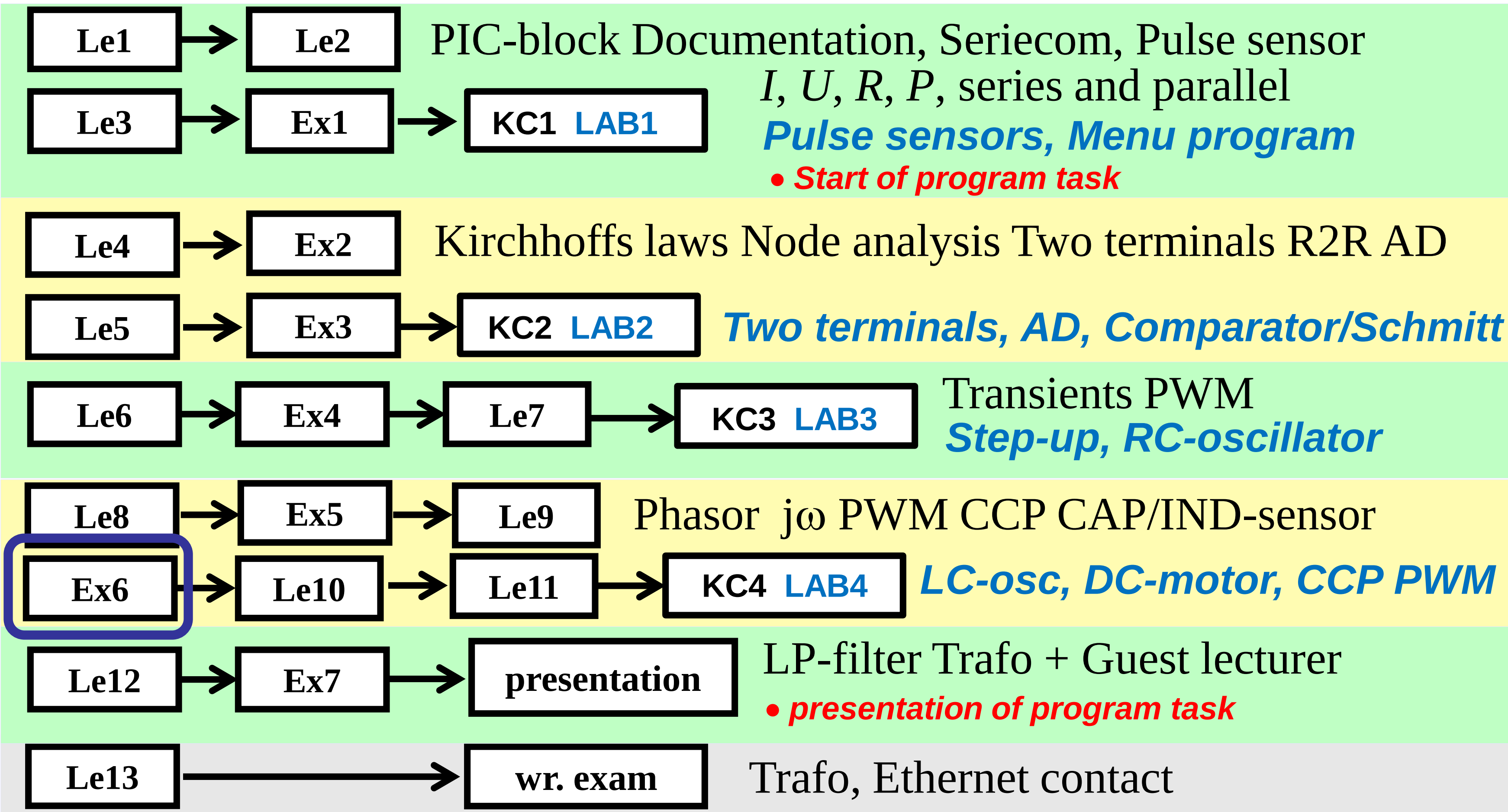
# IE1206 Embedded Electronics

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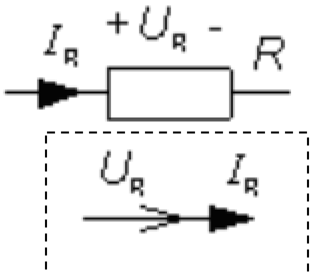
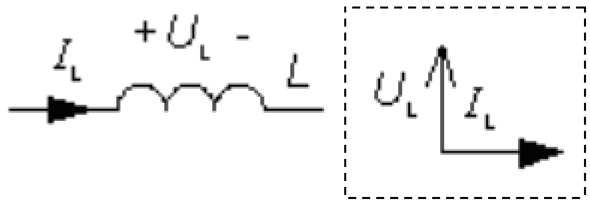
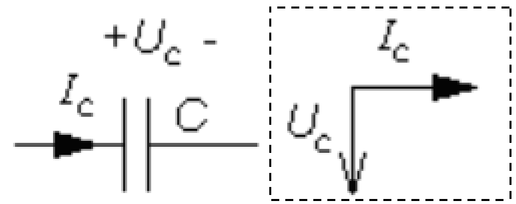
# Phasor - vector




$$\omega = 2\pi f$$

$$\left|X_L\right| = \omega \cdot L$$

$$\left|X_C\right| = \frac{1}{\omega \cdot C}$$

$$\boxed{Z = \frac{U}{I}}$$

# Complex phasors, jω-method

- Complex OHM's law for $R$ $L$ and $C$.

$$\underline{U}_{\mathrm{R}} = \underline{I}_{\mathrm{R}} \cdot R$$

$$\underline{U}_{\mathrm{L}} = \underline{I}_{\mathrm{L}} \cdot \mathrm{j} X_{\mathrm{L}} = \underline{I}_{\mathrm{L}} \cdot \mathrm{j}\omega L \qquad \omega = 2\pi \cdot f$$

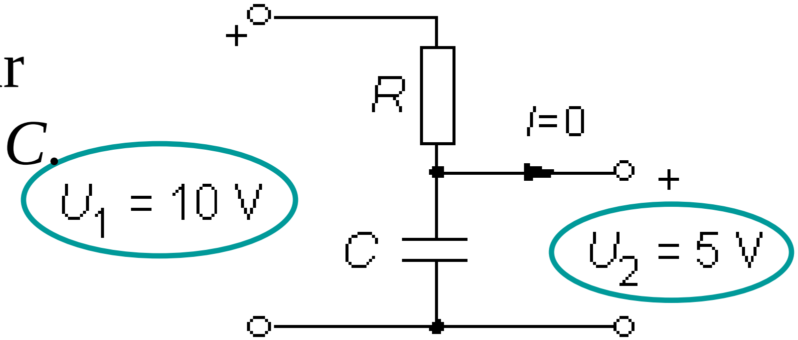
$$\underline{U}_{\mathrm{C}} = \underline{I}_{\mathrm{C}} \cdot \mathrm{j} X_{\mathrm{C}} = \underline{I}_{\mathrm{C}} \cdot \frac{1}{\mathrm{j}\omega C}$$

- Complex OHM's law for $Z$.

$$\boxed{\underline{U} = \underline{I} \cdot \underline{Z}} \qquad Z = \frac{U}{I} \qquad \varphi = \arg(\underline{Z}) = \arctan\left(\frac{\mathrm{Im}[\underline{Z}]}{\mathrm{Re}[\underline{Z}]}\right)$$

# ω for half the voltage? (12.3)

$U_1$ is a sine voltage with the angular frequency ω. Decide the product $R \cdot C$. (No current is consumed at $U_2$).



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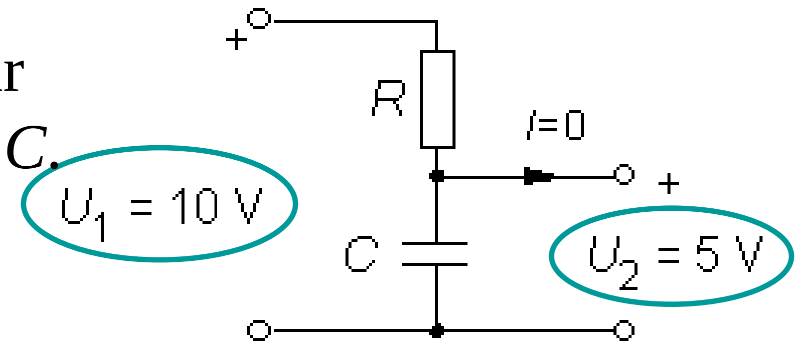


$$\underline{U}_2 = \underline{U}_1 \cdot \frac{\frac{1}{\mathrm{j}\omega C}}{R + \frac{1}{\mathrm{j}\omega C}}$$

