Department of Mathematics

SF1625
Calculus 1
Year 2015/2016

Problem set for seminar 4

See www.kth.se/social/course/SF1625 for information about how the seminars work and what you are expected to do. **At this seminar there will be a written test.** In the test you will be asked to solve a problem like one of the problems below or like one of the recommended exercises from the text book Calculus by Adams och Essex (8:th edition):

Ch 4.1: 5, 7, 9, 16, 17. Ch 4.2: 7, 9. Ch 4.3: 1, 5, 17. Ch 4.4: 3, 14, 29, 35. Ch 4.5: 5, 11, 27, 31. Ch 4.6: 3, 5, 9, 17, 31. Ch 4.8: 1, 7, 13, 21. Ch 4.9: 1, 3, 13, 30. Ch 4.10: 1, 5, 9

PROBLEMS TO SOLVE BEFORE THE SEMINAR

Uppgift 1. Let $f(x) = (2x+1)e^{-x}$.

A. Determine the domain of definition of f.
B. Where is f continuous?
C. On what intervals is f increasing and decreasing?
D. Find all local extreme values of f.
E. Compute the limits $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
F. Sketch the graph $y = f(x)$.

Uppgift 2. Let $h(t) = t + \cos t$.

A. Find all critical points of h.
B. Find all local extreme values of h.
C. Determine whether h assumes a maximum and a minimum value and determine on what intervals h is increasing and decreasing.

Uppgift 3. Find all asymptotes to the curve $y = \dfrac{2+x^2}{x}$.

Uppgift 4. You want to construct a cylindrical can with top and bottom. The total surface area of the can is to be A. How should you choose the radius and the height of the can in order to maximize its volume?

Uppgift 5. Let $f(x) = xe^{-x^2/2}$. On what intervals is f är convex (concave up) and conkave (concave down)?

Uppgift 6. Let $f(x) = e^x$.

A. Find the Taylor polynomial of degree 2 to f around the point $x = 0$.
B. Use the Taylor polynomial to approximate $1/\sqrt{e}$, i.e. $f(-1/2)$.
C. Determine whether the error is less than $1/25$.

Uppgift 7. Låt $g(t) = \sqrt{t}$

A. Find the Taylor polynomial of degree 2 to g around the point $t = 25$.
B. Use the Taylor polynomial to approximate $\sqrt{26}$.
C. Determine whether the error is less than $1/100$.

Uppgift 8. We study the equation $x^5 + x - 1 = 0$

A. Show, using the derivative, that the equation has at most one solution.
B. Show, using intermediate values, that the equation has at least one solution. lbetween 0 och 1.
C. Find an approximation of the solution by chosing a starting point and doing two iterations of Newton-Raphson's method.

Uppgift 9. Compute the limits:

A. $\lim\limits_{h\to 0} \dfrac{1-\sqrt{1+h}}{h}$

B. $\lim\limits_{x\to 0} \dfrac{\cos x - e^{x^2}}{x^2}$

Uppgift 10. A spherical tank with radius 5 dm is filled by water at 3 liters per minute. How fast is the surface rising at the time when the depth of the tank is 2 dm? Hint: The volume V is related to the depth d by

$$V = \pi \frac{15d^2 - d^3}{3}.$$

Uppgift 11. Determine whether $f(x) = x \ln x$ assumes a maximum and a minimum value when x varies in the interval $1 \leq x \leq 5$ and if so, find these maximum and minimum values.

Uppgift 12. Determine whether $f(x) = x - \arcsin x$ assumes a maximum and a minimum value and if so, find these maximum and minimum values.

Uppgift 13. Sketch, using the derivative, the curve $y = \ln\sqrt{1+x^2} - \arctan x$.

EXTRA PROBLEMS TO DISCUSS AT THE SEMINAR

Here are some extra problems. You don't have to write down solutions in advance.

- Decide whether $f(x) = |x-1| + \sqrt{x+1}$ assumes a maximum and a minimum value for x in the interval $[-1, 2]$ and if so, find those.
- An aircraft travels at constant speed 600 km/h och constant height 5 km. At a certain moment it passes right above a house. How fast is the distance between the house and the aircraft changing one minute later?
- Show that the equation $2\arctan x = 6 - 3x$ has exactly one solution and that the solution lies in the interval $[1, 2]$. Use Newton-Raphson's method to approximate the solution.
- Is there a function with domain of definition $\mathbf{R}$ that has a global extreme value at the origin without the derivative at the origin being zero?
- Is there a function with domain of definition $\mathbf{R}$ that has derivative zero at the origin and still does not have a global extreme value at the origin ?
- Is there a function with domain of definition $\mathbf{R}$ that is strictly increasing even though the derivative is zero at some point?
- Is there a function with domain of definition $\mathbf{R}$ that is not strictly increasing even though the derivative is positive everywhere?
- Determine the constants a, b och c in order that
$$|ae^{bx+cx^2} - 2x^2 - 4| \leq 10^{-4} \quad \text{when } |x| \leq 0.1.$$
- Show that $x((\ln x)^3 - 3(\ln x)^2 + 6\ln x) \geq 6(x-1)$ for all $x > 0$.