Department of Mathematics

SF1625
Calculus 1
Year 2015/2016

Problem set for seminar 1

See www.kth.se/social/course/SF1625 for information about how the seminars work and what you are expected to do. **At this seminar there will be a hand-in.** Solve the problems 1-4 below and write down the solution with one problem per sheet of paper. Write name and birthdate on each sheet. When the serminar begins you will be told what problem to hand in. This will take place at the beginning of the seminar, so **don't be late!** Before you begin working on the problems below you should work on the recommended exercises in the text book Calculus by Adams and Essex (8:th edition):

Ch P1: uppg 7, 11, 19, 29, 39. Ch P2: uppg 13, 15, 17, 23. Ch P3: uppg 3, 7, 43, 49. Ch P4: uppg 1, 3, 7, 11, 31, 33, 53. Ch P5: uppg 9, 25. Ch P6: uppg 1, 7, 17. Ch P7: uppg 1, 3, 7, 19, 25, 26, 51. Ch 1.2: uppg 9, 13, 21, 25, 30, 49, 50, 78, 79. Ch 1.3: uppg 3, 6, 11, 13, 53. Ch 1.4: uppg 7, 8, 12, 15, 17, 20, 21, 29. Ch 1.5: uppg 13, 29.

PROBLEMS TO SOLVE BEFORE THE SEMINAR

Uppgift 1. Solve these equations. Be careful to find *all* solutions.

A. $\sin 2x = -1/\sqrt{2}$
B. $\tan 3x = \sqrt{3}$
C. $|2x + 1| = |x|$

Uppgift 2. Determine the domain of definition for each of these functions. Also, decide if these functions are bounded and if they are odd or even.

A. $g(t) = 1/\sqrt{1 - 2t}$
B. $h(t) = 1/(6x^2 + 12x - 48)$

Uppgift 3. Compute the limits:

A. $\lim_{x\to\infty} \dfrac{x^2 + 3x + 5}{5x^2 + 2x + 3}$ B. $\lim_{x\to-3} \dfrac{x + 3}{x^2 - 9}$ C. $\lim_{x\to3} \dfrac{x + 3}{x^2 - 9}$

Uppgift 4. Find k so that the function

$$g(x) = \begin{cases} \dfrac{\sin kx}{x}, & x \neq 0 \\ 4, & x = 0 \end{cases}$$

is continuous at the origin. Is the function then continuous for all x?

Extra problems to discuss at the seminar

Here are some extra problems. You don't have to write down solutions in advance.

- Show that $p(x) = x^5 + x^3 + 1$ has a zero between -1 och 0. Is the zero closer to -1 or to 0? How can you ge sure that there are no more zeros?
- Explain how you can be sure that the function $f(x) = (x^3 + x\tan^9 x)^{47}$ attains a maximum and a minimum value when x varies in the interval $[0, 1]$. What can you say about the same function on the interval $[0, 2]$?
- A parking meter charges you according to the following: the first hour costs 4 kronor and then each started hour costs 2 kronor up to the maximum amount of 10 kronor. Let $h(t)$ be the cost as a function of the time t in hours . Sketch the graph $y = h(t)$ for $0 \leq t \leq 4$. At what points is h continuous?
- Is there a function that is both odd and even?
- Is there a function that is neither odd nor even?
- If f is odd, what is $f(0)$?
- A curve in the i xy-plane is given by $x^2 + 2x + y^2 - 4y = 4$. Sketch the curve! Is the curve the graph of a function, $y = f(x)$, for some function f?
- True or false?

  P1. $x = 2 \implies x^2 = 4.$
  P2. $x^2 = 4 \implies x = 2.$
  P3. $x \sin x = x \implies \sin x = 1.$