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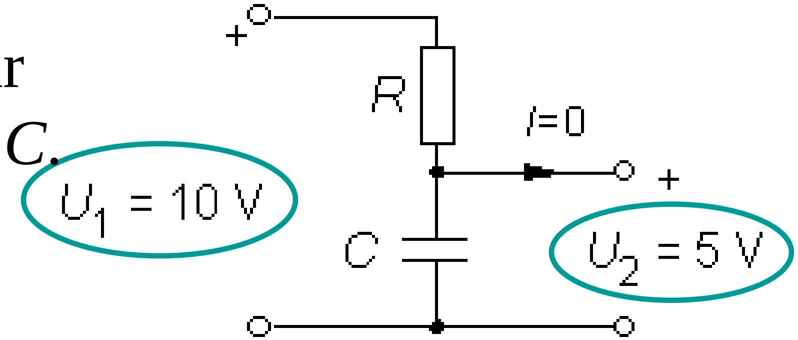


$$\underline{U}_2 = \underline{U}_1 \cdot \frac{\frac{1}{\mathrm{j}\omega C}}{R + \frac{1}{\mathrm{j}\omega C}} \cdot \frac{(\mathrm{j}\omega C)}{(\mathrm{j}\omega C)} =$$

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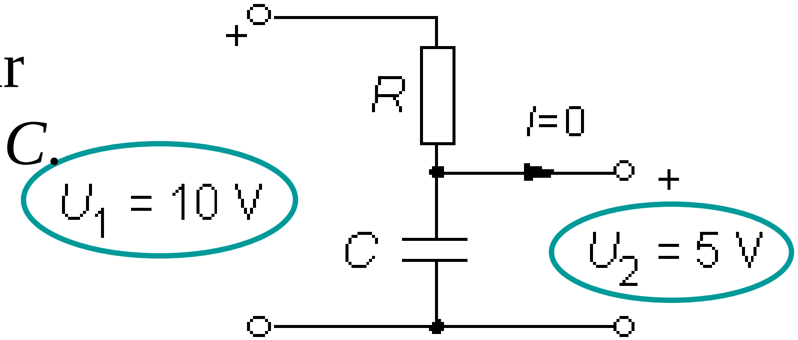


$$\underline{U}_2 = \underline{U}_1 \cdot \frac{\frac{1}{\mathrm{j}\omega C}}{R + \frac{1}{\mathrm{j}\omega C}} \cdot \frac{(\mathrm{j}\omega C)}{(\mathrm{j}\omega C)} = \underline{U}_1 \cdot \frac{1}{1 + \mathrm{j}\omega RC}$$

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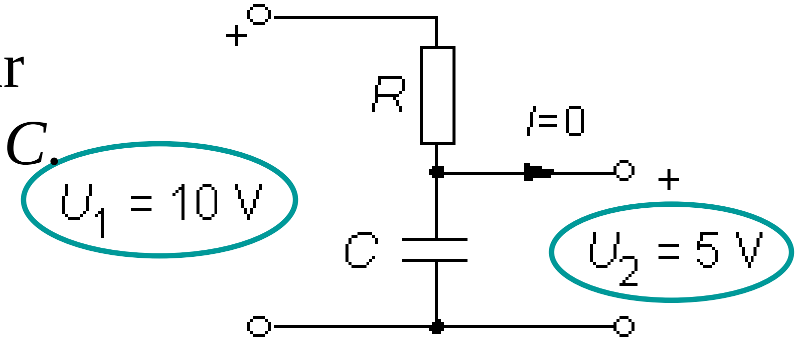


$$\underline{U}_2 = \underline{U}_1 \cdot \frac{\frac{1}{\mathrm{j}\omega C}}{R + \frac{1}{\mathrm{j}\omega C}} \cdot \frac{(\mathrm{j}\omega C)}{(\mathrm{j}\omega C)} = \underline{U}_1 \cdot \frac{1}{1 + \mathrm{j}\omega RC} \quad \Rightarrow \quad \frac{U_1}{U_2} = \sqrt{1 + R^2\omega^2C^2} = \frac{10}{5} = 2$$

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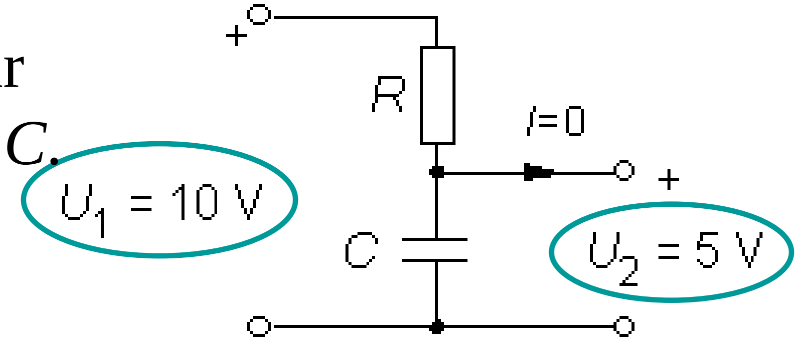
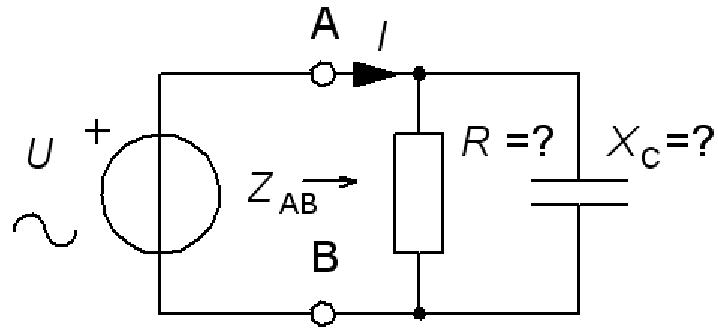



$$\underline{U}_2 = \underline{U}_1 \cdot \frac{\frac{1}{\mathrm{j}\omega C}}{R + \frac{1}{\mathrm{j}\omega C}} \cdot \frac{(\mathrm{j}\omega C)}{(\mathrm{j}\omega C)} = \underline{U}_1 \cdot \frac{1}{1 + \mathrm{j}\omega RC} \quad \Rightarrow \quad \frac{U_1}{U_2} = \sqrt{1 + R^2\omega^2 C^2} = \frac{10}{5} = 2$$

$$1 + R^2\omega^2 C^2 = 4 \quad \Leftrightarrow \quad R\omega C = \sqrt{3} \quad \Leftrightarrow \quad \boxed{RC = \frac{\sqrt{3}}{\omega}}$$

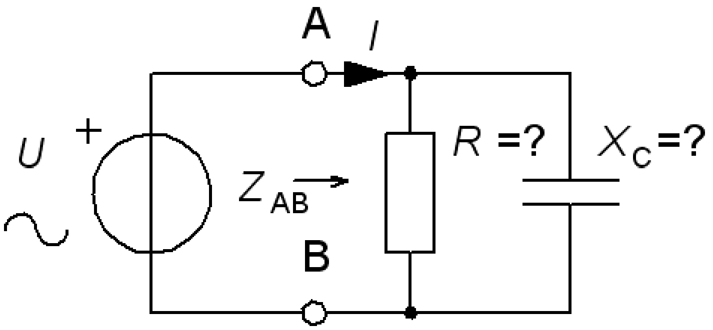
# *Try yourself ...* (12.1)

Set up the complex expression for current $I$ expressed with $U$ $R$ $C$ $\omega$. Let $U$ be reference phase, real. Answer with a expression of the form $a$+j$b$.



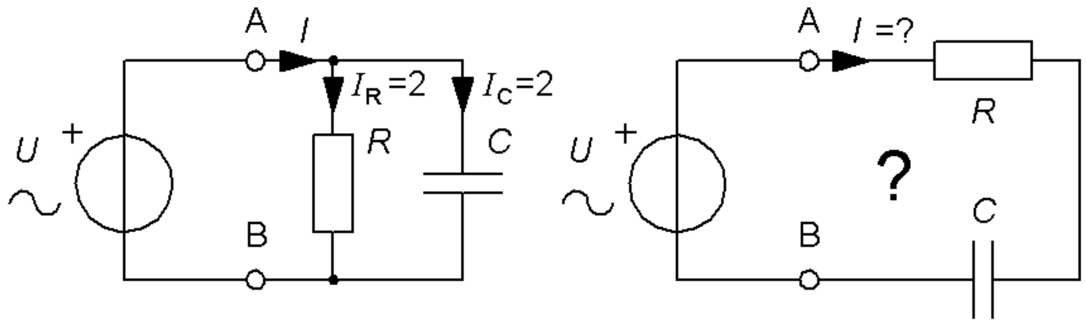
# *Try yourself ...* (12.1)

Set up the complex expression for current *I* expressed with *U R C* ω. Let *U* be reference phase, real. Answer with a expression of the form *a*+j*b*.




$$\underline{I} = \underline{I}_{\mathrm{R}} + \underline{I}_{\mathrm{C}} = \frac{U}{R} + \frac{U}{\dfrac{1}{\mathrm{j}\omega C}} = \frac{U}{R} + \mathrm{j}\omega C \cdot U$$

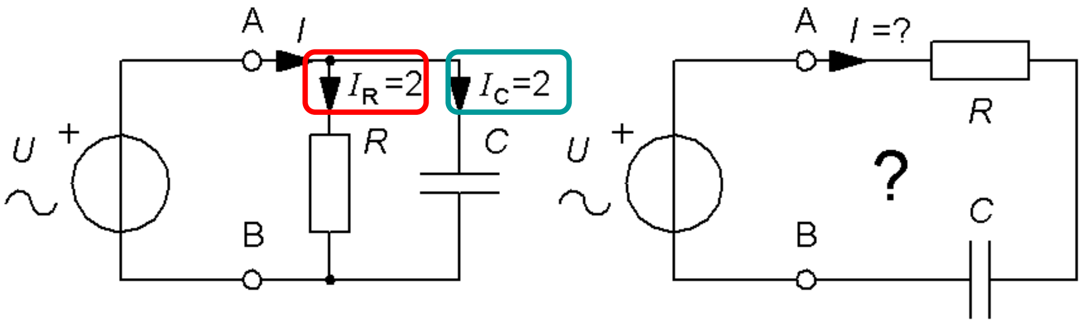
# Compare serial with parallel (12.5)




When a resistor *R* and a capacitor *C* is connected in parallel to a voltage source *U* each of them get the current 2A.

