IE1206 Embedded Electronics

Le1 → Le2 — PIC-block Documentation, Seriecom Pulse sensors

Le3 → Ex1 → KC1 LAB1 — *I*, *U*, *R*, *P*, serial and parallel

Pulse sensors, Menu program

- ***Start of programing task***

Le4 → Ex2 — Kirchhoffs laws Node analysis Two-terminals R2R AD

Le5 → Ex3 → KC2 LAB2 — ***Two-terminals, AD, Comparator/Schmitt***

Le6 → Ex4 → Le7 → KC3 LAB3 — Transients PWM

Step-up, RC-oscillator

Le8 → Ex5 → Le9 — Phasor jω PWM CCP CAP/IND-sensor

Ex6 → Le10 → Le11 → KC4 LAB4 — ***LC-osc, DC-motor, CCP PWM***

Le12 → Ex7 → Display — LP-filter Trafo

- ***Display of programing task***

Le13 → Written exam — Trafo, Ethernet contact

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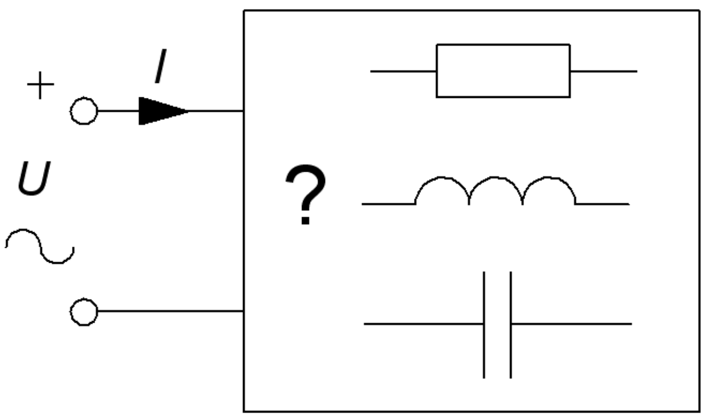
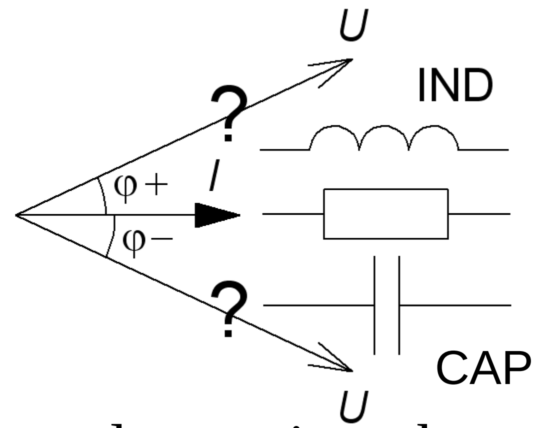
R L C





An impedance which contain inductors and capacitors have, depending on the frequency, either inductive character **IND**, or capacitive character **CAP**.

An important special case occurs at the frequency where capacitances and inductances are equally strong, and their effects cancel each other out. The impedance becomes purely resisistiv. The phenomenon is called the **resonance** and the frequency on which this occurs is the **resonant frequency**.

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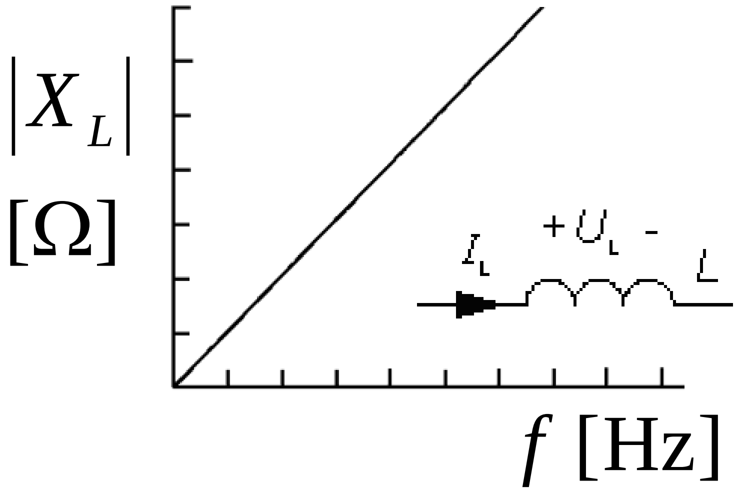
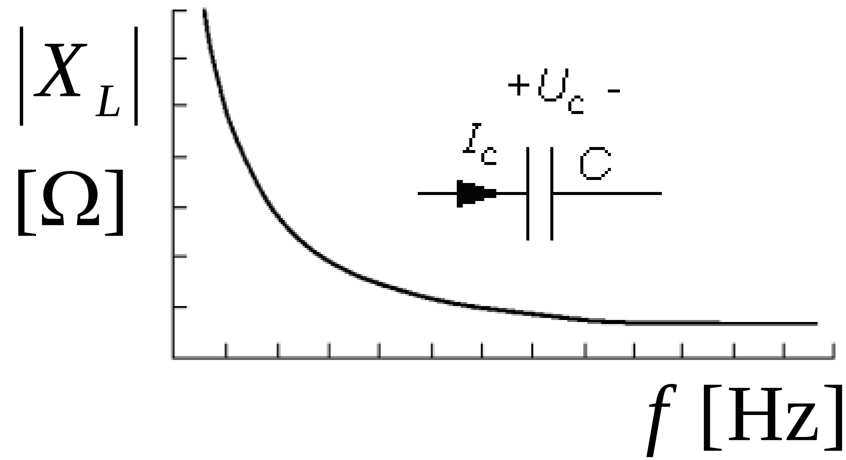
Reactance frequency dependency

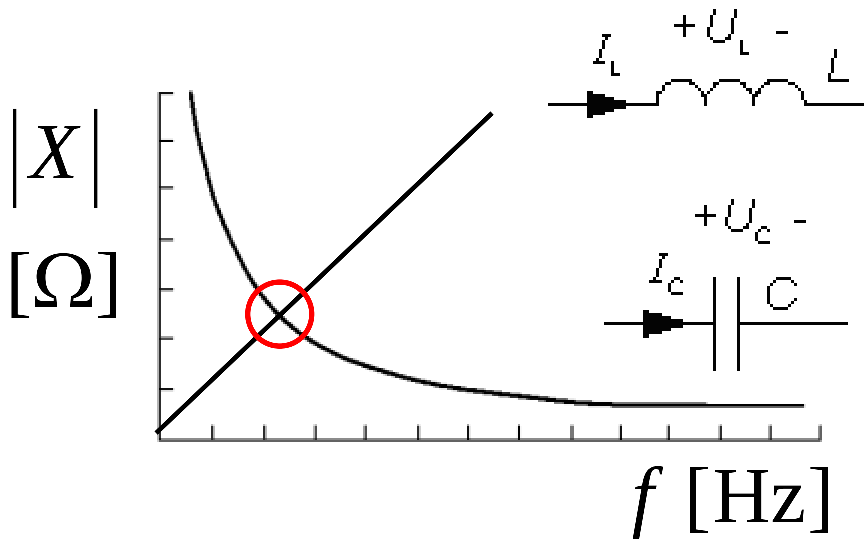




$$|X_L| = \omega \cdot L \qquad |X_C| = \frac{1}{\omega \cdot C}$$

$$\omega = 2\pi\, f$$

R L C impedances



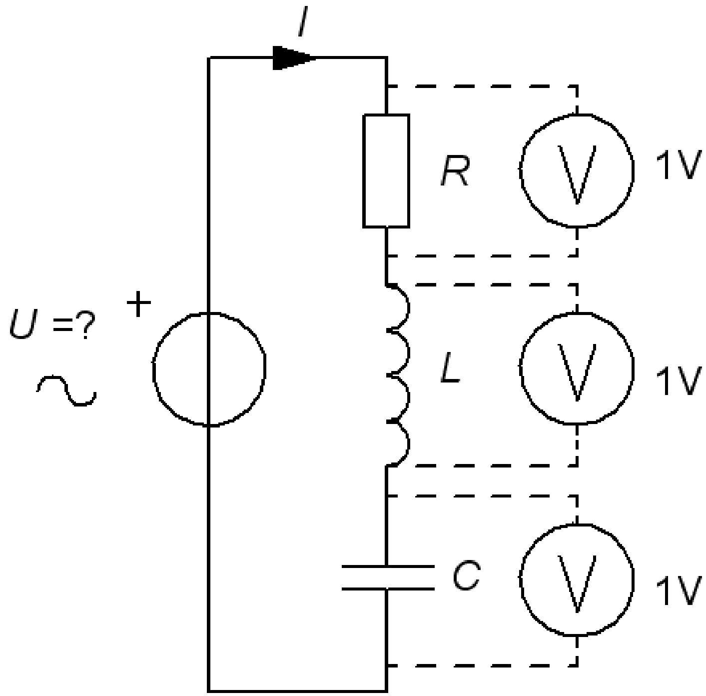
- At a certain frequence X_L and X_C has the same amount.

