Department of mathematics

SF1625
Calculus 1
Year 2015/2016

Module 1: Functions, Limits, Continuity

This module includes Chapter P and 1 from Calculus by Adams and Essex and is taught in three lectures, two tutorials and one seminar.

Important concepts. The three most important concepts are **function**, **limit** and **continuity**. Note how they are defined. Along with the concept of a function are several other concepts that are important: *domain of definition, range, graph, tangent, normal, bounded function, odd function, even function, piecewise defined function, composition of functions, the absolute value function*. You should learn how they are defined and learn how to use them in problem solving.

The definition of **limit** is done in order to state clearly what the concept of a limit means. You seldom use this definition to compute limits – for this you hav other methods (that build on the definition).

There is a large class of functions, sometimes called **elementary functions**, that are contiuous in all of their domains of definition. To compute the limit of such a fuction you only need to compute the value of the function. It is only at points where these functions are not defined that you have to investigate the limit in som other more advanced way. These functions are *polynomials , rational functions, root functions, exponential functions, logarithmic functions, trigonometric functions, inverse trigonometric functions – and all combinations of these using + - x / and composition.*

For **continuous functions** defined on **closed and bounded intervals** there are a couple of important theorems, saying that such functions always assumes maximum and minimum values and also that they assume all intermediate values.

This is how you work. Prepare for **lectures** by reading and watching videos.After each lecture you read in the book and solve exercises. At **tutorials** you solve exercises from the book. Before the **seminar** you solve the problems in the problem set.

Observe that for each hour in class you should spend one or two hours of work at home, reading and solving exercises, continuously through the course. A minimum of two hours each day is probably required. Those who put in the hours usually pass the course, while most of those who don't, fail.

You have to master **pre-calculus** before you take this course. Exponents, logarithms, roots, trigonometry polynomials, straight lines, etcetera.

Do not underestimate the difficulties. Even though you recognize some of the material you might find that the demands are much higher now than in high school.

Recommended exercises from Calculus are Ch P1: 7, 11, 19, 29, 39. Ch P2: 13, 15, 17, 23. Ch P3: 3, 7, 43, 49. Ch P4: 1, 3, 7, 11, 31, 33, 53. Ch P5: 9, 25. Ch P6: 1, 7, 17. Ch P7: 1, 3, 7, 19, 25, 26, 51. Ch 1.2: 9, 13, 21, 25, 30, 49, 50, 78, 79. Ch 1.3: 3, 6, 11, 13, 53. Ch 1.4: 7, 8, 12, 15, 17, 20, 21, 29. Ch 1.5: 13, 29.

CAN YOU SOLVE THESE EXERCISES?

Exercise 1. Equations for straight lines.

A. Find an equation for the straight line through $(5, -1)$ with slope -2.
B. Find an equation for the straight line through $(1, -3)$ and $(-2, 5)$.
C. Are the lines $8x + 16y + 5 = 0$ and $x = -2y + 33$ parallell?
D. Are the lines $8x + 9y + 5 = 0$ and $9x - 8y + 15 = 0$ orthogonal?
E. What is the point-slope equation?

Exercise 2. Solve the equations:

A. $\sin 2x = -\dfrac{1}{\sqrt{2}}$
B. $|2x + 1| = 2$

Exercise 3. Compute the limits:

A. $\lim_{x\to 1} \dfrac{x-2}{x^2-4}$ B. $\lim_{x\to 2} \dfrac{x-2}{x^2-4}$

C. $\lim_{x\to -2} \dfrac{x-2}{x^2-4}$ D. $\lim_{x\to \infty} \dfrac{x-2}{x^2-4}$

Exercise 4. Compute the limits:

A. $\lim_{x\to 0} \frac{x - \sin x}{x}$

B. $\lim_{x\to \infty} \frac{x - \sin x}{x}$

Exercise 5. Let $f(x) = \frac{5x-1}{\cos 2x}$.

A. Find the domain of definition of f.
B. At what points is f continuous?
C. Is f odd or even?
D. Is f bounded?

Exercise 6. Let $g(t) = \sqrt{1 - \frac{1}{t+1}}$.

A. Find the domain of definition of g.
B. At what points is g continuous?
C. Is g odd or even?
D. Is g bounded?

Exercise 7. Låt $h(x) = |x| - |x+1|$.