How big would the current in the resistor be if the two were series connected to the voltage source?

# Compare serial with parallel (12.5)

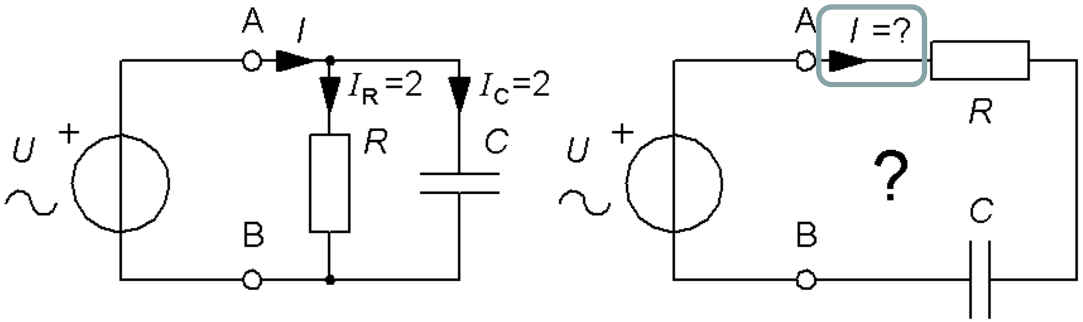



- Parallell connection:

$$\underline{I} = \underline{I}_{\mathrm{R}} + \underline{I}_{\mathrm{C}} = \frac{U}{R} + \mathrm{j}U\omega C \quad \underline{I} = 2 + 2\,\mathrm{j}$$

$$I_R = \frac{U}{R} = 2 \quad I_C = U\omega C = 2 \quad \Rightarrow \quad R = \boxed{\frac{U}{2}} \quad \frac{1}{\omega C} = \boxed{\frac{U}{2}}$$

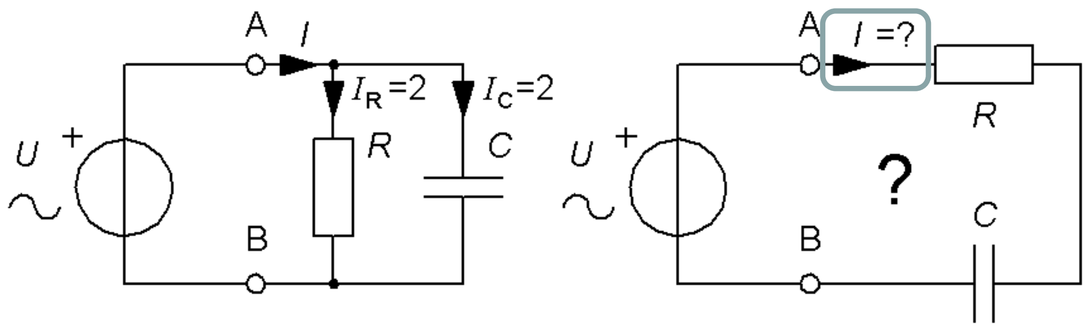
# Compare serial with parallel (12.5)




- Series connected:

$$\underline{I} = \frac{U}{R + \frac{1}{\mathrm{j}\omega C}} \quad \Rightarrow \quad I = \frac{U}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

# Compare serial with parallel (12.5)




- Serial connection:

*as before …*

$$I=\frac{U}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}=\left\{R=\frac{U}{2}\quad\frac{1}{\omega C}=\frac{U}{2}\right\}=\frac{U}{\sqrt{\left(\frac{U}{2}\right)^2+\left(\frac{U}{2}\right)^2}}=$$

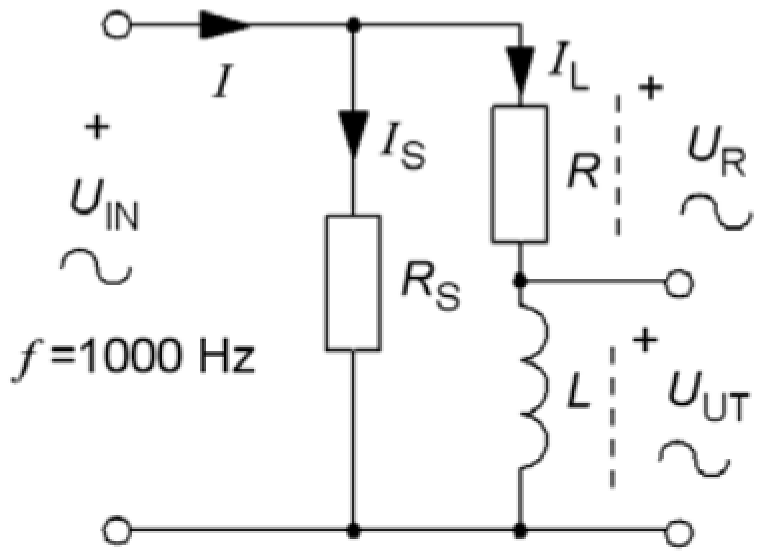
$$=\frac{U}{U\sqrt{\frac{1}{4}+\frac{1}{4}}}=\sqrt{2}=1{,}414\text{ A}$$

Paralell $I_R$= 2A
Serial $I_R$= 1,4A

# AC circuit with inductor (12.11)

An AC voltage $U_{IN}$ with the frequency $f = 1000$ Hz feeds a circuit with an inductance $L = 10$ mH in series with a resistor $R = 50\ \Omega$. In parallel with these are a resistor $R_S = 100\ \Omega$.
Given is voltage $U_{UT} = 6{,}28$ V.

**a)** Calculate $I_L$
**b)** Calculate $U_R$
**c)** Calculate $U_{IN}$
**d)** Calculate $I$



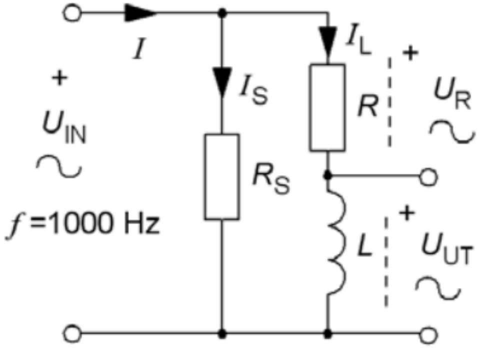
# **a**) Calculate $I_{\mathrm{L}}$ (12.11)

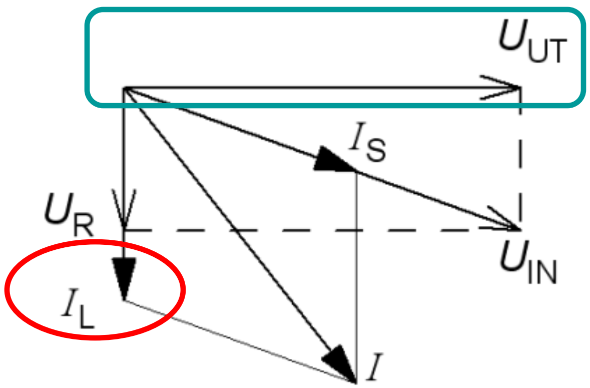
a) $\underline{U}_{UT}$ are chosen as reference phase, $\arg(\underline{U}_{UT}) = 0$

$\underline{U}_{UT} = j\omega L \cdot \underline{I}_1 \quad \underline{U}_{UT} = U_{UT} = 6{,}28$

$$\underline{I}_L = \frac{\underline{U}_{UT}}{j\omega L} = \frac{6{,}28}{j \cdot 2\pi \cdot 1000 \cdot 10 \cdot 10^{-3}} - 0{,}1j$$

