SF1611 Introductory course in mathematics I. 1.5 cr
Exam, August 29, 2014. Duration: 60 minutes. No aids allowed

The problems are worth 1 credit each and you are only required to provide answers, not complete derivations. In order to pass, you must get at least 5 credits.

Name:..Pers.no...................................Program.............

Result:

1	2	3	4	5	6	7	8	Σ	Grade

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1. Write in words how the following statement is pronounced.

$$\forall x \in \mathbb{R} \ \left(\sqrt{x} \in \mathbb{Q} \Leftrightarrow \sqrt{x} \in \mathbb{N}\right)$$

   Answer:

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2. Write the set $\{x \in \mathbb{R} \mid x \geq x^2\}$ as an interval.
   Answer:

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3. Find a quadratic polynomial whose constant term is 2 and whose zeros are -1 and 1.
   Answer:

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4. Perform the division

$$\frac{2x^3 - x + 1}{x + 1}.$$

   Answer:

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5. Find an integer $n < 10$ such that $|n + 1| > 10$.
   Answer:

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6. Simplify $\ln\sqrt{e^3}$ as much as possible.
Answer:

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7. Find all real solutions to the equation $\sin^2 x = 1$.
Answer:

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8. Fill in the gap in the following proof that $1\cdot2+2\cdot3+3\cdot4+\cdots+(n-1)n = \frac{1}{3}(n-1)n(n+1)$ for any positive integer n.

We will argue by induction over n. If $n = 1$ the statement is true because the sum has no terms at all and the right-hand side vanishes. Under the supposition that the statement holds for n, our task is to show that it holds for $n + 1$. We have $1 \cdot 2 + 2 \cdot 3 + \cdots + n \cdot (n + 1) = \big(1 \cdot 2 + 2 \cdot 3 + \cdots + (n - 1) \cdot n\big) + n(n + 1)$ which, by the induction assumption, equals

Factoring out $\frac{1}{3}n(n + 1)$ we obtain $\frac{1}{3}n(n + 1)\big((n - 1) + 3\big) = \frac{1}{3}n(n + 1)(n + 2)$, so the statement holds for $n + 1$ too.