A. Find the domain of definition of h. At what points is h continuous?
B. Write h as a piecewise defined function without the use of the absolute value sign.
C. Sketch the graph $y = h(x)$. What is the range of h.
D. Is h bounded?

Exercise 8. We study the function

$$s(x) = \begin{cases} x^2+1, & x < 0 \\ x+2, & 0 \le x < 2 \\ x^2, & x \ge 2 \end{cases}$$

A. Find the domain of definition of s.
B. At what points is s continuous?
C. Make a simple sketch of $y = s(x)$.

Exercise 9. The unit circle is given by $x^2 + y^2 = 1$.

A. Is it the graph $y = f(x)$of some function f ? If yes, which one? If no, why not?
B. Is the upper half (where $y \ge 0$) the graph $y = f(x)$
of some function f ? If yes, which one? If no, why not?

Exercise 10. Show that

$$x^4 - x^2 - 2x - 1 = 0$$

has at least two solutions in $-1 < x < 2$.

Exercise 11. At what points is

$$f(t) = \begin{cases} \dfrac{\sin 2t}{t}, & t \neq 0 \\ 2, & t = 0 \end{cases}$$

continuous?

Exercise 12. Explain how you can be sure that

$$f(x) = \frac{\sin 47x - \cos^3 x}{x^{23} + 2x + 1}$$

attains a maximum and a minimum value for $0 \leq x \leq 3$.

FACIT OCH LÖSNINGSTIPS

1. A. $y + 1 = -2(x - 5)$. Kan också skrivas $y = -2x + 9$
1. B. $y = \frac{8}{3}x - \frac{1}{3}$
1. C. Ja (ty de har samma riktningskoefficient)
1. D. Ja (ty $k_2 = -1/k_1$, där k_1 och k_2 är riktningskoefficienterna)
1. E. Läs i boken eller i er gymnasiebok (och kika gärna också på svaret till 4A)

2. A. $x = -\pi/8 + n\pi$, där n är ett godtyckligt heltal, eller $x = 5\pi/8 + n\pi$, där n är ett godtyckligt heltal.
2. B. Lösningarna är $x = 1/2$ och $x = -3/2$

3. A. 1
3. B. $1/4$
3. C. Gränsvärde saknas (det är INTE heller ∞)
3. D. 0

4. A. 0
4. B. 1

5. A. Alla $x \neq \frac{\pi}{4} + n\frac{\pi}{2}$, n godtyckligt heltal. Tips: problemet är när nämnaren är noll.
5. B. Alla $x \neq \frac{\pi}{4} + n\frac{\pi}{2}$, n godtyckligt heltal.
5. C. f är varken udda eller jämn.
5. D. Nej

6. A. Alla tal som är större än eller lika med 0 och alla tal mindre än -1. Tips: undvik negativt under rottecknet och undvik division med noll.
6. B. Samma svar som i 2A.
6. C. Varken udda eller jämn.

6. D. Nej

7. A. Definitionsmängden är alla reella tal x. Funktionen är kontinuerlig överallt.
7. B. För $x \geq 0$ är $h(x) = -1$.
För x mellan -1 och 0 är $h(x) = -2x - 1$.
För $x \leq -1$ är $h(x) = 1$.
7. C. Det är lätt att rita grafen med hjälp av informationen i B. Värdemängden består av alla tal y sådana att $-1 \leq y \leq 1$
7. D. Ja, $|h(x)| \leq 1$ för alla x

8. A. Alla x
8. B. $x \neq 0$
8. C. Se sidorna 36 och 37 i boken för exempel på denna typ av funktion.

9. A. Nej. För varje x mellan -1 och 1 finns två olika möjliga värden på y.
9. B. Ja. $f(x) = \sqrt{1 - x^2}$

10. Funktionen $f(x) = x^4 - x^2 - 2x - 1$ är kontinuerlig överallt och $f(-1) = 1,\ f(0) = -1,\ f(2) = 7$, så det följer av satsen om mellanliggande värden att funktionen har ett nollställe mellan -1 och 0 och ytterligare ett mellan 0 och 2.

11. Funktionen är kontinuerlig i alla punkter på reella axeln.

12. Funktionen är kontinuerlig i varje punkt av det slutna och begränsade intervallet $[0, 3]$. Det följer att största och minsta värde finns, enligt the max-min theorem (sid 83).