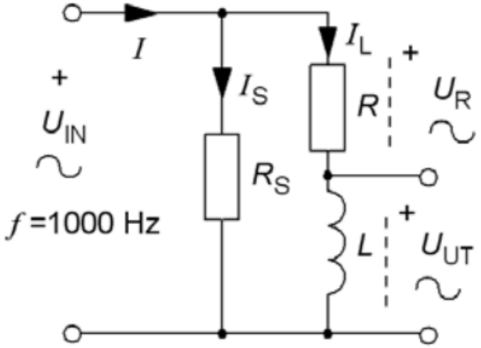
$I_L = 0{,}1\ A$

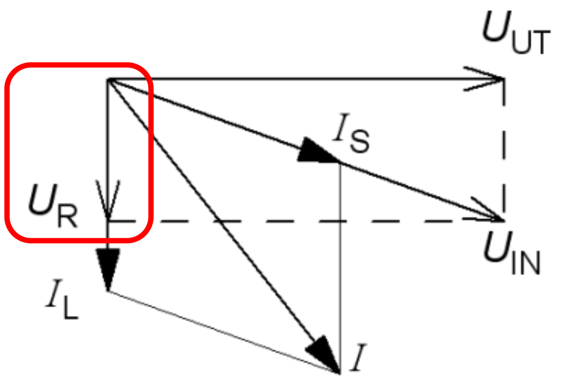


# **b**) Calculate $U_{\text{R}}$ (12.11)

b) $\underline{U}_R = R \cdot \underline{I}_L = -50 \cdot 0{,}1j = -5j \quad U_R = 5\ \text{V}$

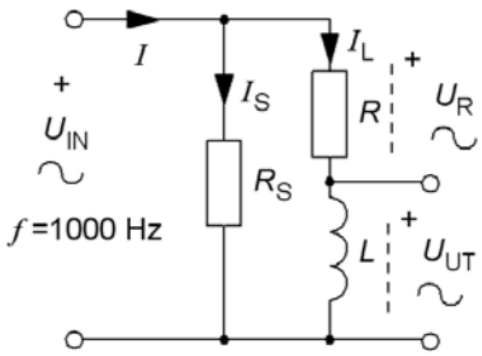


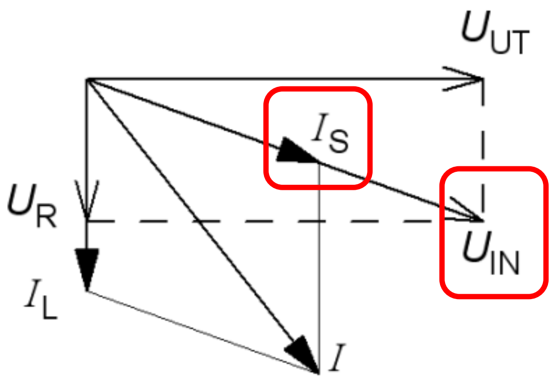



# **c**) Calculate $U_{\mathrm{IN}}$ (12.11)

$$\text{c)}\quad \underline{U}_{IN} = \underline{U}_R + \underline{U}_{UT} = 6,28 - 5j \qquad U_{IN} = \sqrt{6,28^2 + 5^2} = 8,0\ \text{V}$$

$$\underline{I}_S = \frac{\underline{U}_{IN}}{R_S} = \frac{6,28 - 5j}{100} = 0,063 - 0,05j$$

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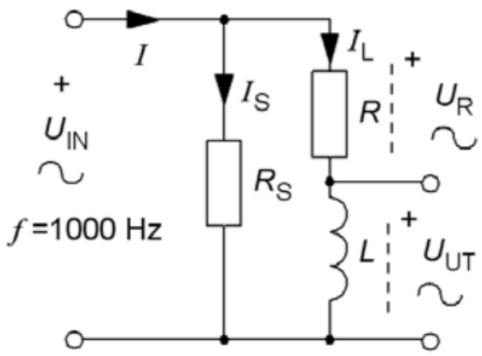
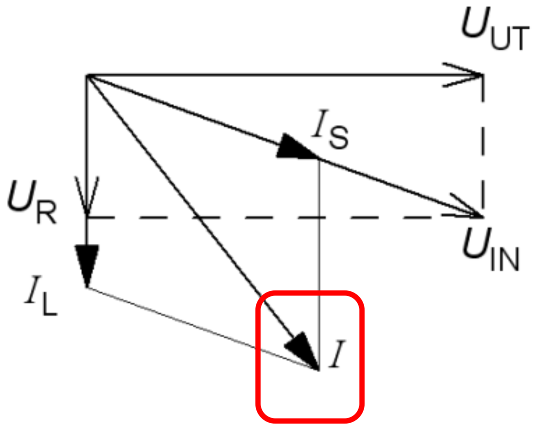
# **d**) Calculate $I$ (12.11)

$$\underline{I}_S = \frac{\underline{U}_{IN}}{R_S} = \frac{6,28 - 5j}{100} = 0,063 - 0,05j$$

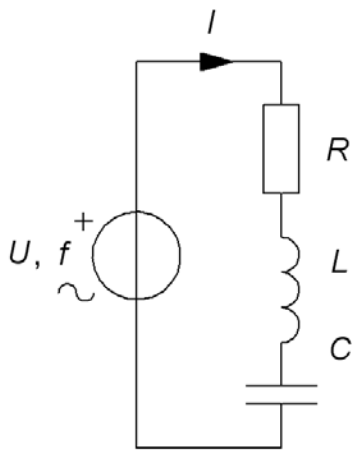
$$\text{d)} \quad \underline{I} = \underline{I}_L + \underline{I}_S = -0,1j + 0,063 - 0,05j = 0,062 - 0,15j$$

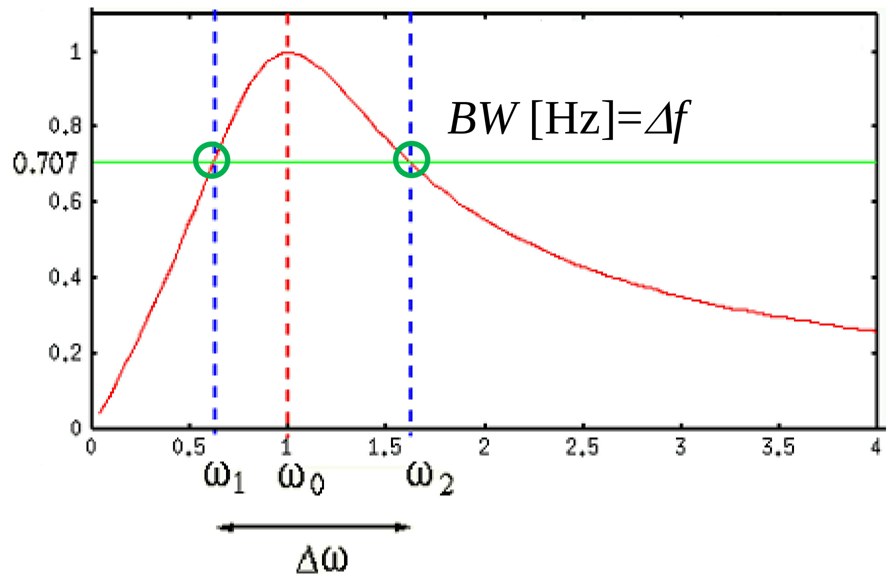
$$I = \sqrt{0,063^2 + 0,15^2} = 0,16 \text{ A}$$



# Series resonance




$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}} \qquad \boxed{Q = \frac{2\pi f_0 L}{r}} \qquad \frac{\Delta\omega}{\omega_0} = \frac{1}{Q} \Rightarrow \boxed{\frac{\Delta f}{f_0} = \frac{1}{Q}}$$

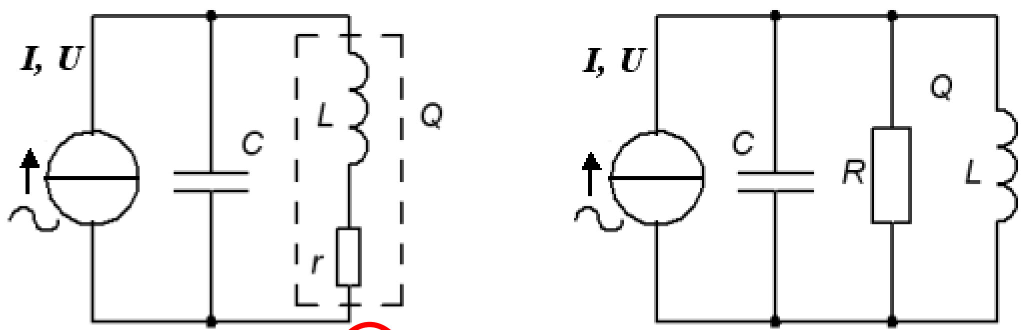
If $Q$ is high, no significant error is done if the bandwidth is divided equally on both sides of $f_0$.

$$f_2, f_1 \approx f_0 \pm \frac{\Delta f}{2}$$

# Parallel resonance

The **parallel resonance circuit** in manual computation for simplicity one usually uses the formulas of the ideal resonant circuit. At high Q and close to the resonance frequency $f_0$ the deviations becomes insignificant. At **Q >10** are the two circuits "interchangeable".