$$|X_L| = \omega \cdot L \qquad |X_C| = \frac{1}{\omega \cdot C}$$

$$\omega = 2\pi f$$

How big is *U* ? (13.1)

The three volt meters show the same, 1V, how much is the alternating supply voltage *U* ? (*Warning*, *teaser*)



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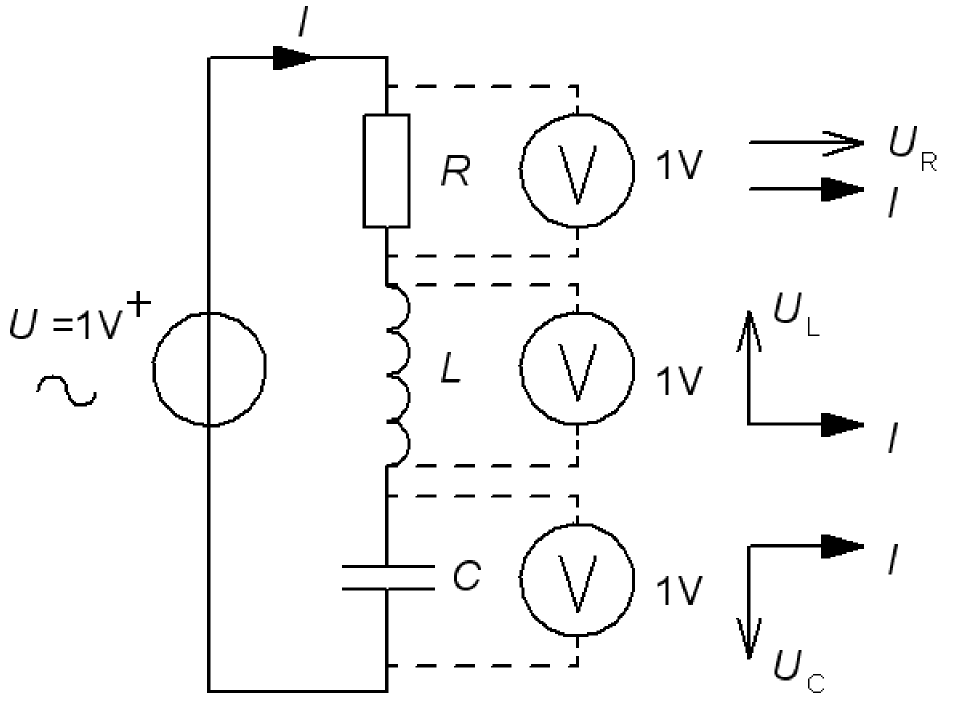

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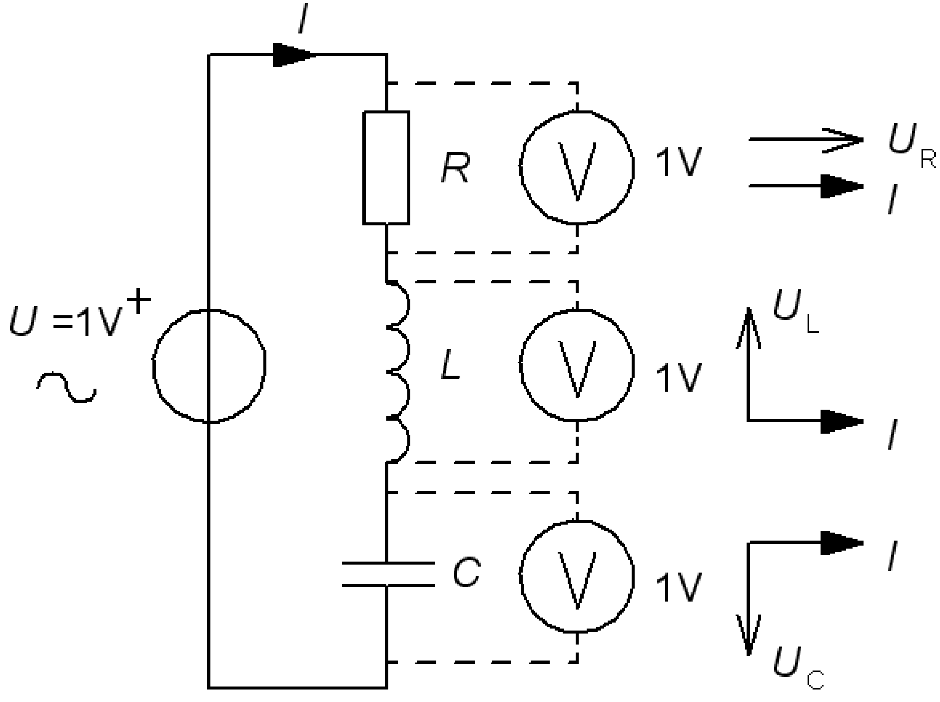


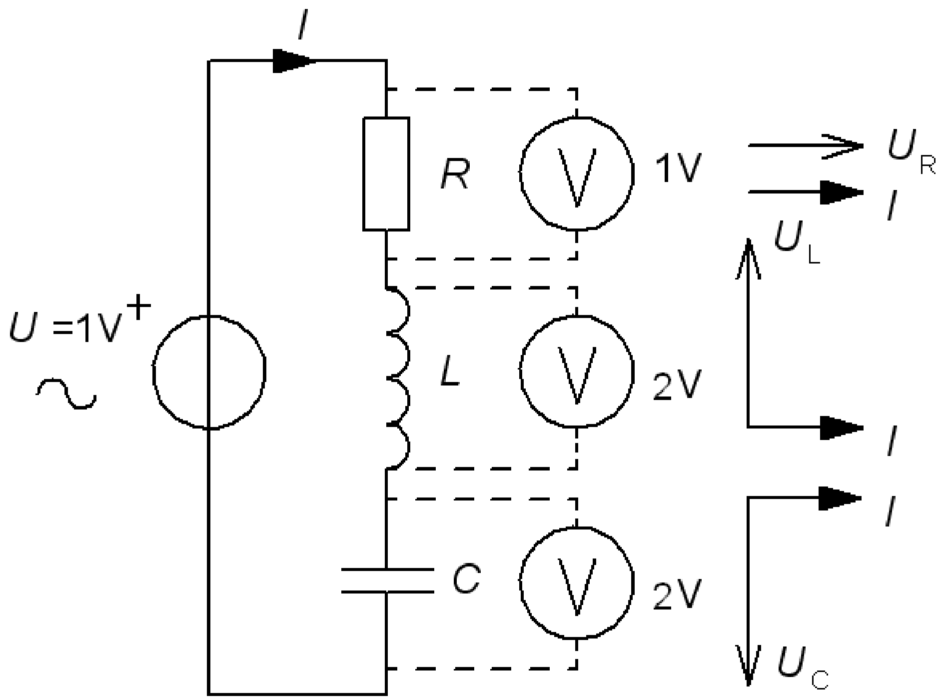
Since volt meters show the "same" and the current *I* is common:

$$R = |X_L| = |X_C| \qquad R = \omega L = \frac{1}{\omega C}$$

If $|X_L|=|X_C|=2R$?

