Department of Mathematics

SF1625
Calculus 1
Year 2015/2016

Problem set for seminar 2

See www.kth.se/social/course/SF1625 for information about how the seminars work and what you are expected to do. **At this seminar there will be a written test.** In the test you will be asked to solve a problem like one of the problems below or like one of the recommended exercises from the text book Calculus by Adams och Essex (8:th edition):

Ch 2.1: uppg 5, 7. Ch 2.2: uppg 1, 3, 11, 26, 27, 40, 41, 42, 43, 44, 45, 47. Ch 2.3: uppg 1, 7, 11, 17, 25, 33, 35, 47. Ch 2.4: uppg 3, 5, 11, 18, 23, 30, 31, 37. Ch 2.5: uppg 13, 15, 23, 29, 31, 35, 45, 62. Ch 2.6: uppg 3, 9. Ch 2.7: uppg 1, 3, 11, 13, 23, 29. Ch 2.8: uppg 5, 13, 21, 27. Ch 2.9: uppg 3, 9, 13. Ch 2.9: uppg 3, 9, 13. Ch 2.11: uppg 5, 7, 13, 16, 17, 18, 19.

Problems to solve before the seminar

Uppgift 1. Differentiate these functions with respect to x and state for what x they are differentiable. Do some of these functions fail to be differentiable at some points of their domains of definition?

A. $f(x) = \tan^2 x$

B. $g(x) = \dfrac{ax+b}{cx+d}$

C. $h(x) = 2\sin\sqrt{x}$

D. $k(x) = |x|\cos x$

E. $r(x) = \sqrt{1+x}$

F. $s(x) = \left(\dfrac{1}{(1-x)^2}\right)^{1/3}$

Uppgift 2. Let $g(x) = x^2 \sin x$.

A. Determine the domain of definition of g.
B. At what points is g continuous?
C. Compute $g'(x)$.
D. At what points is g differentiable?

Uppgift 3. Let $h(t) = |1 + t|(1 + 2\sin t)^5$.

A. Determine the domain of definition of h.
B. At what points is h continuous?
C. Compute $h'(t)$.
D. At what points is h differentiable?

Uppgift 4. Find equations for the tangent and the normal at the point $(2, 16)$ to the curve $y = x^4$.

Uppgift 5. On what intervals is $f(x) = x^4 - 4x^3 + 4x^2$ strictly increasing? Strictly decreasing?

Uppgift 6. Show using the derivative that $f(x) = 2\sin^2 x + \cos 2x$ is constant.

Uppgift 7. Let $f(x) = \sin x^2$. Compute $f^{(n)}(0)$ for $n = 1, 2, 3$.

Uppgift 8. Find an equation for the tangent to $y = \tan x$ at the point on the curve where $x = \pi/4$. Can you use this to find an approximate value of $\tan(\pi/5)$?

Uppgift 9. Find an equation for the tangent to $x^3 + y^3 + y + x = 0$ at the point $(-1, 1)$. Hint: implicit differentiation.

Uppgift 10. Theory: Show using the definition that $\dfrac{d}{dx}\left(\dfrac{1}{x}\right) = -\dfrac{1}{x^2}$

Uppgift 11. Theory: Prove the product rule!

Uppgift 12. Theory: Prove that a function differentiable at a must be continuous at a. Give an example showing that the converse is not true.

Extra problems to discuss at the seminar

Here are some extra problems. You don't have to write down solutions in advance.

- Give an example of a function with domain of definition $\mathbb{R}$, that is neither continuous nor differentiable at $x = 1$.
- Give an example of a function with domain of definition $\mathbb{R}$, that is continuous but not differentiable at $x = 1$.
- Is there a function that is differentiable but not continuous at $x = 1$?
- Let $h(x) = |x| - |x + 1|$. Compute $h'(x)$ and state at what points h is differentiable. What does the graph look like at points where h is not differentiable?
- Let $U(t)$ be the Heaviside function given by

  $$U(t) = \begin{cases} 1 & \text{om } t \geq 0 \\ 0 & \text{om } t < 0 \end{cases}$$

  Compute $U'(t)$. At what points is U differentiable?
- Let U be as in the previous exercise and put $f(t) = (U(t-\pi) - U(t-3\pi)) \sin t$. Make a simple sketch of $y = f(x)$. At what points is f continuous? Compute $f'(t)$. At what points is f differentiable? What does the graph look like at points where h is not differentiable?