$$\omega_0 \approx \frac{1}{\sqrt{LC}}$$

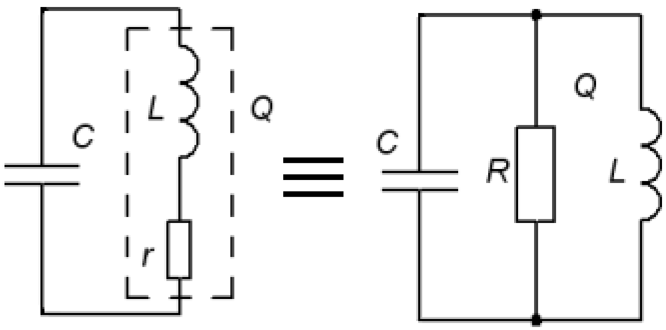



Alternative definition of $Q$ with $R_P$

$$\boxed{Q = \frac{\omega_0 L}{r_S} = \frac{R_P}{\omega_0 L}} \Rightarrow \boxed{R_P = Q^2 \cdot r_S}$$

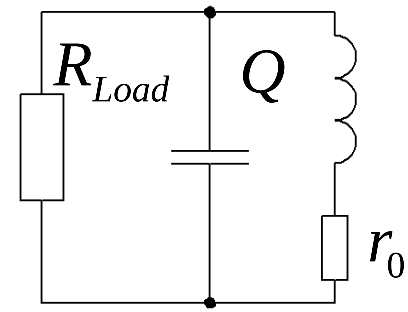
(*applies approximately for* ***Q >10*** )

# Loaded parallel resonance

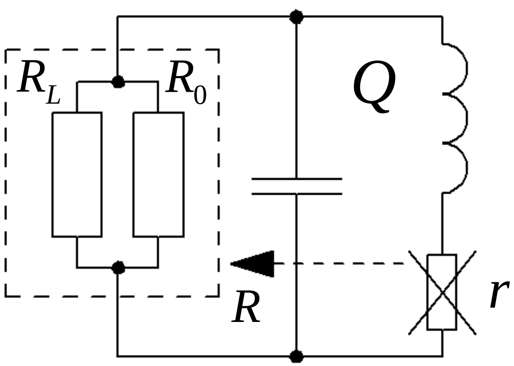



$$Q = \frac{X_L}{r} = \frac{R}{X_L} \qquad R = Q^2 \cdot r$$

Usually the resonance circuit is loaded!



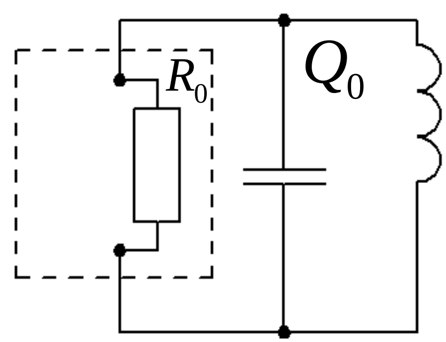

If the loaded resonance circuit is to become the wanted $Q$ value one has to have an inductor with a much better $Q_0$!




$$R_L \parallel R_0 = R$$

$$R = Q^2 \cdot r$$

- $Q_0$ unloaded Q-value




$$Q_0 = \frac{R_0}{X_L} > Q$$

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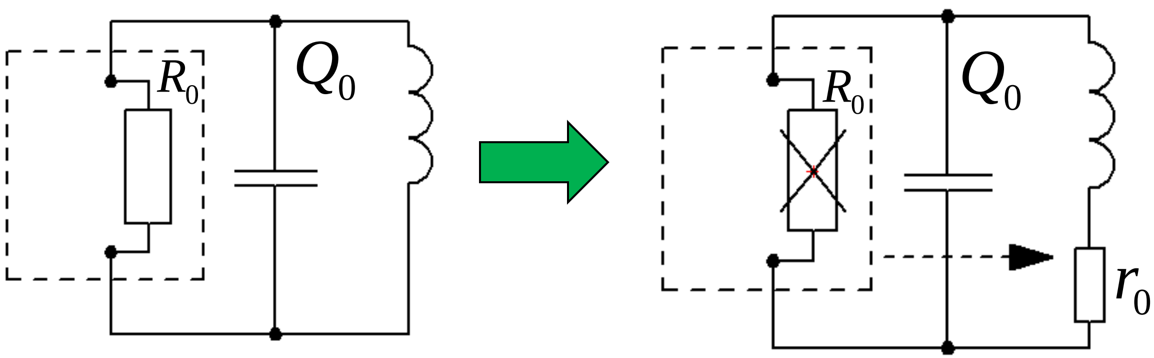
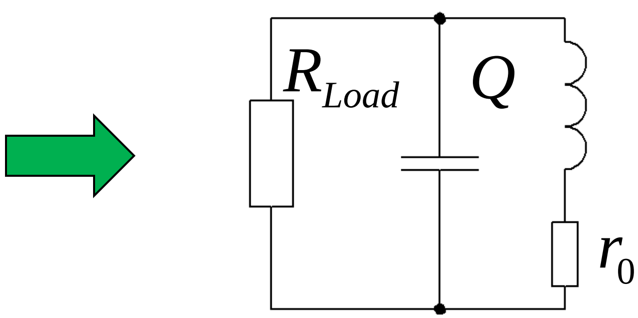
# Loaded parallel resonance




$$r_0 = \frac{R_0}{Q_0^2}$$

We will need an inductor with $Q_0$!




- When the circuit is loaded with $R_{\mathrm{Load}}$ the Q-value will change from $Q_0$ to $Q$!

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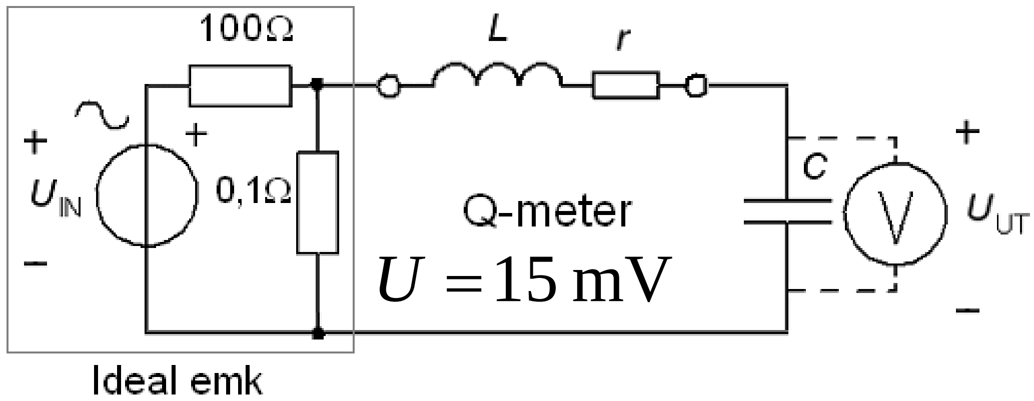
# To measure Q-value (13.9)

**Radio controled clock** is a clock that is automatically synchronized with a time code from a radio transmitter in Germany, on longvawe 77,5 kHz. The time signal consists of pulses encoded digitally. The signal strength is weak so such a receiver uses a tuned resonant circuit with *L* and *C*. The coil has a ferrite core, and this is also used as an antenna.
In a project we have to measure the Q-value this resonance circuit. How will this be done? Other values:

$L$ = 1,5 mH
$C$ = 2,8 nF

# To measure Q-value (13.9)

$U_{IN} = 15\ \text{V}$




$U_{UT} = 1{,}73\ \text{V}$

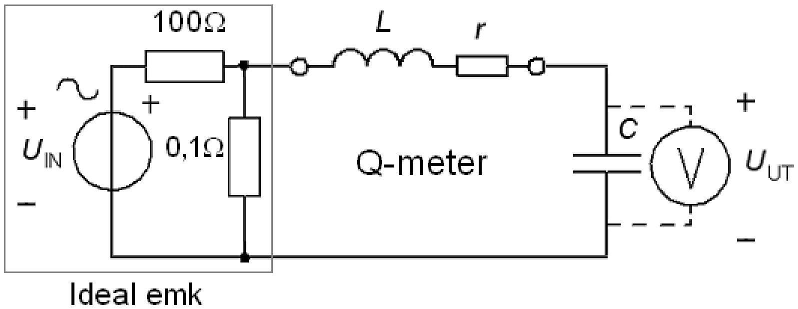
*This is how to measure the inductor's Q-value.*

$U_{IN}$ = 15V is a sine voltage with the frequency 77,5 kHz (the resonance frequency) which is voltage divided to 15 mV. Over the capacitor we then measures the much bigger voltage $U_{UT}$ = 1,73 V.

a) What is the inductor's *Q*-value?

b) What is the value of the inductor's internal resistance *r* (will also include other losses)?