Suppose the AC voltage *U* still 1 V, but the reactances are *twice* as big. What will the voltmeters show?

$$\omega L = \frac{1}{\omega C} = 2 \cdot R$$



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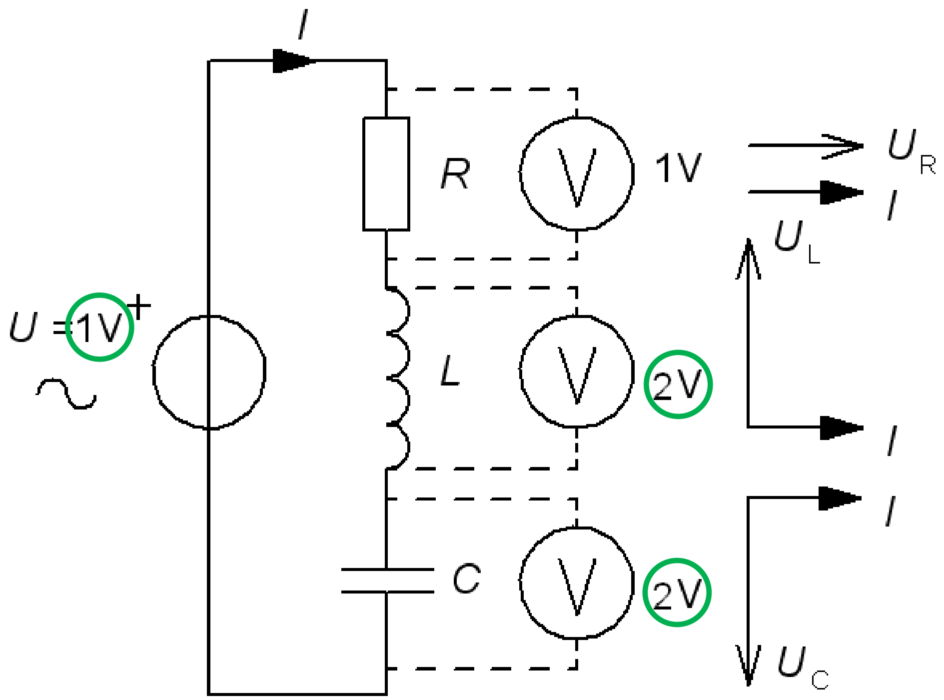

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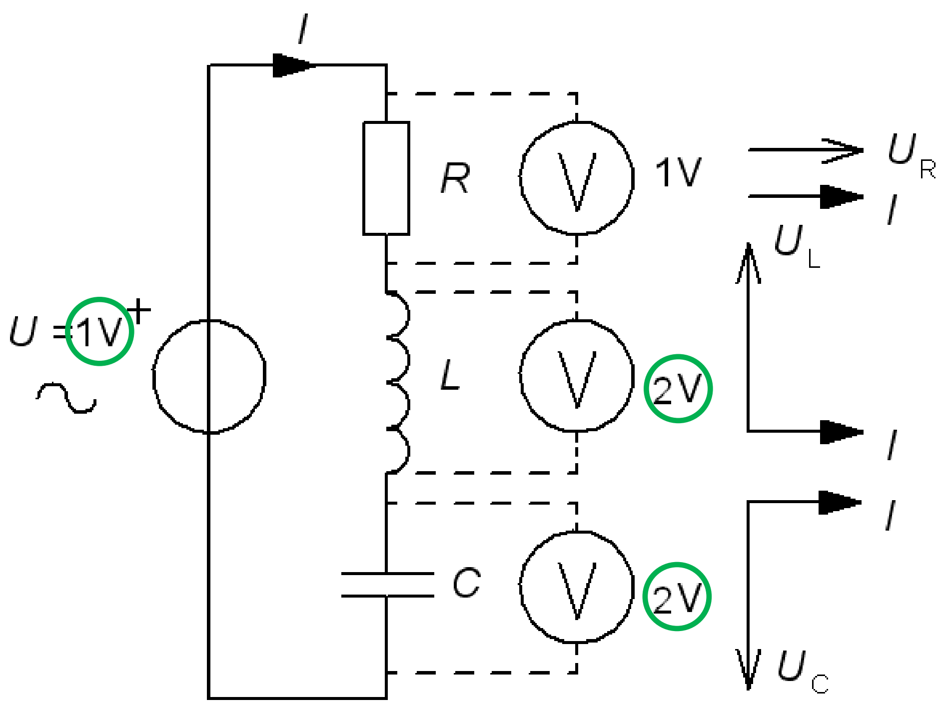

At resonance, the voltage over the reactances can be many times higher than the AC supply voltage.

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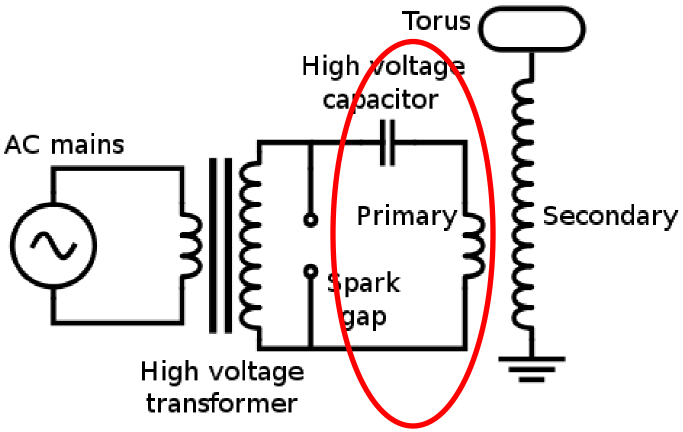
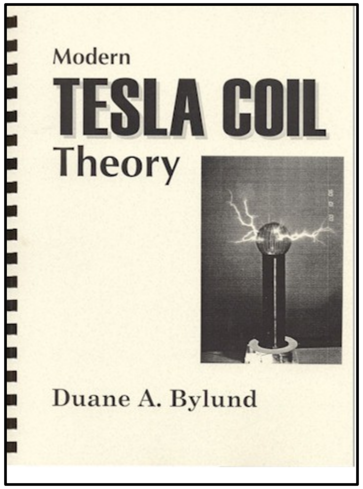
Tesla coil

Many builds "Tesla" coils to gain some excitement in life…





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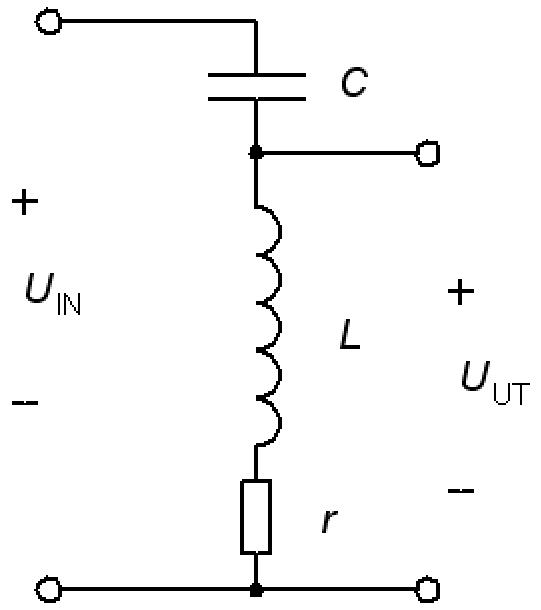
Inductor quality factor *Q*

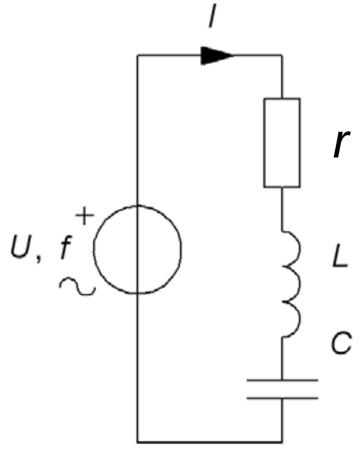
Usually it is the internal resistance of the coil which is the resistor in the RLC circuit. The higher the coil AC resistance ωL is in relation to the DC resistance r, the larger the voltage across the coil at a resonance get. This ratio is called the coil quality factor Q.
(or Q-factor).



$$Q = \frac{X_L}{r} = \frac{\omega L}{r} \quad \Rightarrow \quad \boxed{U_{UT} \approx Q \cdot U_{IN}}$$

Series resonance

$$\underline{U} = \underline{I} \cdot \left(r + \mathrm{j}\omega L + \frac{1}{\mathrm{j}\omega C} \right) = \underline{I} \cdot \left(r + \mathrm{j}(\omega L - \frac{1}{\omega C}) \right)$$



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Series resonance

=0

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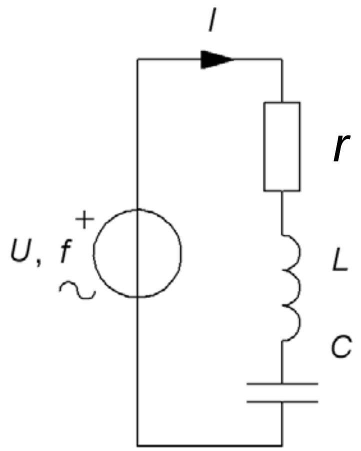

The Impedance is real when the imaginary part is ”0”. This will happen at angular frequency ω_0 (frequency f_0).

