Institutionen för Matematik

SF1625
Calculus 1
Year 2015/2016

Problem set for Seminar 6

See www.kth.se/social/course/SF1625 for information about how the seminars work and what you are expected to do. **At this seminar there will be a written test.** In the test you will be asked to solve a problem like one of the problems below or like one of the recommended exercises from the text book Calculus by Adams och Essex (8:th edition):

Ch 6.3: uppg 1, 3, 9. Ch 6.5: uppg 1, 3, 5, 15, 23, 33, 34, 35. Ch 7.1: uppg 1, 3, 5, 13, 19, 21. Ch 7.2: uppg 1, 3. Ch 7.3: uppg 3, 11, 21. Ch 7.4: uppg 1, 3, 5. Ch 7.6: uppg 1, 7. Ch 7.7: uppg 1, 5.

PROBLEMS TO SOLVE BEFORE THE SEMINAR

Uppgift 1. State why these integrals are improper and compute them.

A. $\displaystyle\int_0^1 \frac{1}{\sqrt{1-x}}\,dx$

B. $\displaystyle\int_0^\infty \frac{1}{1+x^2}\,dx$

C. $\displaystyle\int_0^\infty \frac{x}{(1+x^2)^{5/2}}\,dx$

D. $\displaystyle\int_0^\infty xe^{-x}\,dx$

Uppgift 2. Determine whether these integrals are convergent or divergent. You do not have to compute them.

A. $\displaystyle\int_0^1 \frac{1}{x\sqrt{2-x}}\,dx,$

B. $\displaystyle\int_1^\infty \frac{1+x+\ln x}{1+e^x}\,dx$

Uppgift 3. Compute the integral $\int_0^1 \sqrt{1-x^2}\,dx$ (use $x = \sin t$)

Uppgift 4. Compute the integral $\int_e^{e^2} \frac{\ln(\ln x)}{x}\,dx$ (start by putting $\ln x = t$)

Uppgift 5. Compute the integral $\int_0^{1/2} \arcsin x\,dx$ (to begin with, integrate by parts)

Uppgift 6. Compute the integral $\int_0^1 \arctan x\,dx$ (to begin with, integrate by parts)

Uppgift 7. Compute the integral $\int_0^1 \frac{x-1}{x^2-5x+6}\,dx$ (use partial fractions)

Uppgift 8. Compute the volume that is generated when the area between the x-axis and the curve $y = e^{-x}$ for $0 \le x \le 1$ is rotated around

A. x-axis
B. y-axis

Uppgift 9. Compute the volume that is generated when the area between the x-axis and the curve $y = e^{-x^2}$ for $0 \le x \le 1$ is rotated around the y-axis.

Uppgift 10. Compute the length of the curve $y = \ln(1-x^2), \quad 0 \le x \le 0.5$.

Uppgift 11. For a certain spring, the force it takes to change the length of the spring x meter is $x/4$ N. How much work does it take to change the length of the spring $1/5$ meter?

EXTRA PROBLEMS TO DISCUSS AT THE SEMINAR

Here are some extra problems. You don't have to write down solutions in advance.

- Give an example of a function that is not integrable and explain why.
- The function $f(x) = e^{x^2}$ is continuous on the compact interval $[0, 1]$ and is therefore integrable on that interval (ses Th 2 in Ch 5.3). Still you can not write down an anti-derivative in the usual fashion. Is there a contradiction?

- A ball with radius r has its center on the surface of another ball with radius R, where $R > r$. Compute the volume of that part of the smaller ball which is inside the bigger ball.
- Is the integral $\displaystyle\int_{-\infty}^{\infty} \frac{x}{x^2+1}\,dx$ convergent or divergent?
- Beräkna $\displaystyle\lim_{n\to\infty} \frac{1}{\sqrt{n}e^n} \int_n^{n+1} \sqrt{x}e^x\,dx$