# To measure Q-value (13.9)




$$f_0 = \frac{1}{2\pi\sqrt{L \cdot C}} = \frac{1}{2\pi\sqrt{1{,}5 \cdot 10^{-3} \cdot 2{,}8 \cdot 10^{-9}}} = 77{,}5 \cdot 10^3$$

Check of resonance frequency 77,5 KHz

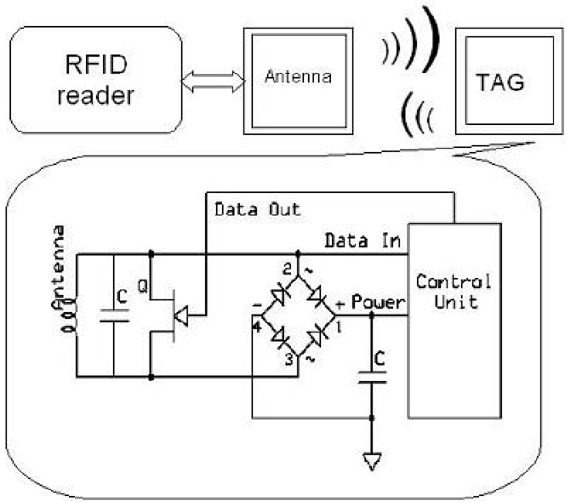
The voltage divider: $U_r = 15 \frac{0{,}1}{100} = 0{,}015 \text{ V}$

a) $$Q = \frac{2\pi f \cdot L}{r} \cdot \frac{I}{I} = \frac{U_L}{U_r} = \{U_L = U_C = U_{UT}\} = \frac{U_{UT}}{U_r} = \frac{1{,}73}{0{,}015} = \boxed{115}$$

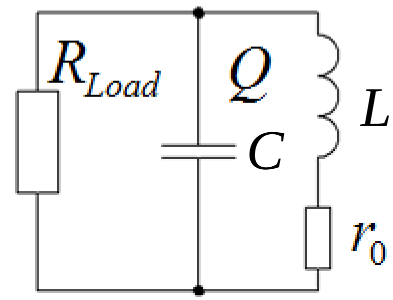
b) $$r = \frac{2\pi f \cdot L}{Q} = \frac{2\pi \cdot 77{,}5 \cdot 10^3 \cdot 1{,}5 \cdot 10^{-3}}{115} = \boxed{6{,}33\ \Omega}$$

Big compared with 0,1Ω from voltage divider.

# SL’s access card (13.7)




$$r_0 = ?$$



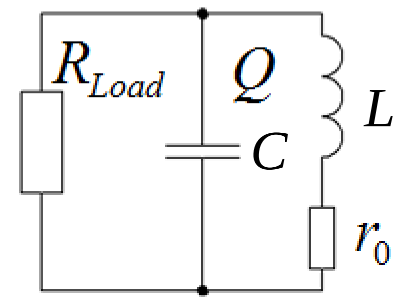

SL access-card contains a RFID-tag that communicates with the turnstyle reader on the frequency 13,56 MHz and uses the data transfer speed of 70 KHz.

To be able to read data in that speed then the resonance circuits inside the reader and the card must have a bandwidth at least twice this data speed, eg. **200 kHz**.

# SL's access card (13.7)

RFID-tag in the card concists of a parallel resonance circuit $C||(L+r_0)||R_{\mathrm{Load}}$. The processor in the card consumes current from the resonance circuit. This is symbolized with the resistance $R_{\mathrm{Load}}$= 30000 Ω.

$r_0 = ?$




$f_0$ = 13,56 MHz

$BW$ = 200 kHz

$L$ = 2,5 μH

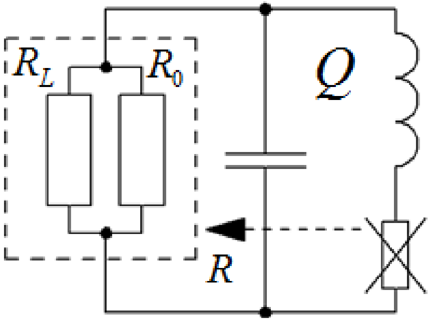
$C$ = 55 pF

$R_{\mathrm{L}}$= 30000 Ω

# SL's access card (13.7)

- Wanted Q-value:

$$Q = \frac{f_0}{\Delta f} = \frac{13{,}56 \cdot 10^6}{200 \cdot 10^3} = 68$$




- Total parallel resistance for bandwidth 200 kHz

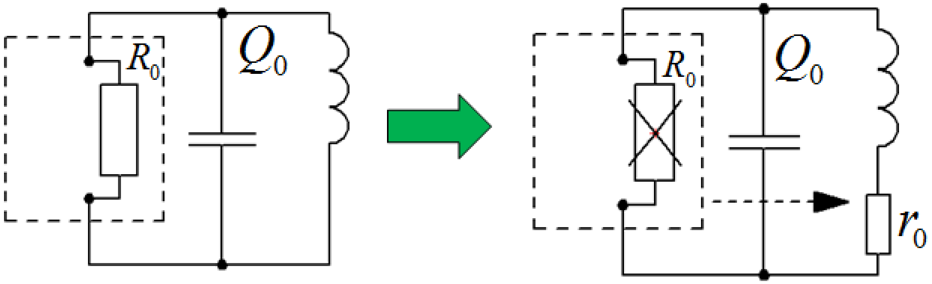
$$R = Q \cdot X_L = Q \cdot 2\pi f_0 L = 68 \cdot 2\pi \cdot 13{,}56 \cdot 10^6 \cdot 2{,}5 \cdot 10^{-6} = 14469\ \Omega$$

$$R_L = 30000\ \Omega \qquad R_L > R$$

$$R = R_L \parallel R_0 \quad \Rightarrow \quad R_0 = \frac{R_L \cdot R}{R_L - R} = \frac{30000 \cdot 14469}{30000 - 14469} = 27947\ \Omega$$

$$Q_0 = \frac{R_0}{X_L} = \frac{27947}{2\pi \cdot 13{,}56 \cdot 10^6 \cdot 2{,}5 \cdot 10^{-6}} = 131$$

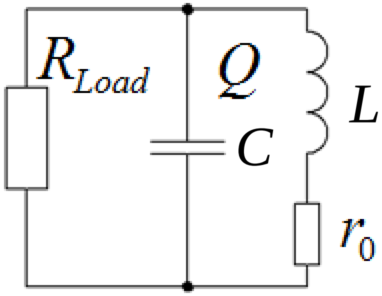
# SL's access card (13.7)




- The inductor Q-value: **131**!
- The inductor resistance $r_0$

$$r_0 = Q_0^2 \cdot R_0 = 131^2 \cdot 27947 = 1{,}63\ \Omega$$

- The loaded resonance circuit




$$Q = 68$$

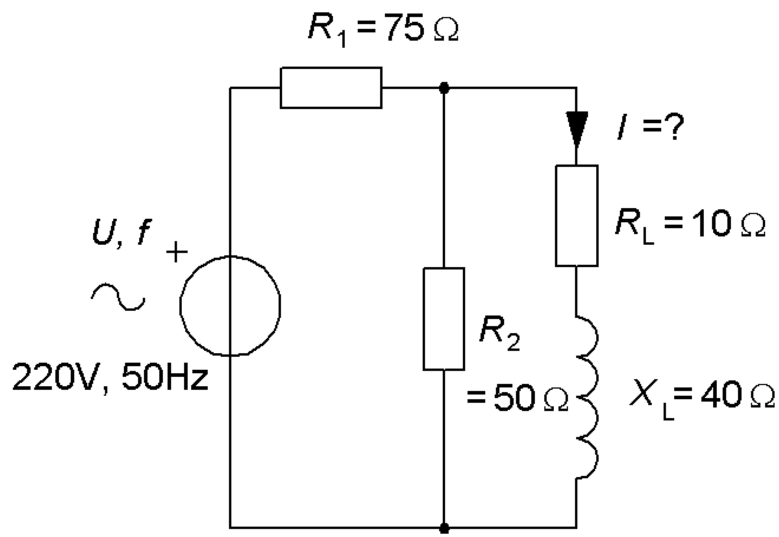
$$R_L = 30000\ \Omega$$

$$r_0 = \boxed{1{,}63\ \Omega}$$

# Thevenine equivalent with inductor (12.4)

Determine the value of the current *I*.

Use Thevenine equivalent.



# Thevenine equivalent with inductor (12.4)

Determine the value of the current *I*.

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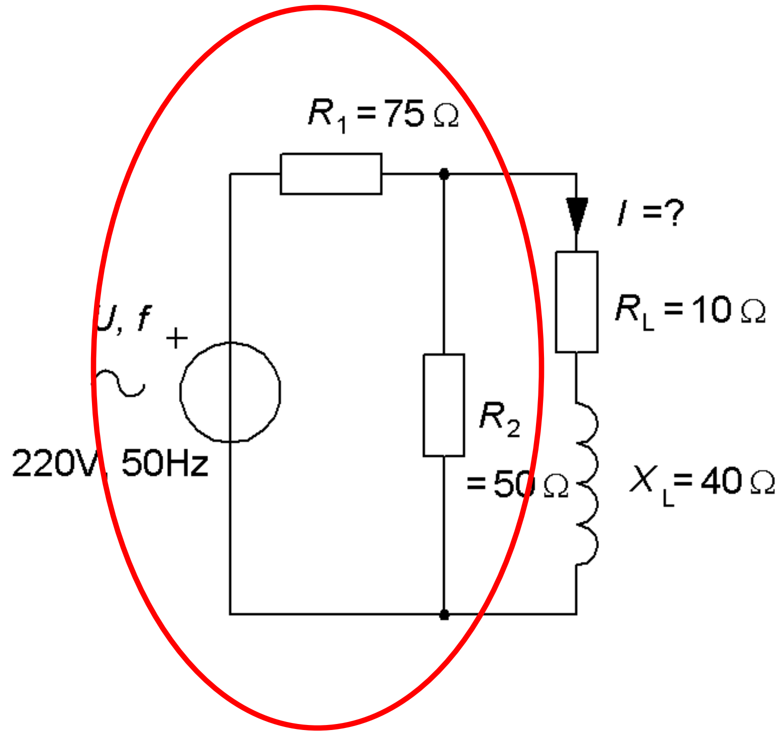
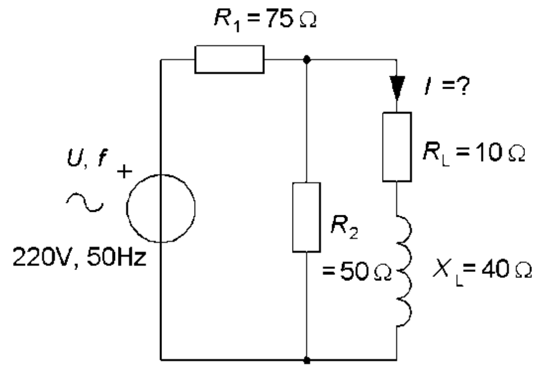