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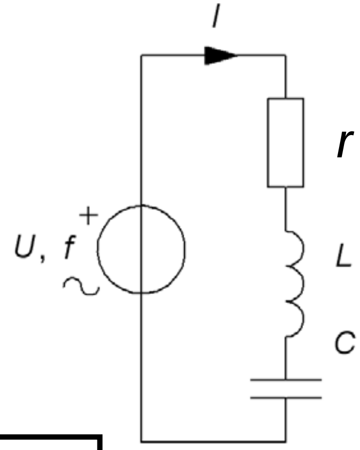

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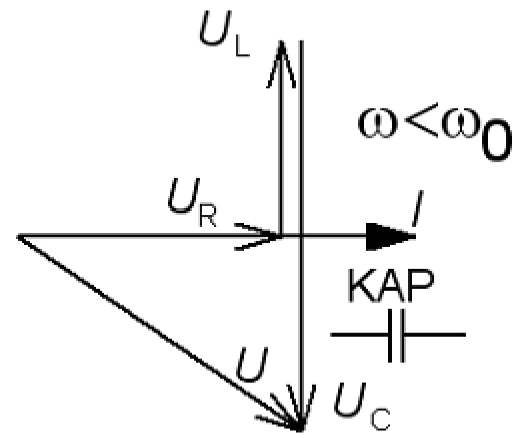
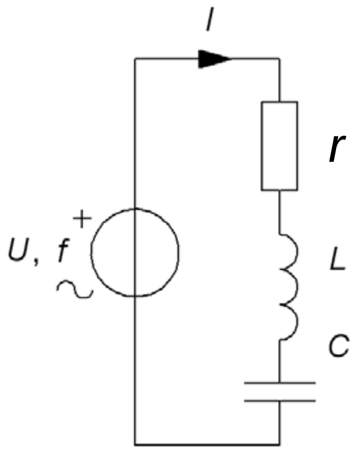
This will happen at angular frequency ω_0 (frequency f_0).

$$\mathrm{Im}\left[\underline{Z}\right] = \omega L - \frac{1}{\omega C} = 0 \quad \Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}} \qquad \boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

Series resonance phasor diagram

$$\underline{U} = \underline{I} \cdot \left(r + \mathrm{j}(\omega L - \frac{1}{\omega C}) \right)$$





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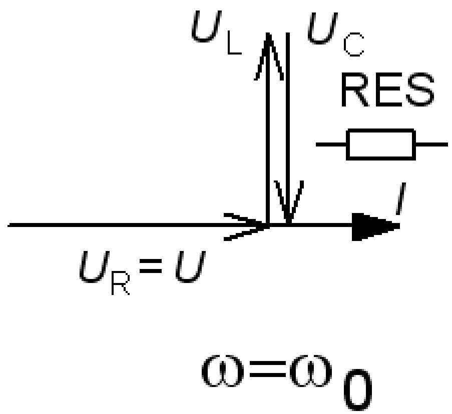

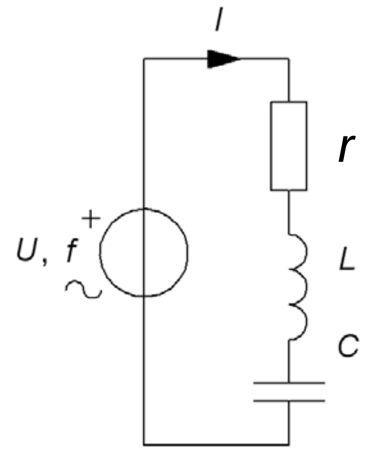

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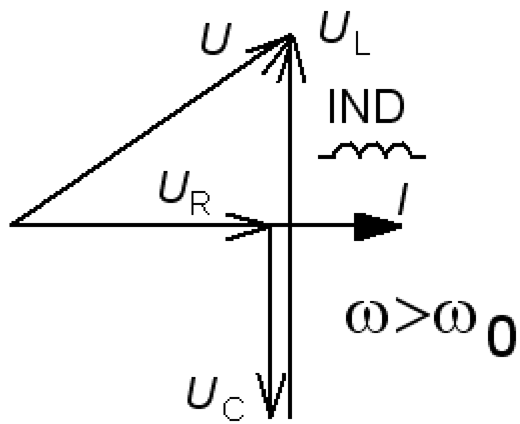
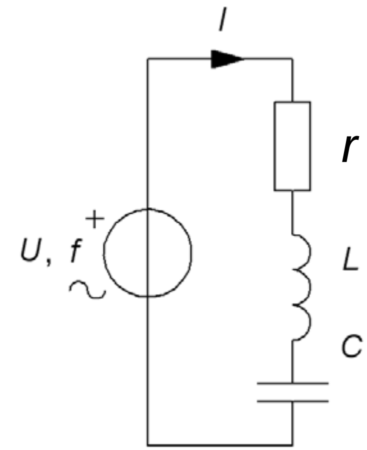
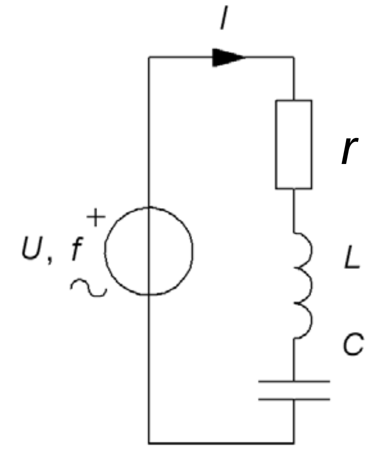




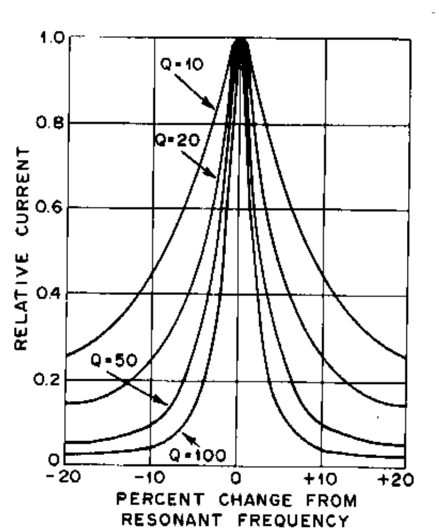
Series resonance circuit Q

It is the resistance of the resonant circuit, usually coil internal resistance, which determines how pronounced resonance phenomenon becomes. It is customary to "normalize" the relationship between the different variables by introducing the resonance angular frequency $\boldsymbol{\omega_0}$ together with the peak current $\boldsymbol{I_{max}}$ in the function $I(\omega)$ with parameter $\boldsymbol{Q}$:



$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Q = \frac{\omega_0 L}{r}$$

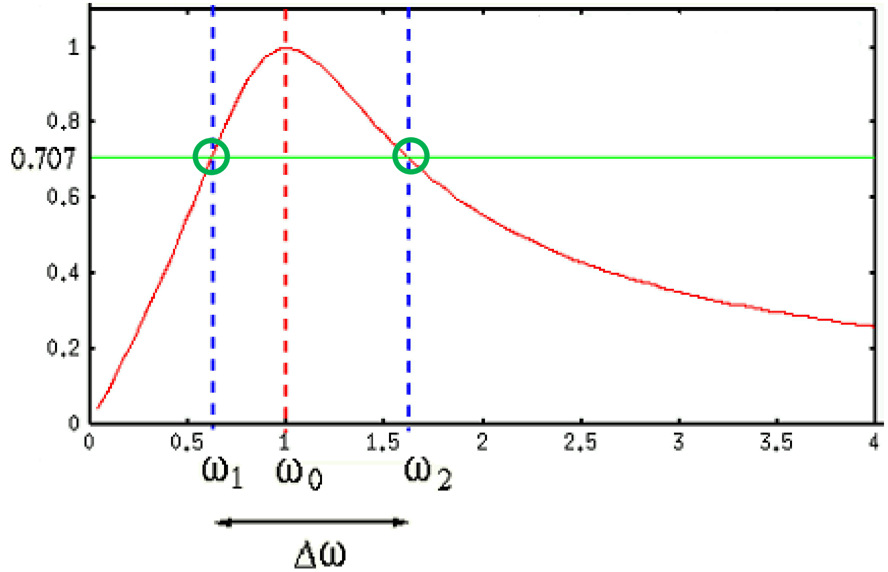
$$\underline{I} = \frac{I_{\max}}{\left(1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right)}$$



Normalized chart of the series resonant circuit. A high Q corresponds to a narrow resonance peak.

Bandwidth *BW*

At two different angular frequencies becomes imaginary Im and real part Re in the denominator equal. I is then $I_{max}/\sqrt{2}$ (≈71%).
The **Bandwidth** $BW = \Delta\omega$ is the distans between those two angular frequencies.



