SF1611 Introductory course in mathematics I. 1.5 cr
Suggested solutions to the exam on August 29, 2014. Duration: 60 minutes. No aids allowed

The problems are worth 1 credit each and you are only required to provide answers, not complete derivations. In order to pass, you must get at least 5 credits.

Name:...Pers.no...................................Program.............

Result:

1	2	3	4	5	6	7	8	Σ	Grade

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1. Write in words how the following statement is pronounced.

$$\forall x \in \mathbb{R} \ \ (\sqrt{x} \in \mathbb{Q} \Leftrightarrow \sqrt{x} \in \mathbb{N})$$

Answer: *For any real x, the square root of x is rational if and only if the square root of x is a natural number.*

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2. Write the set $\{x \in \mathbb{R} \mid x \geq x^2\}$ as an interval.
Answer: $[0, 1]$

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3. Find a quadratic polynomial whose constant term is 2 and whose zeros are -1 and 1.
Answer: $-2x^2 + 2$

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4. Perform the division

$$\frac{2x^3 - x + 1}{x + 1}.$$

Answer: $2x^2 - 2x + 1$

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5. Find an integer $n < 10$ such that $|n + 1| > 10$.
Answer: $n = -12$ (or any smaller integer)

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6. Simplify $\ln\sqrt{e^3}$ as much as possible.
Answer: $3/2$

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7. Find all real solutions to the equation $\sin^2 x = 1$.
Answer: $x = \pi\big(\frac{1}{2} + n\big)$, where n runs over all integers.

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8. Fill in the gap in the following proof that $1\cdot2+2\cdot3+3\cdot4+\cdots+(n-1)n = \frac{1}{3}(n-1)n(n+1)$ for any positive integer n.

We will argue by induction over n. If $n = 1$ the statement is true because the sum has no terms at all and the right-hand side vanishes. Under the supposition that the statement holds for n, our task is to show that it holds for $n+1$. We have $1\cdot2+2\cdot3+\cdots+n\cdot(n+1) = \big(1\cdot2+2\cdot3+\cdots+(n-1)\cdot n\big)+n(n+1)$ which, by the induction assumption, equals

$$\frac{1}{3}(n-1)n(n+1)+n(n+1).$$

Factoring out $\frac{1}{3}n(n+1)$ we obtain $\frac{1}{3}n(n+1)\big((n-1)+3\big) = \frac{1}{3}n(n+1)(n+2)$, so the statement holds for $n+1$ too.
