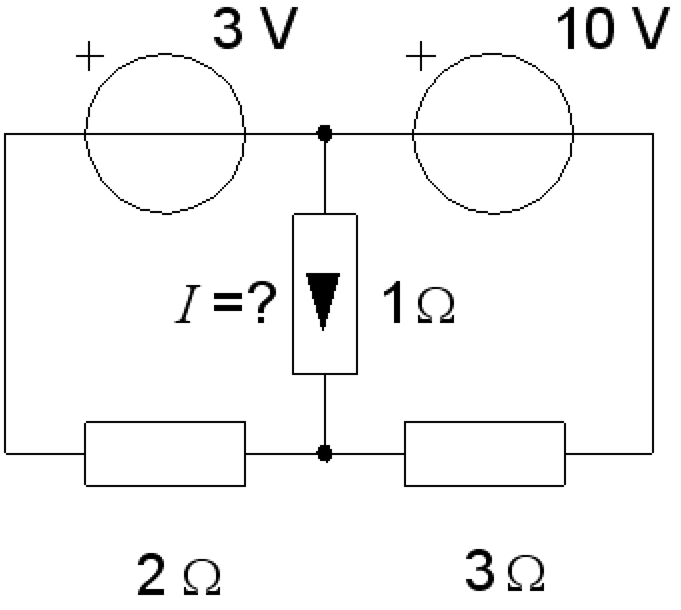
Superposition Principle

If the components and the relationships are linear and independent then the superposition principle is valid.

Nonlinear components such as diodes or nonlinear relationships as power prevents the use of superposition.

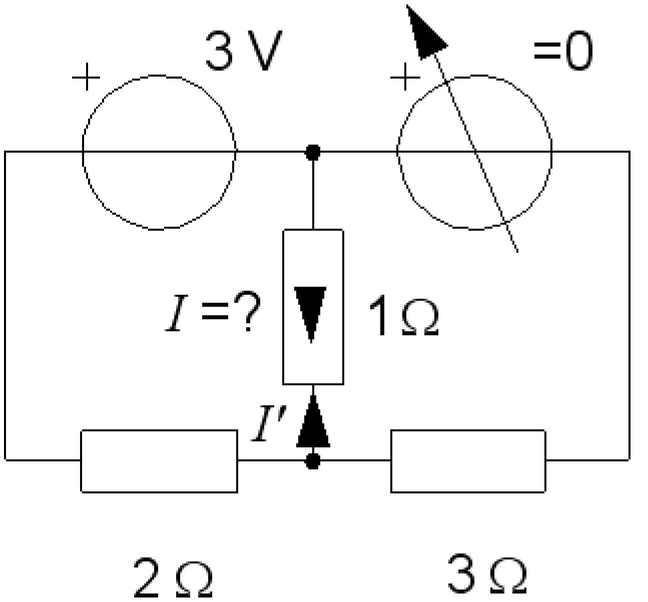
Superposition, only 3V-emf

Turn down the 10V emf to "0" and calculate the contribution *I'* from 3V emf to *I*.



Superposition, only 3V-emk

Turn down the 10V emf to "0" and calculate the contribution *I'* from 3V emf to *I*.