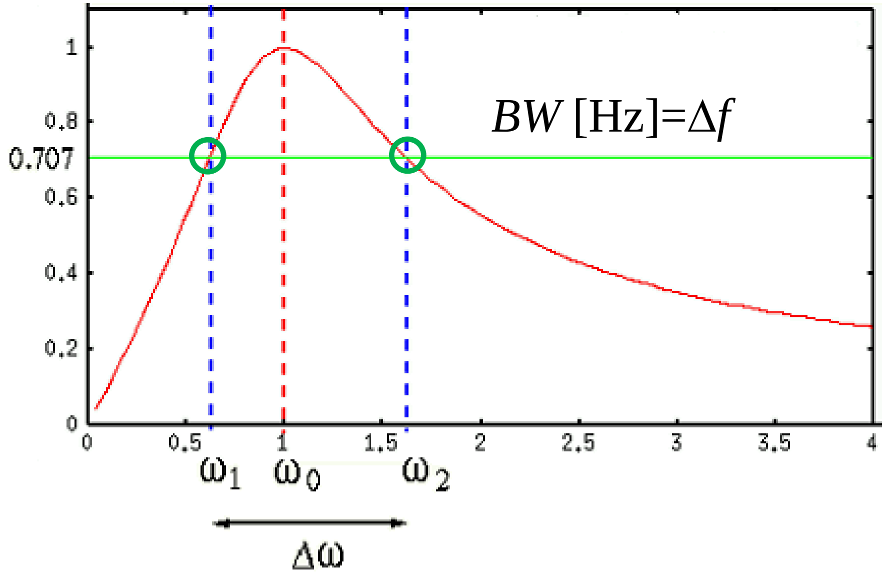
$$\underline{I} = \frac{I_{max}}{\left(\boxed{1} + j\boxed{Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}\right)}$$

Re = Im

The equations give :

$$BW\left[\text{rad/s}\right] = \Delta\omega = \omega_2 - \omega_1 = \frac{\omega_0}{Q} \qquad \omega_0^2 = \omega_2 \cdot \omega_1 \qquad \omega_2, \omega_1 = \omega_0\left(\pm\frac{1}{2Q} + \sqrt{\frac{1}{(2Q)^2} + 1}\right)$$

• More convenient formulas



$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$Q = \frac{2\pi f_0 L}{r}$$

$$\frac{\Delta\omega}{\omega_0} = \frac{1}{Q} \Rightarrow \frac{\Delta f}{f_0} = \frac{1}{Q}$$

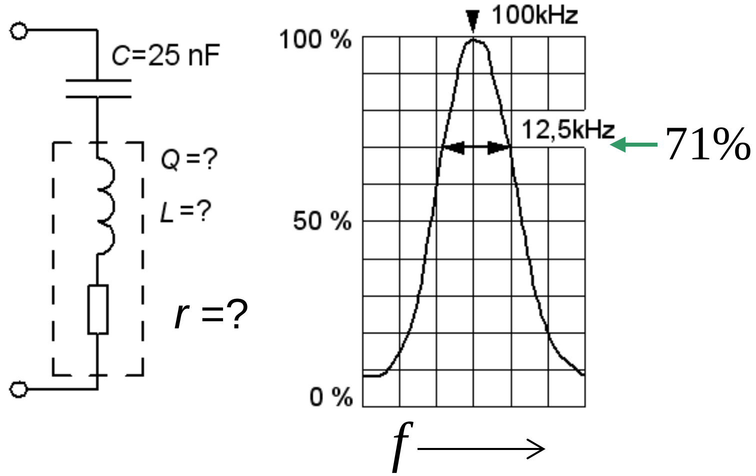
If Q is high, no significant error is done if the bandwidth is divided equally on both sides of f_0.

$$f_2, f_1 \approx f_0 \pm \frac{\Delta f}{2}$$

Example, series resonance circuit

C = 25 nF
f_0 = 100 kHz
$BW = \Delta f$ = 12,5 kHz

Q = ? L = ? r = ?



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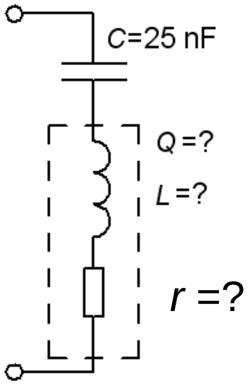

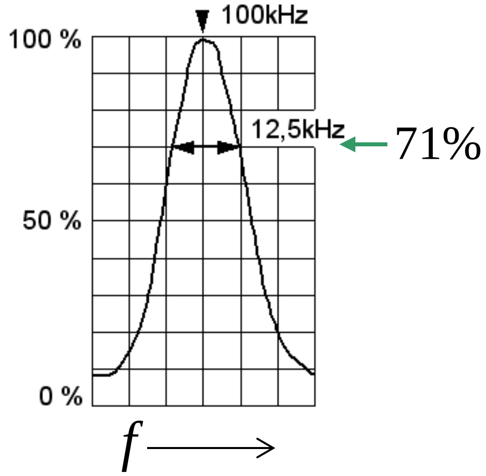

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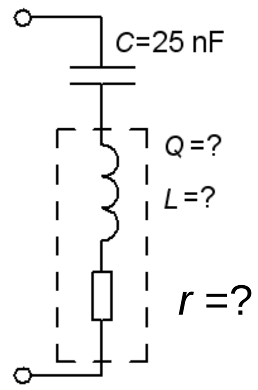

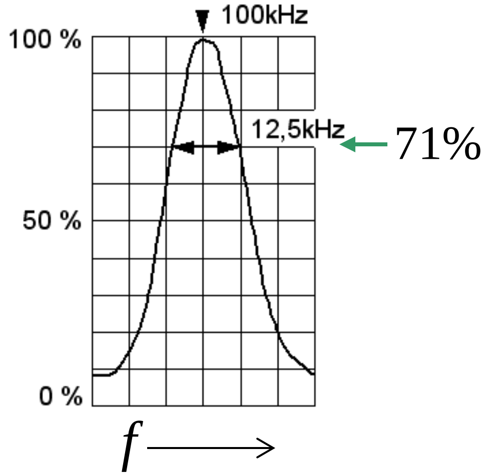

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$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \Rightarrow \quad L = \frac{1}{(2\pi f_0)^2 C} = \frac{1}{(2\pi \cdot 100 \cdot 10^3)^2 \cdot 25 \cdot 10^{-9}} = 0{,}1\,\text{mH}$$

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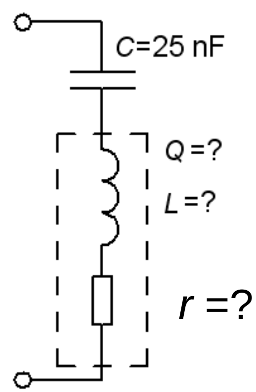

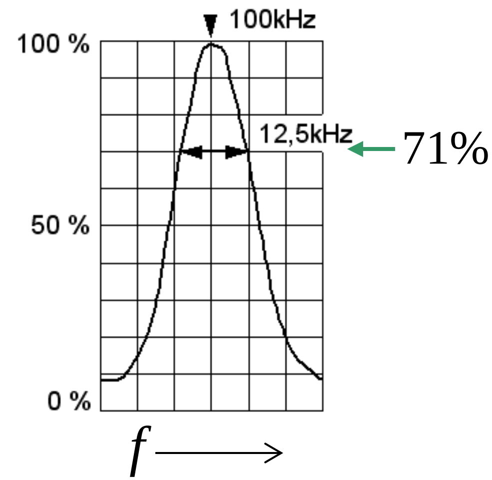

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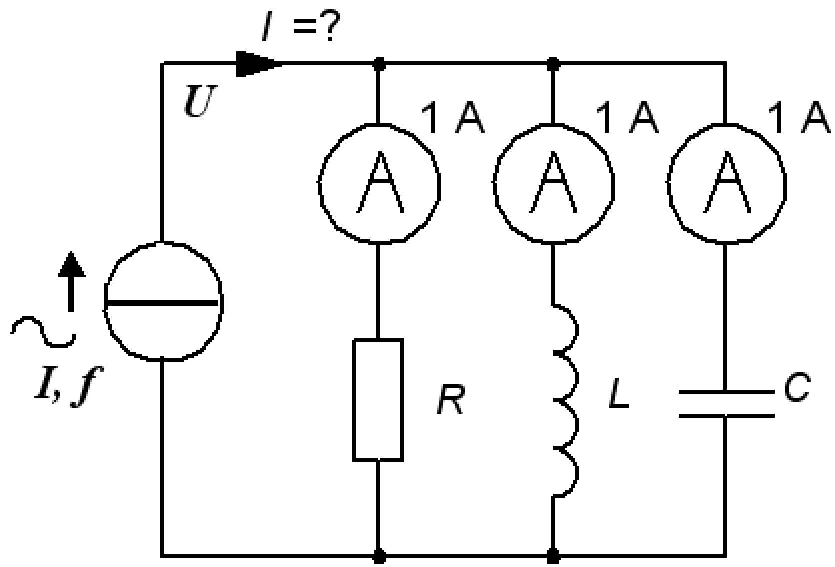
$$Q = \frac{X_L}{r} = \frac{2\pi f_0 \cdot L}{r} \quad \Rightarrow \quad r = \frac{2\pi f_0 \cdot L}{Q} = \frac{2\pi \cdot 100 \cdot 10^3 \cdot 0{,}1 \cdot 10^{-3}}{8} \approx 8\,\Omega$$

