Applied exercises from old exams

Exercise 1. From three identical boards of wood, 1 meter long and 10 cm broad, you want to construct a trough. At what angle should you place the planks in order for the trough to hold as much as possible?

Exercise 2. A carton of milk at $4°$ C is taken out of the fridge and placed in a room of $20°$ C. After 12 minutes the temperature of the milk is $12°$ C. After how much more time is the temperature of the milk $18°$ C ?
Use Newton's law of cooling

Exercise 3. A sperical container with radius R m is filled with water at v m^3 per minute. How fast, i.e. with how many meters per minute, is the surface rising at the time when it is $R/4$ m above the lowest point of the container?

Exercise 4. A particle starts from rest and then moves with the acceleration $100 \cos t$ m/s^2 at time t s. What is its velocity and position after $t = 3$ s?

Exercise 5. A reservoire has been contaminated. A natural cleaning process occurs as clean water enters the reservoire while the contaminated water exits it. According to a mathematical model of this phenomenon the concentration $K(t)$ of the poisonous substance at time t satisfies the differential equation

$$\frac{dK(t)}{dt} = -\frac{K(t)}{1500}.$$

How long does it take, according to the model, for the concentration of the substance to reach half of its original concentration?

Exercise 6. A vehicle starts from rest and drives for 30 minutes straight ahead with the acceleration $2 + 60t$ km/h^2. What is its speed after 30 minutes? How far has it driven?

Exercise 7. A vehicle is to be driven 10 km. Suppose the speed after x kilometers is $v(x)$ km/h. How long does it take for the vehicle to travel the 10 km?

Exercise 8. A 2 m long cylindrical rod with radius 0.1 m is made out of a material with variable density. If we place the rod along an x-axis between the points 0 and 2 the density ρ at the point x is

$$\rho(x) = 1 - \frac{(x-1)^2}{4} \text{ kg/m}^3,.$$

Compute the mass of the rod.

Exercise 9. A car starts driving at a traffic light and increases it speed with constant acceleration until the speed is 25 m/s. Then the car continues going at constant speed 25 m/s. After 23 s the car has travelled 500 m. At what time did the car reach the speed 25 m/s?

Exercise 10. An object with mass m is moving straight ahead with speed $v(t)$. There is force of friction $-kv(t)$, where k is a positive constant. If no other forces affects the body we must have

$$m\frac{dv}{dt} = -kv(t).$$

Determine the speed as a function of time if $k = 1$, $m = 2$ kg and $v(0) = 5$ m/s. How far does the object travel after the time $t = 0$?

Exercise 11. Tjhe differential equation $\dfrac{du}{dt} = -\dfrac{u}{RC}$ describes the voltage u at time t when a capacitor with capacitance C is hooked up to a resistor R. Solve the differential equation and determine how long it takes for u reach half its original value.

Exercise 12. The populations of the countries A and B are growing exponentially. In A the population doubles every 50 years and in B every 150 years. Today twice as many live in B than in A. How long does take before the population in A is greater than that of B ?

FACIT OCH LÖSNINGSTIPS

1. $4\pi/3$ (Lösningsförslag finns, tentamen 2013-10-26)

2. Efter ytterligare 24 minuter (Lösningsförslag finns, tentamen 2014-01-11)

3. Vattenytan stiger med hastigheten $16v/(7\pi R^2)$ meter per minut (Lösningsförslag finns, tentamen 2012-12-10)

4. Hastigheten är $100 \sin 3$ m/s och positionen är $100(1 - \cos 3)$ m.

5. $1500 \ln 2$ s. (Lösningsförslag finns, tentamen 2012-02-11)

6. Efter en halvtimme är hastigheten 8.5 km/h och fordonet har kört 1.5 km.

7. Tiden det tar är $\int_0^{10} \dfrac{1}{v(x)}\, dx$ h.

8. Stången massa är $11\pi/600$ kg. (Lösningsförslag finns, tentamen 2011-12-15)

9. 6 sekunder. (Lösningsförslag finns, tentamen 2011-10-18)

10. $v(t) = 5e-t/2$. Sträckan är 10 m. (Lösningsförslag finns, tentamen 2011-08-25)

11. $u(t) = ke^{-t/RC}$ där k är en godtycklig konstant. Spänningen har halverats efter tiden $RC \ln 2$. (Lösningsförslag finns, tentamen 2014-10-24)

12. 75 år. (Lösningsförslag finns, tentamen 2009-12-19)