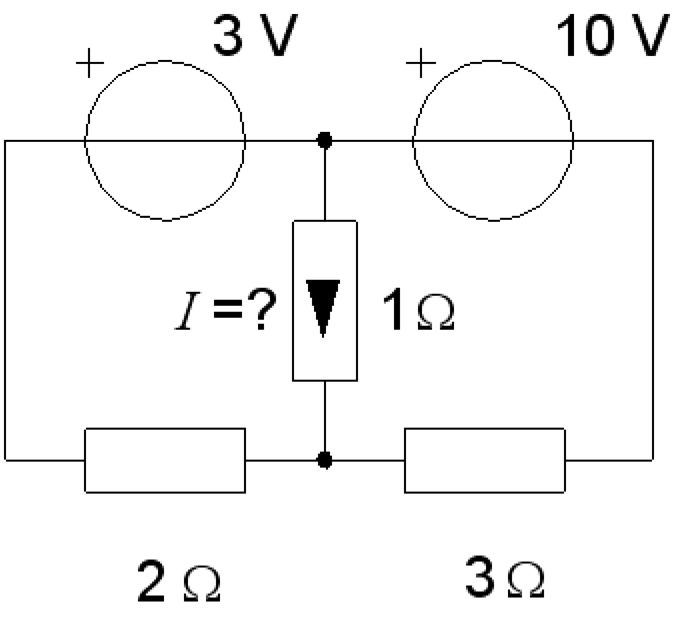


Voltage divider formula:

$$U_{1\Omega} = 3\frac{\frac{1\cdot 3}{1+3}}{\frac{1\cdot 3}{1+3}+2} = 0{,}82 \quad \Rightarrow \quad I' = \frac{U_{1\Omega}}{1} = 0{,}82$$

Superposition, only 10V-emf

Turn down the 3V emf to ”0” and calculate the contribution *I*” from 10V emf to *I*.



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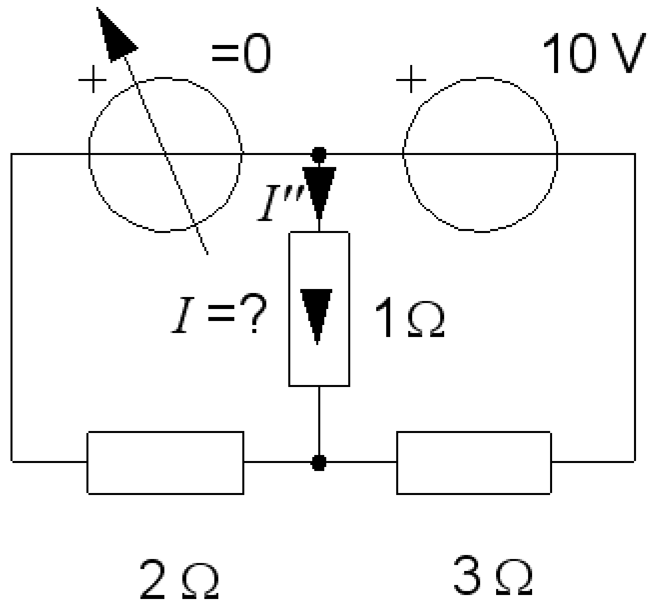

Voltage divider formula:

$$U_{1\Omega} = 10\frac{\frac{1\cdot 2}{1+2}}{\frac{1\cdot 2}{1+2}+3} = 1{,}82 \quad \Rightarrow \quad I'' = \frac{U_{1\Omega}}{1} = 1{,}82$$

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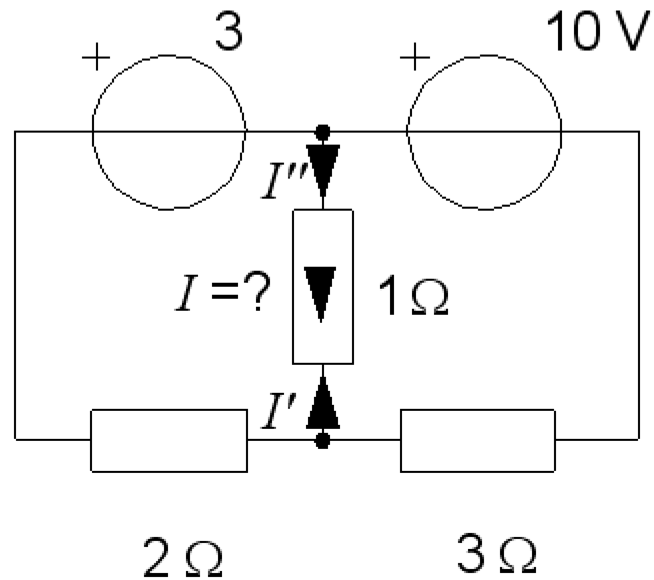
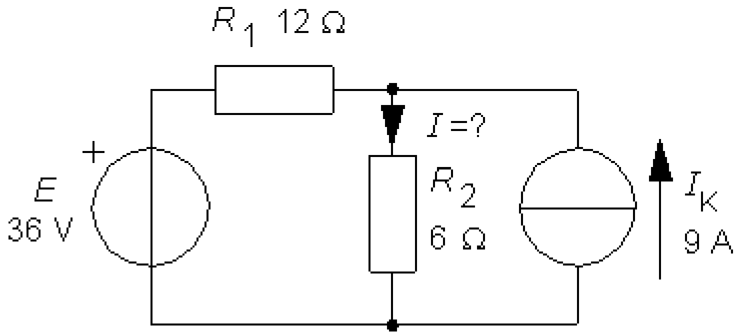
Superposition – adding the contributions