How big is *I* ? (13.2)

The three ammeters show the same, 1A, how much is the AC supply current *I* ?   (*Warning*, *teaser*)



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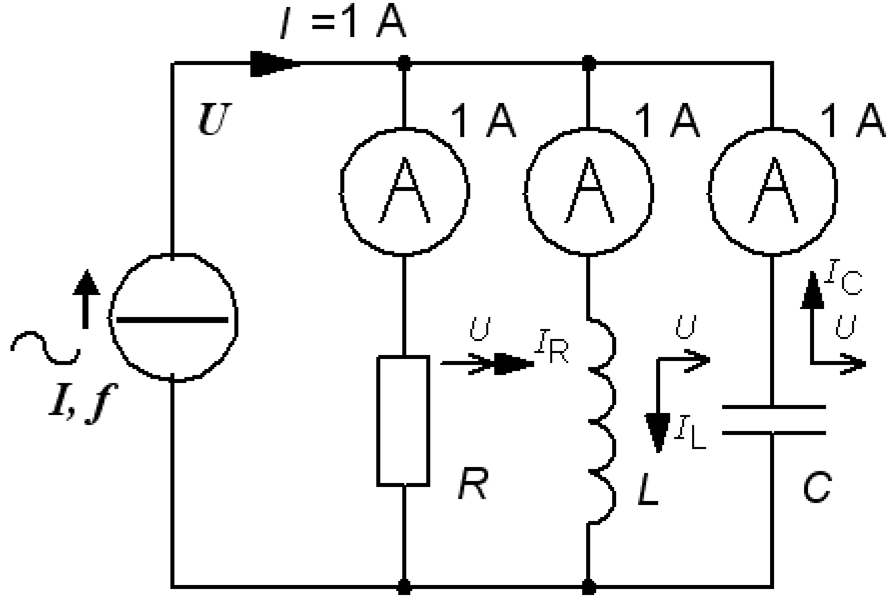

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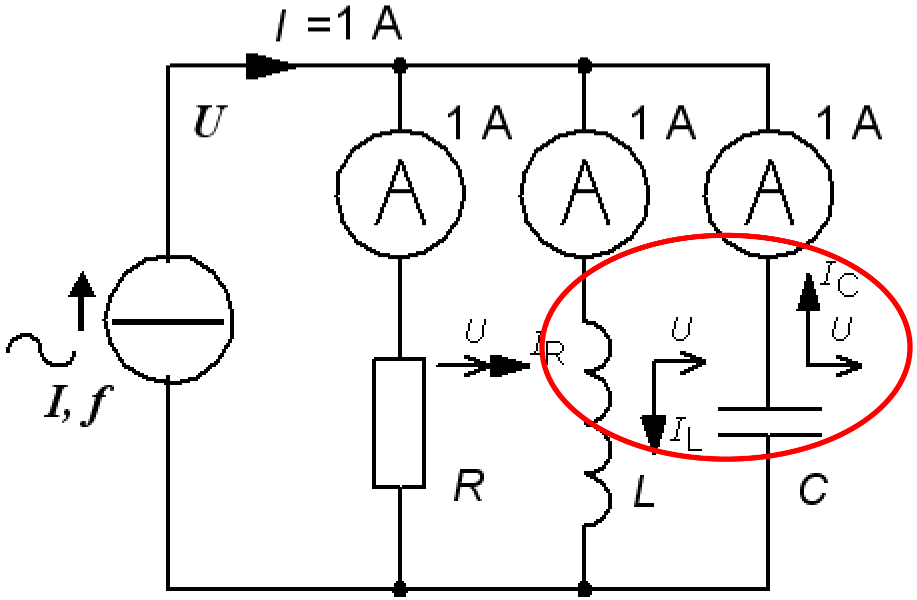
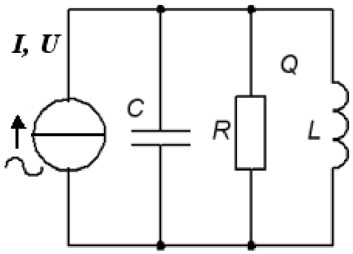


I_L and I_C becomes a circulating current decoupled from I_R. I_L, I_C can be *many times bigger* than the supply current $I = I_R$. This is parallel resonance.

Ideal parallel resonance circuit

$$\underline{Z} = R \parallel L \parallel C = \frac{1}{\frac{1}{R} + \frac{1}{j\omega L} + j\omega C} = \frac{1}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)}$$

=0



The resonance frequency has exactly the same expression as for the series resonant circuit, but otherwise the circuit has **reverse character**, **IND** at low frequencies and **CAP** at high. At resonance, the impedance is real = *R*.

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

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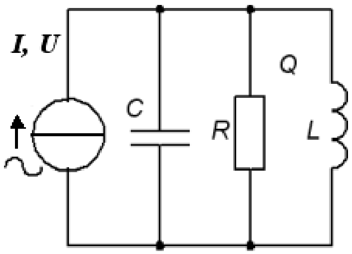

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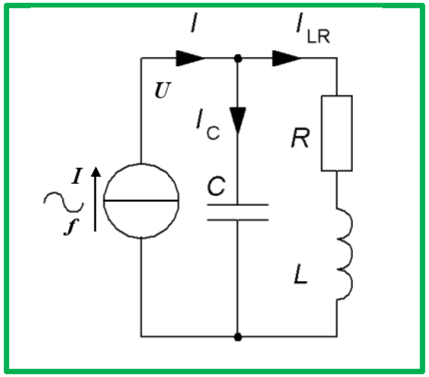
Actual parallel resonant circuit

Actual parallel resonant circuits has a series resistance inside the coil. The calculations become more complecated and the resonance frequency will also differ slightly from our formula.

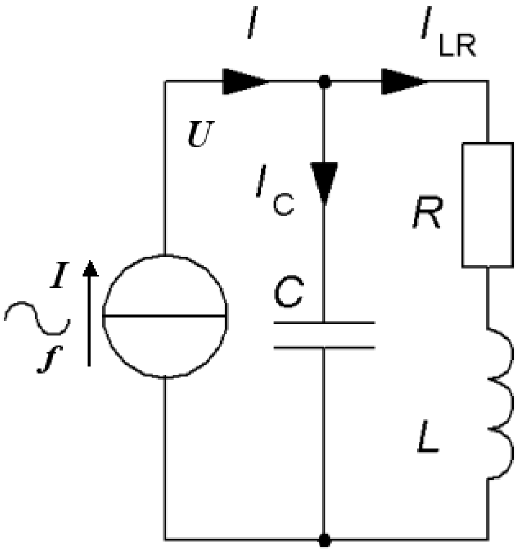


Example, actual circuit (13.3)

$$\underline{I} = \underline{I}_{\mathrm{C}} + \underline{I}_{\mathrm{LR}} = \frac{U}{\dfrac{1}{\mathrm{j}\omega C}} + \frac{U}{r + \mathrm{j}\omega L} \cdot \frac{(r - \mathrm{j}\omega L)}{(r - \mathrm{j}\omega L)} = U \cdot \left(\mathrm{j}\omega C + \frac{r - \mathrm{j}\omega L}{r^2 + (\omega L)^2} \right) =$$

$$= U \cdot \left(\frac{r}{r^2 + (\omega L)^2} + \mathrm{j}\left(\omega C - \frac{\omega L}{r^2 + (\omega L)^2} \right) \right)$$

