Department of Mathematics

SF1625
Calculus 1
Year 2015/2016

Module 7: Plane curves, sequences, series

Sections l 8.1, 8.2, 8.5 and 9.1-9.3 in Calculus by Adams och Essex. Three lectures and two tutorials. ’

Important concepts. This module concerns **plane curves** and **sequences and series**. Plane curves will be studied only briefly. That topic will be important in later courses. More important now are sequences and series. The concepts of **convergence** and **divergence** are important. To determine whether a series is convergent or divergent we will often use comparison criteria. An important theorem is Cauchy’s integral criterion.

Recommended exercises from Calculus. Ch 8.1: 1, 3, 5. Ch 8.2: 1, 7. Ch 8.5: 9, 13. Ch 9.1: 1, 3, 17. Ch 9.2: 1, 5. Ch 9.3: 1, 3, 27, 29, 35.

CAN YOU SOLVE THESE EXERCISES?

Exercise 1. Find an equation for the ellips with semi-axes 3 och 4. Is there more than one possibility?

Exercise 2. Draw these curves:

A. $x^2 + 2y^2 = 4$
B. $x + y^2 = 1$
C. $x^2 = 1 + y^2$

Exercise 3. Parametrize these curves:

A. $x^2 + y^2 = 2$
B. $x^2 + y^2 - 2x - 3 = 0$
C. $y^2 = x + 1$

Exercise 4. Are these series convergent or divergent? Hint: do the terms approach 0?

A. $\displaystyle\sum_{k=1}^{\infty} \cos\frac{\pi}{k}$

B. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\arctan k}$

Exercise 5. Convergent or divergent? Can you compute these series?

A. $\displaystyle\sum_{k=0}^{\infty} \frac{1}{2^k}$

B. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{2^k}$

C. $\displaystyle\sum_{j=1}^{\infty} e^{-j}$

Exercise 6. Are these series convergent or divergent? You dont have to compute them.

A. $\displaystyle\sum_{k=2}^{\infty} \frac{10}{k\sqrt{k}}$

B. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{1+e^k}$

C. $\displaystyle\sum_{j=4}^{\infty} \frac{1+j+\ln j}{j^2-1}$

Exercise 7. Use the integral criterion to settle if this series is convergent or divergent.
$\displaystyle\sum_{k=2}^{\infty} \frac{1}{k \ln k}$

Exercise 8. Show that

$$\frac{1}{2} < \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(n+1)} < \frac{\pi+1}{2}.$$

FACIT OCH LÖSNINGSTIPS

1. $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$ (Det finns fler alternativ)

2. Rita på. Det är en ellips, en parabel och en hyperbel

3. A. TEx: $x = \sqrt{2}\cos t,\ y = \sqrt{2}\sin t,\ t \in \mathbb{R}$
3. B. $x = 1 + 2\cos t,\ y = 2\sin t,\ 0 \le t < 2\pi$
3. C. $x = t^2 - 1,\ y = t,\ t \in \mathbb{R}$

4. Båda serierna är divergenta

5. Alla tre serierna är konvergenta
5. A. 2
5. B. $1/2$. Tips: bryt ut $1/2^2$ och använd formeln för geom summa på det som är kvar.
5. C. $\frac{1}{e-1}$. Tips: $e^{-j} = (1/e)^j$

6. A. Konvergent. 6B. Konvergent. 6.C. Divergent

7. Divergent

8. Använd integraluppskattning. De båda bilderna i beviset av sats 8 i Ch 9.3 på sidan 511 säger precis vad man ska göra här!