$$I = I'' - I' = 1{,}82 - 0{,}82 = 1 \text{ A}$$

Calculate *I* with superposition (8.7)



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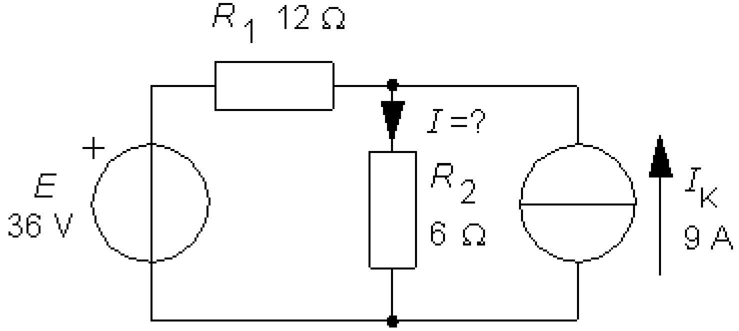
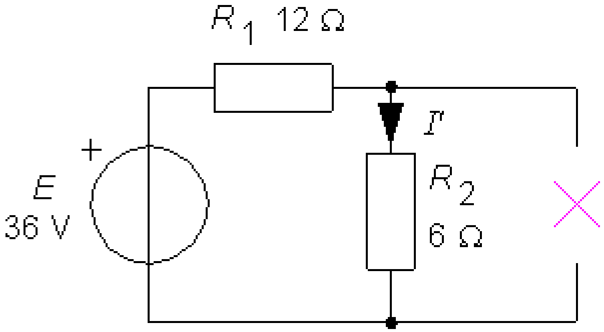


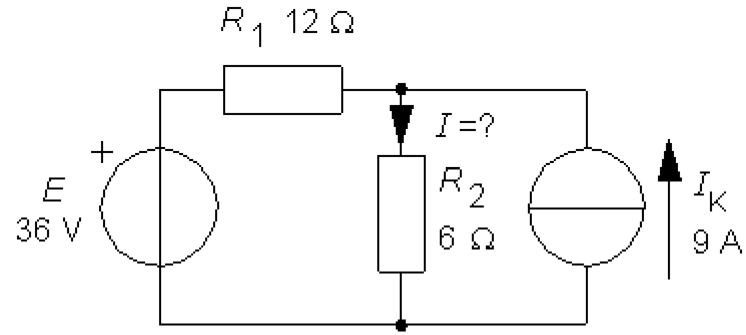


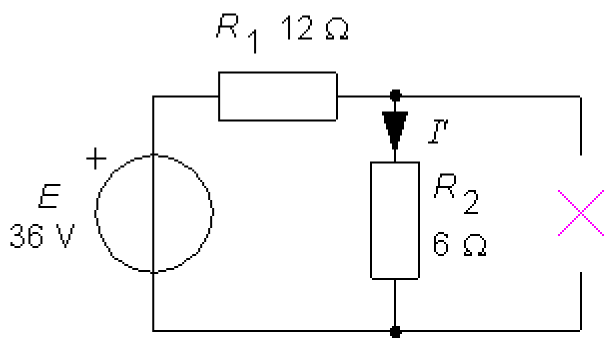
$$I' = \frac{E}{R_1 + R_2} = \frac{36}{12+6} = 2$$