=0



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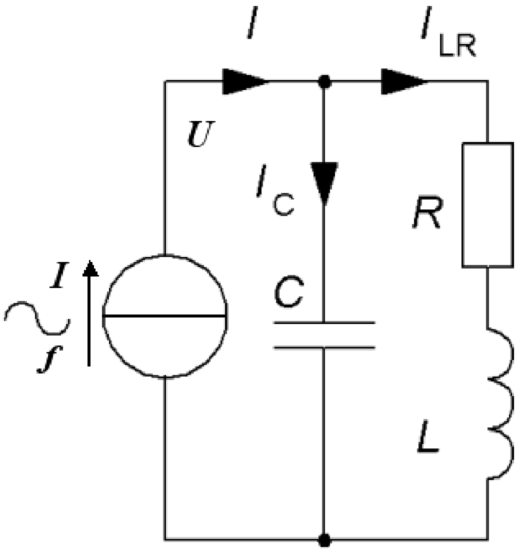
=0



$$\omega_0 C = \frac{\omega_0 L}{r^2 + (\omega_0 L)^2} \quad \Rightarrow \quad \omega_0^{\,2} = \frac{1}{LC} - \frac{r^2}{L^2} \quad \omega_0 = 2\pi f \quad \Rightarrow \quad f_0 = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{r^2}{L^2} \right)}$$

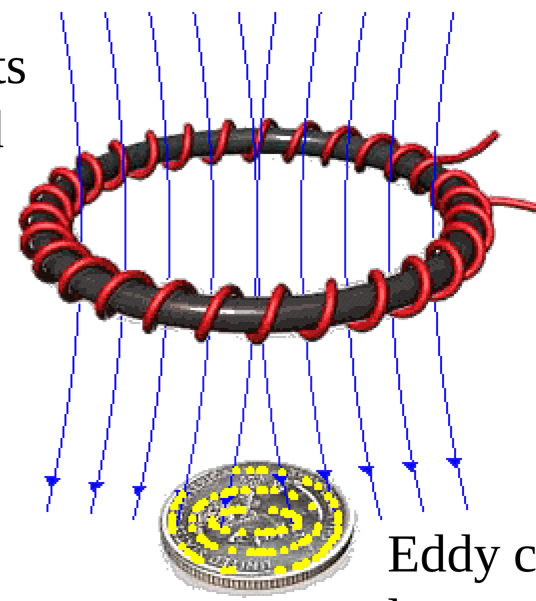
Metal Detector

$$f_0 = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{r^2}{L^2}\right)}$$

Any "losses" (even eddy-current losses in all kinds of metals) are summarized by the symbol r !

Iron objects affects the magnetic field and thus also L !

The parallel resonant frequency is affected by the coil losses. That’s how hidden treasures are found!



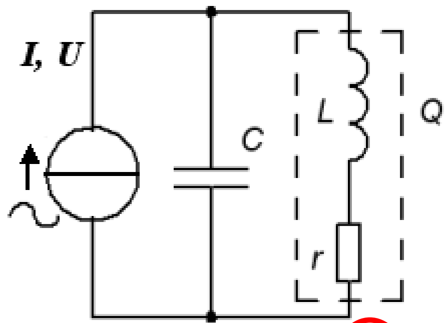
Eddy current losses

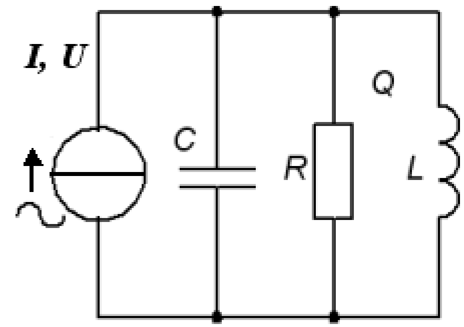
Series or parallel resistor

In manual computation for simplicity one usually uses the formulas of the ideal resonant circuit. At high Q and close to the resonance frequency f_0 the deviations becomes insignificant.

At **Q >10** are the two circuits "interchangeable".

$$\omega_0 \approx \frac{1}{\sqrt{LC}}$$





Alternative definition of Q with R_P

$$Q = \frac{\omega_0 L}{r_S} = \frac{R_P}{\omega_0 L} \Rightarrow R_P = Q^2 \cdot r_S$$

(*applies approximately for* ***Q >10***)

Example, parallel circuit

Parallel circuit.

$C = 25$ nF

$f_0 = 100$ kHz

$BW = 1250$ Hz

$L = ?$ $r = ?$

Example, parallel circuit

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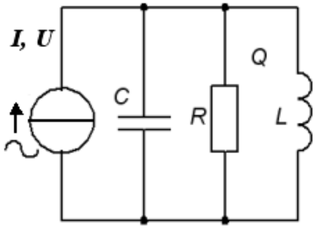
L = ? r = ?

$$Q = \frac{f_0}{\Delta f} = \frac{100 \cdot 10^3}{1250} = 80$$

Example, parallel circuit



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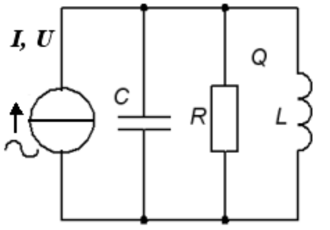

L = ? r = ?

$$Q = \frac{f_0}{\Delta f} = \frac{100 \cdot 10^3}{1250} = 80$$

80 > 10 justifying counting with the ideal model.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \Rightarrow \quad L = \frac{1}{(2\pi f_0)^2 C} = \frac{1}{(2\pi \cdot 100 \cdot 10^3)^2 \cdot 25 \cdot 10^{-9}} = 0{,}1\,\text{mH}$$

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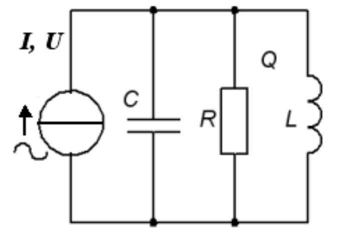

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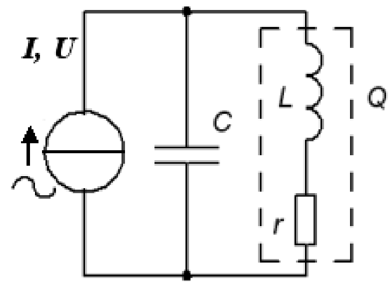
$$Q = \frac{R_\text{P}}{X_\text{L}} = \frac{R_\text{P}}{2\pi f_0 \cdot L} \quad \Rightarrow \quad R_\text{P} = 2\pi f_0 \cdot L \cdot Q = 2\pi \cdot 100 \cdot 10^3 \cdot 0{,}1 \cdot 10^{-3} \cdot 80 \approx 5027\ \Omega$$

Example, parallel circuit

Parallel circuit.
C = 25 nF
f_0 = 100 kHz
BW = 1250 Hz

Answer with a series resistor!



L = ? r = ?

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$$Q = \frac{R_P}{X_L} = \frac{R_P}{2\pi f_0 \cdot L} \quad \Rightarrow \quad R_P = 2\pi f_0 \cdot L \cdot Q = 2\pi \cdot 100 \cdot 10^3 \cdot 0{,}1 \cdot 10^{-3} \cdot 80 \approx 5027\ \Omega$$

$$r_S = \frac{1}{Q^2} R_P = \frac{1}{80^2} 5027 \approx 0{,}8\ \Omega$$

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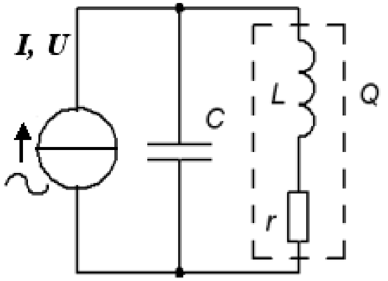


L = ? r = ?

$$Q = \frac{f_0}{\Delta f} = \frac{100 \cdot 10^3}{1250} = 80$$

80 > 10 justifying counting with the ideal model.

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}} \Rightarrow L = \frac{1}{(2\pi f_0)^2 C} = \frac{1}{(2\pi \cdot 100 \cdot 10^3)^2 \cdot 25 \cdot 10^{-9}} = 0{,}1\,\text{mH}$$

$$Q = \frac{R_P}{X_L} = \frac{R_P}{2\pi f_0 \cdot L} \Rightarrow R_P = 2\pi f_0 \cdot L \cdot Q = 2\pi \cdot 100 \cdot 10^3 \cdot 0{,}1 \cdot 10^{-3} \cdot 80 \approx 5027\,\Omega$$

$$r_S = \frac{1}{Q^2} R_P = \frac{1}{80^2} 5027 \approx 0{,}8\,\Omega$$

Luckily we did not have to use this formula to calculate the L

$$f_0 = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{r^2}{L^2}\right)}$$

Nowdays there are help …

WolframAlpha computational knowledge engine

solve L f=(1/(2*pi))*sqrt(1/(L*C) - r^2/L^2)

Examples Random

Input interpretation:

solve $f = \frac{1}{2\pi}\sqrt{\frac{1}{L\,C} - \frac{r^2}{L^2}}$ for L

Results:

$$L = \frac{1 - \sqrt{1 - 16\,\pi^2\,C^2\,f^2\,r^2}}{8\,\pi^2\,C\,f^2}$$

$$L = \frac{\sqrt{1 - 16\,\pi^2\,C^2\,f^2\,r^2} + 1}{8\,\pi^2\,C\,f^2}$$

Even if the transformation of R to r is not necessary, by computational resons, anymore – so these are still important concepts when engineers "reason".