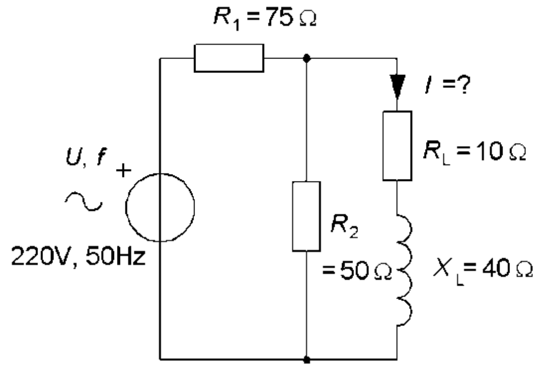
# Thevenin equivalent with inductor (12.4)

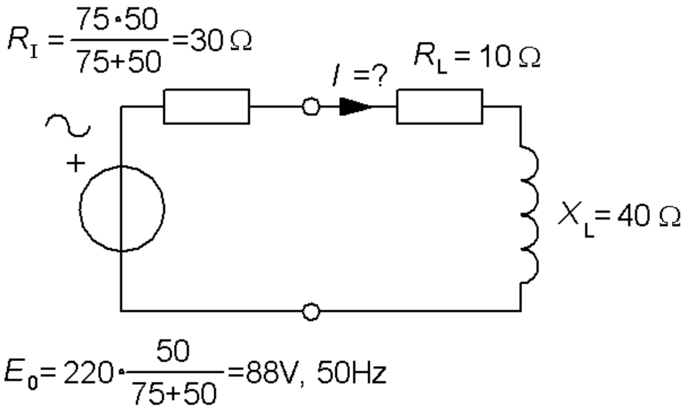
Calculate the Thevenin equivalent $E_0$ and $R_I$ of this circuit.



# Thevenine equivalent with inductor (12.4)

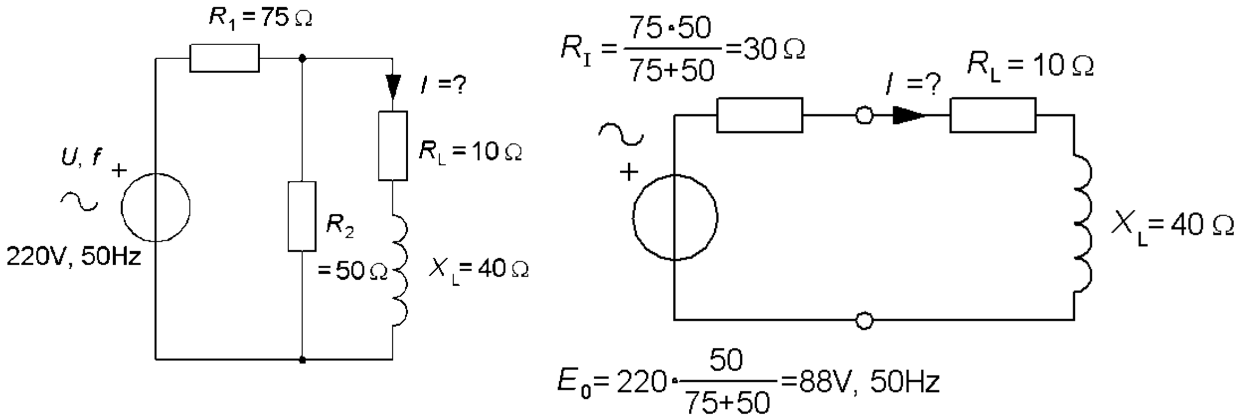
Calculate the Thevenin equivalent $E_0$ and $R_I$ of this circuit.



# Thevenine equivalent with inductor (12.4)

Calculate the Thevenin equivalent $E_0$ and $R_\mathrm{I}$ of this circuit.




The emf and resistors – this time as with DC circuits …

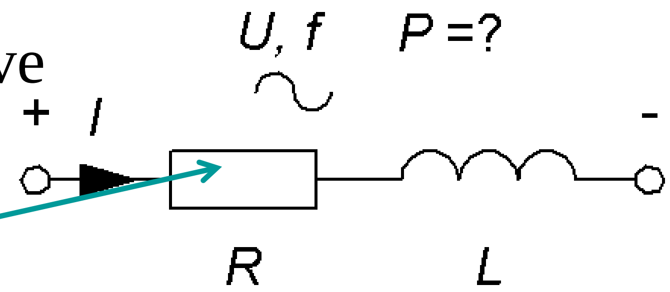
$$R_I = \frac{75 \cdot 50}{75+50} = 30\,\Omega \quad E_0 = 220\frac{50}{75+50} = 88\text{ V}$$

The inductor – now it must be considered an AC circuits …

$$\underline{I} = \frac{U}{\underline{Z}} \quad \Rightarrow \quad I = \frac{88}{\left|(30+10)+\mathrm{j}40\right|} = \frac{88}{\sqrt{(30+10)^2+40^2}} = 1{,}56\text{ A}$$

# Active power in impedance

Set up an expression of the active power $P$ for this impedance.




There will only be power in the resistors.

$U$ reference phase, real.

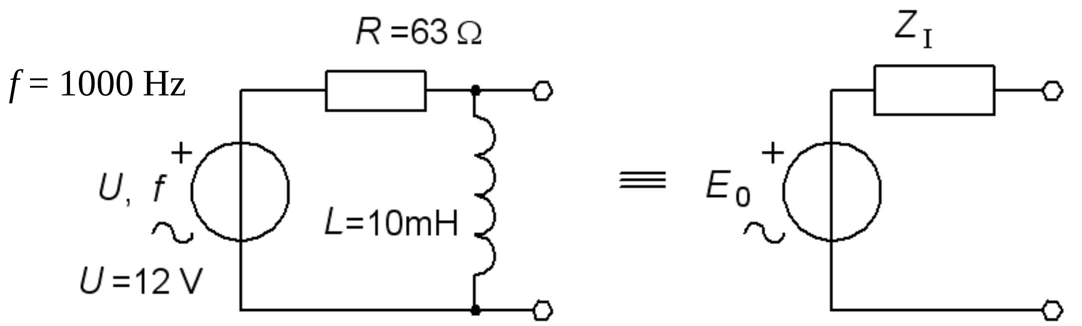
$$P = I^2 \cdot R \qquad \underline{I} = \frac{U}{\underline{Z}} = \frac{U}{R + \mathrm{j}\omega L} \quad \Rightarrow \quad I = \frac{U}{\sqrt{R^2 + (\omega L)^2}}$$

$$P = R \cdot \frac{U^2}{R^2 + (\omega L)^2} = \boxed{\frac{RU^2}{R^2 + (\omega L)^2}}$$

$$\omega \to \infty \Rightarrow P \to 0$$

$$\omega \to 0 \Rightarrow P \to \frac{U^2}{R}$$

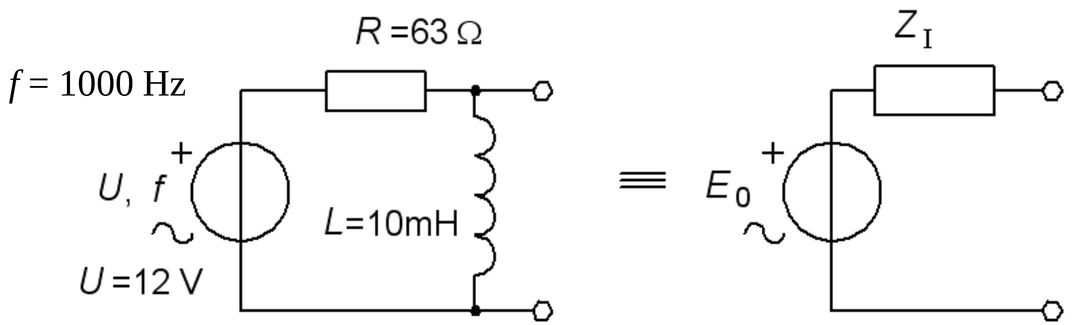
# Example. Complex equivalent




a) Derive the equivalent complex circuit with $E_0$+$Z_I$.

b) Suppose that we can load the circuit with an arbitrary chosen impedance – how should this be composed if one wishes the power in the load to be the maximum? (Maximum power transfer theorem).