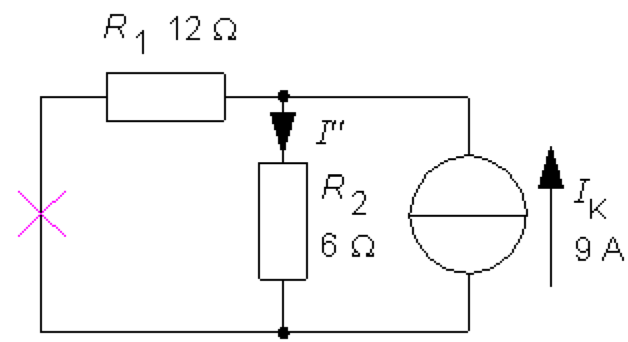
OHM's law

- *A turned down current generator becomes a Break!*

Calculate *I* with superposition (8.7)







$$I' = \frac{E}{R_1 + R_2} = \frac{36}{12+6} = 2$$

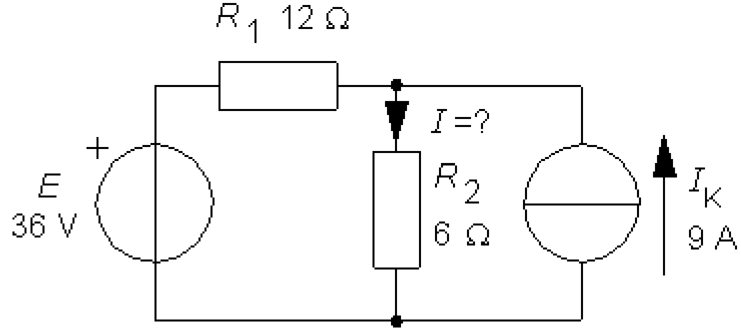
OHM's law

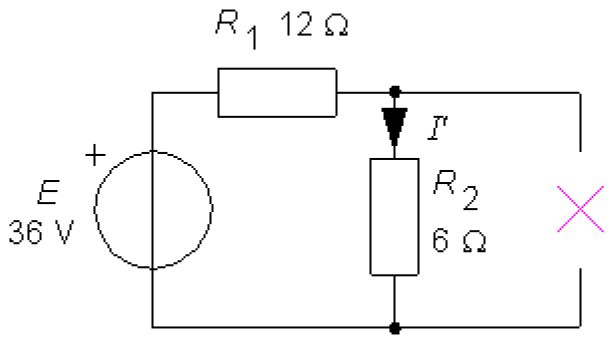
$$I'' = I_K \cdot \frac{R_1}{R_1 + R_2} = 9 \cdot \frac{12}{12+6} = 6$$

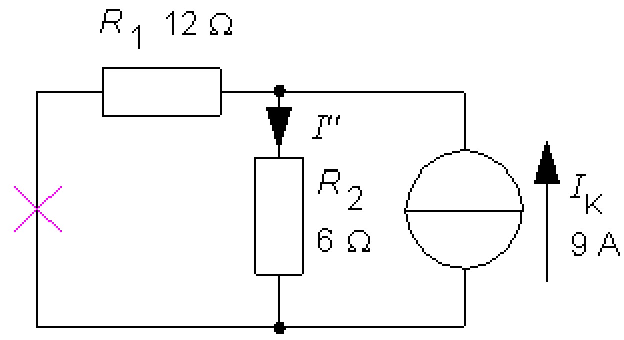
Current branching

- *A turned down voltage source becomes a short circuit!*

Calculate *I* with superposition (8.7)







$$I' = \frac{E}{R_1 + R_2} = \frac{36}{12+6} = 2$$

OHM's law

$$I'' = I_K \cdot \frac{R_1}{R_1 + R_2} = 9 \cdot \frac{12}{12+6} = 6$$

Current branching

$$I = I' + I'' = 2 + 6 = 8 \text{ A}$$

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