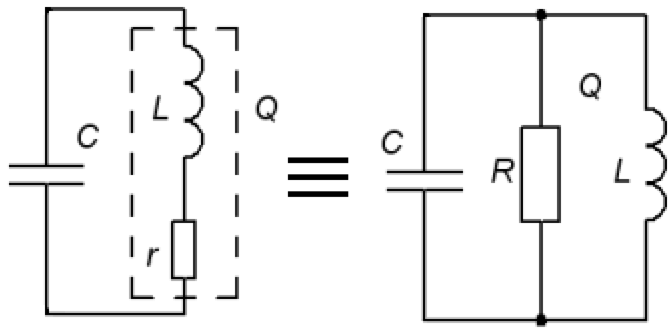
We chose the positive solution …

Nowdays there are help …

$$L = \frac{\sqrt{1 - 16\pi^2 C^2 f_0^2 r^2} + 1}{8\pi^2 C f_0^2} =$$

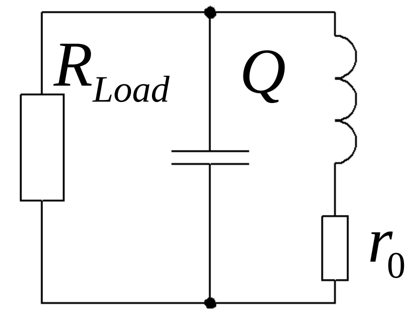
$$= \frac{\sqrt{1 - 16\pi^2 (25 \cdot 10^{-9})^2 \cdot (100 \cdot 10^3)^2 \cdot 0{,}8^2} + 1}{8\pi^2 \cdot 25 \cdot 10^{-9} \cdot (100 \cdot 10^3)^2} = 0{,}1\,\mathrm{mH}$$

Loaded resonance circuit

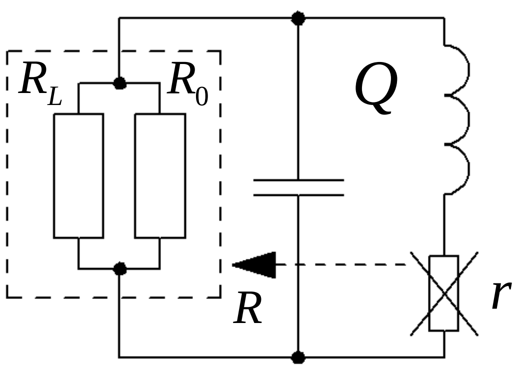


$$Q = \frac{X_L}{r} = \frac{R}{X_L} \qquad R = Q^2 \cdot r$$

Usually the resonance circuit is loaded!



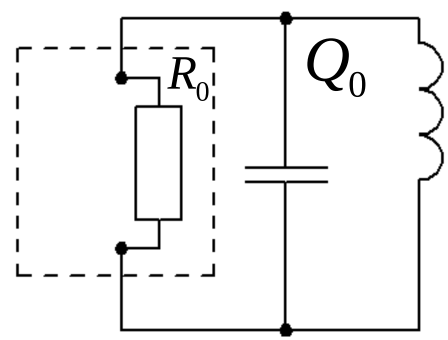
If the loaded resonance circuit is to become the wanted Q value one has to have an inductor with a much better Q_0!



$$R_L \parallel R_0 = R$$

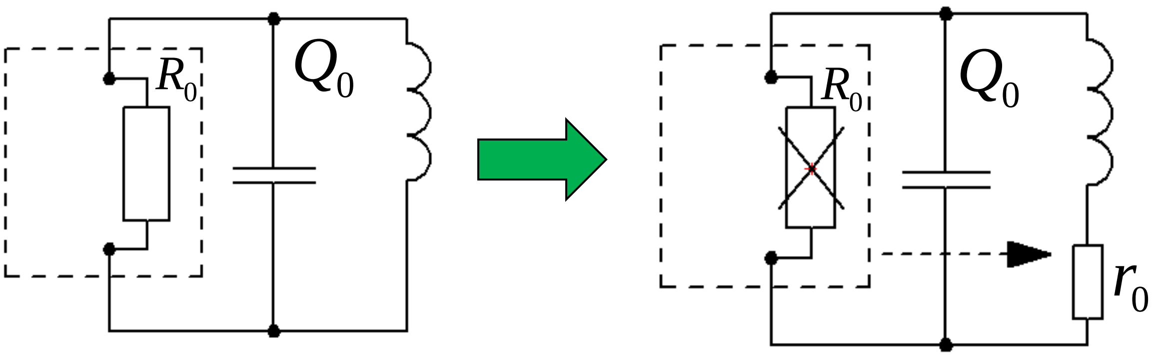
$$R = Q^2 \cdot r$$

- Q_0 unloaded Q-value



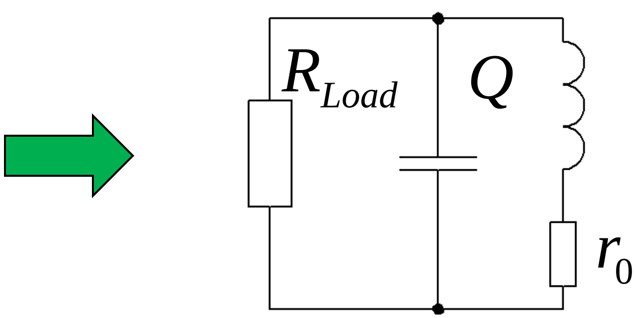
$$Q_0 = \frac{R_0}{X_L} > Q$$

Loaded resonance circuit



$$r_0 = \frac{R_0}{Q_0^2}$$

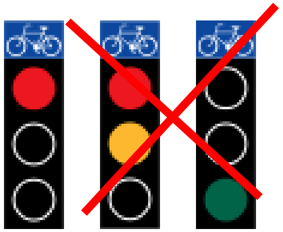
We will need an inductor with Q_0!

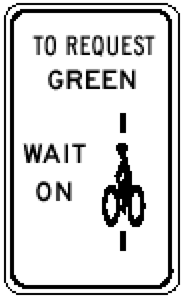


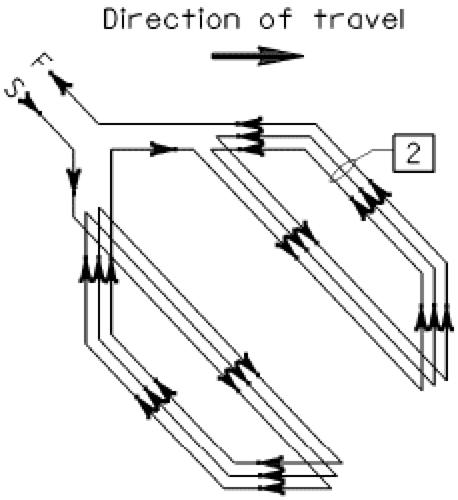
- When the circuit is loaded with R_{Load} the Q-value will change from Q_0 to Q!

The inductive sensor is a rugged sensor type available in many types.

Cyclists who request green?



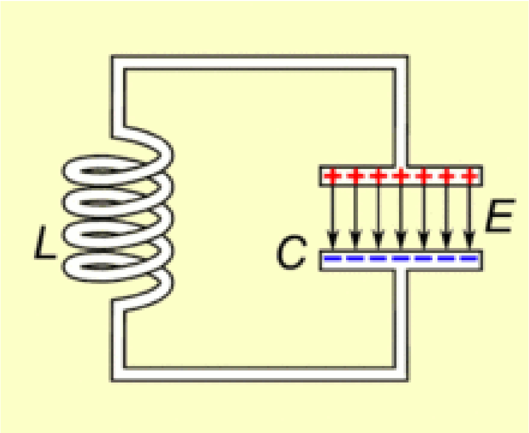




Inductive sensor for bicycle

*Sorry! The Sensor does not work for **all** bicycles?*

William Sandqvist william@kth.se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