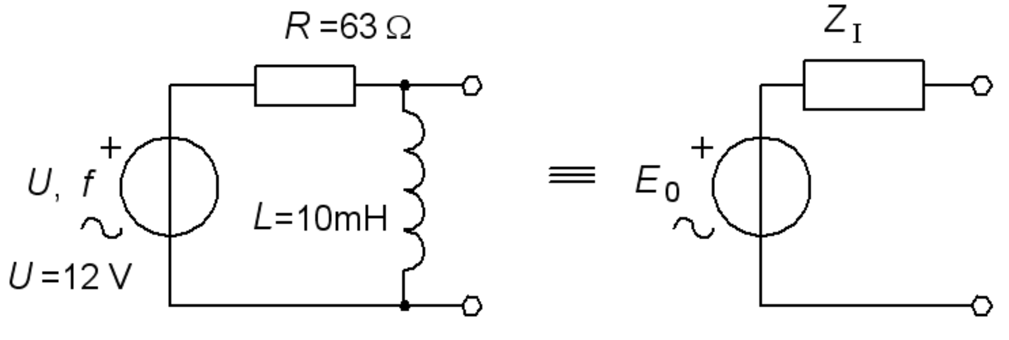
# Example. Complex equivalent, $E_0$




$E_0$ is calculated as the divided voltage. If $U$ is the reference phase we get $E_0$ 8,47 V and gets the phase 45° to $U$.
If there are no other voltage sources or current sources in the circuit then we don't have to keep track on the phase, as $E_0$ might as well become the network's new reference phase!

$$\underline{E}_0 = U\frac{\mathrm{j}\omega L}{R+\mathrm{j}\omega L} = 12\frac{\mathrm{j}2\pi 1000\cdot 0,01}{63+\mathrm{j}2\pi 1000\cdot 0,01} = 6+6\mathrm{j} \quad E_0 = \sqrt{6^2+6^2} = 8,48\ \mathrm{V}$$

# Example. Complex equivalent, $Z_\mathrm{I}$

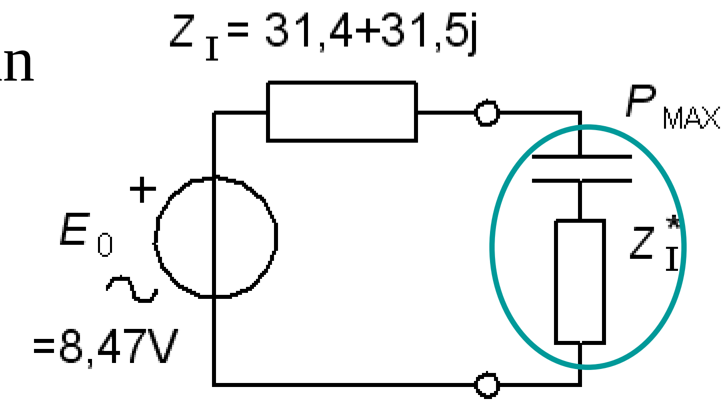



$Z_\mathrm{I}$ is the impedance we see if we turn down $U$.

$$\underline{Z}_I = \frac{R \cdot \mathrm{j}\omega L}{R + \mathrm{j}\omega L} = \frac{63 \cdot \mathrm{j}2\pi 1000 \cdot 0{,}01}{63 + \mathrm{j}2\pi 1000 \cdot 0{,}01} = 31{,}4 + 31{,}5\mathrm{j}$$

# Maximum power, *X*

The equivalent circuit is 8,57 V an emf with internal impedance $Z_I$ = 31,4+31,5j.



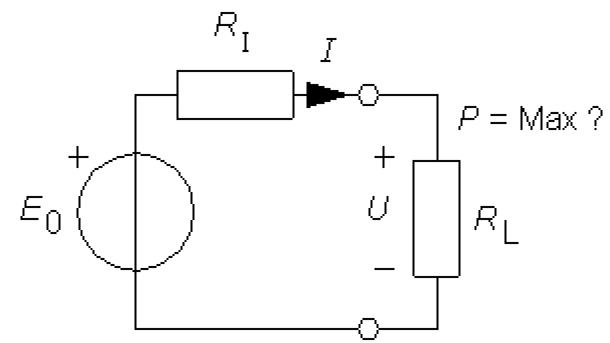

- **Maximum power**.

At resonance inductance and capacitance cancel each other. This will maximize the power in the load. Therefore, the load this time should be capacitive (-31,5j).

When the two reactances cancel each other the circuit becomes completely resistive. What load resistance will give the maximum power?

# Maximum power, $R_I$

$$P = R_L \cdot I^2 \quad I = \frac{E_0}{R_I + R_L} \quad \Rightarrow \quad P = E_0^2 \cdot \frac{R_L}{(R_I + R_L)^2}$$




When do $P(R_L)$ have a maximum? (You get simpler calculations if you turn to the question to "where is 1/P minimum").

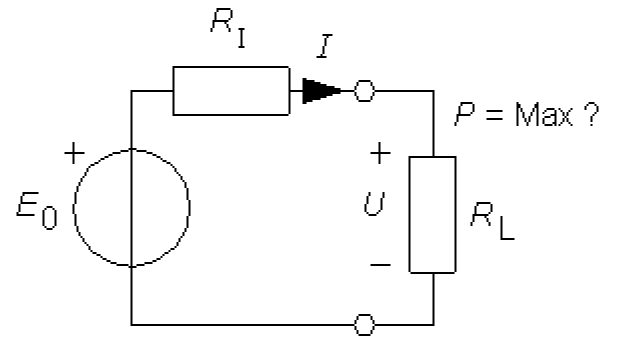
$$\frac{1}{P} = \frac{1}{E_0^2} \cdot \left(\frac{R_L^2}{R_L} + \frac{R_I^2}{R_L} + 2 \cdot \frac{R_I \cdot R_L}{R_L}\right) = \frac{1}{E_0^2} \cdot \left(R_L + 2 \cdot R_I + \frac{R_I^2}{R_L}\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}R_L}\left(\frac{1}{P}\right) = \frac{\mathrm{d}}{\mathrm{d}R_L}\left(\frac{1}{E_0^2} \cdot \left(R_L + 2 \cdot R_I + \frac{R_I^2}{R_L}\right)\right) = 1 - \frac{R_I^2}{R_L^2} = 0 \quad \Rightarrow \quad R_L = R_I$$

Maximum transfered power if you chose $R_L = R_I$.
($R_L$ = 31,4 Ω).

# The maximum power

How big is the power for $R_L = R_I$ (Maximum power)?




$$P = E_0^2 \cdot \frac{R_L}{(R_I + R_L)^2} \qquad R_I = R_L \quad \Rightarrow \quad P_{MAX} = \frac{E_0^2}{4 \cdot R_I}$$

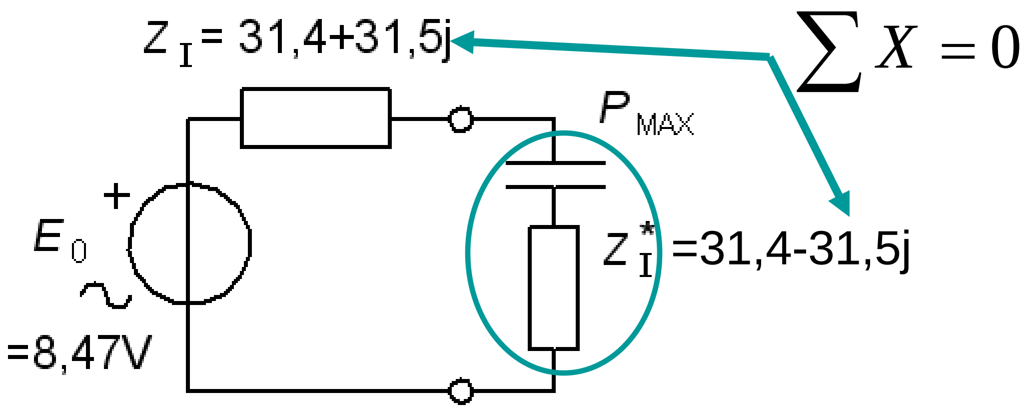
How big are the losses inside the equivalent circuit?

If $R_L = R_I$ the power is divided equal between the internal resistance and the load. This means that the Thermal efficiency will be 50% (= bad).

Maximum power transfer, impedance matching, is only used when neccessary, such as for radio transmitters.

# Maximum power transfer

$$\boxed{\begin{array}{c} P_{\max} \Rightarrow \\ \underline{Z} = \underline{Z}_{\mathrm{I}}^{*} \end{array}}$$




At power match with a load equal to the complex conjugate of the internal impedance, the effect:

$$\boxed{P_{\max} = \frac{\left|\underline{E}_0\right|^2}{4 \cdot \mathrm{Re}[\underline{Z}_{\mathrm{I}}]}}$$